Academic Arithmetic

Wells

With Answers

AN
ACADEMIC ARITHMETIC

FOR

ACADEMIES, HIGH AND COMMERCIAL SCHOOLS.

BY

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PREFACE.

THE present work is intended to furnish a thorough course in all those portions of Arithmetic which are required for admission to any college or scientific school.

The pupil is assumed to have already studied the more elementary parts of the subject in a text-book of lower grade; and only enough examples are given in the earlier chapters to afford material for a review.

Great pains have been taken, in the selection of examples and problems, to illustrate every important arithmetical process; and in Chapter XXV. there will be found a set of miscellaneous problems of somewhat greater difficulty than those in the preceding chapters, furnishing a complete review of the entire subject.

The chapters on the Metric System have been arranged in such a way that they may be taken after the other portions of the work have been studied, or omitted altogether, at the option of the teacher. No examples involving a knowledge of the Metric System are given, except in Chapter XIII., and Arts. 256, 257, and 378.

The Appendix contains topics of minor importance to the majority of pupils, but still liable to be called for in college entrance examinations.

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CONTENTS.

ARITHMETIC.

I. NOTATION AND NUMERATION.

1. Let us consider a collection of things of the same kind; for example, a collection of books.

In order to find out *how many* books there are in the collection, we proceed to *count* them, as follows:

We take any book, and call it *one;* we then take another and call it *two;* the next we call *three;* the next *four;* then *five, six, seven, eight, nine, ten, eleven, twelve,* and so on until all have been taken.

The expressions *one, two, three,* etc., used in the above process are called **Whole Numbers, or Integers.**

2. Notation signifies the representation of numbers by means of symbols.

Numeration signifies the reading of numbers when expressed in symbols.

THE ARABIC SYSTEM OF NOTATION.

3. In the **Arabic System,** the numbers *one, two, three, four, five, six, seven, eight,* and *nine,* are represented by the symbols 1, 2, 3, 4, 5, 6, 7, 8, and 9, respectively.

The symbol 0, read *zero, cipher,* or *naught,* when standing by itself, signifies *nothing.*

The above *symbols* are called **Figures.**

Zero, and the *numbers* one, two, three, four, five, six, seven, eight, and nine, aro called **Digits.**

4. Numbers greater than *nine* are represented by writing side by side two or more of tho above figures.

The last figure at the right is said to be in the *first place*, and denotes *ones* or *units*.

Tho figure just beforo the last is said to be in the *second place*, and denotes *tens*.

Thus, 10 signifies *one ten* and *no ones;* that is, *ten*.

11 signifies *one ten* and *one one;* that is, *eleven*.

12 signifies *one ten* and *two ones;* that is, *twelve*.

In like manner, 13, 14, 15, 16, 17, 18, and 19 represent the next seven numbers in order; that is, *thirteen*, *fourteen*, *fifteen*, *sixteen*, *seventeen*, *eighteen*, and *nineteen*.

Two tens are called *twenty*, and represented by 20.

The next nine numbers in order are represented by 21, 22, and so on to 29; and read *twenty-one*, *twenty-two*, and so on to *twenty-nine*, respectively.

Three tens are called *thirty;* four tens, *forty;* five tens, *fifty;* six tens, *sixty;* seven tens, *seventy;* eight tens, *eighty;* nine tens, *ninety;* and represented by 30, 40, 50, 60, 70, 80, and 90, respectively; and in each case the next nine numbers in order are named and represented in a manner similar to that employed for numbers between 20 and 30.

5. *Ten tens* are called *one hundred*.

A figure in the *third place* denotes *hundreds*.

Thus, 100 signifies *one hundred, no tens*, and *no ones;* that is, *one hundred*.

101 signifies *one hundred, no tens,* and *one one;* and is read *one hundred and one*.

783 signifies *seven hundreds, eight tens,* and *three ones;* and is read *seven hundred and eighty-three*.

In like manner, any number from *one hundred* to *nine hundred and ninety-nine*, may be represented by three figures.

6. The following table gives the signification of each of the first seven places:

1st; *ones.*
2d; *tens.*
3d; *hundreds.*
4th; tens of hundreds, or *thousands.*
5th; *tens of thousands.*
6th; tens of tens of thousands, or *hundreds of thousands.*
7th; tens of hundreds of thousands, or *millions.*

Thus, 7306592 signifies 7 millions, 3 hundreds of thousands, no tens of thousands, 6 thousands, 5 hundreds, 9 tens, and 2 ones.

Note. It will be understood hereafter that, when the *digits* of a number are spoken of, we mean the *numbers represented by its figures, without regard to the places which they occupy.*

Thus, the digits of 352 are 3, 5, and 2, and not 300, 50, and 2.

7. The general law exemplified in Art. 6 may be stated as follows:

Any place signifies tens of the numbers signified by the next place to the right.

8. For convenience of reading, places are divided into *periods* of three places each.

The first, second, and third places form the *first* or *units'* period; the fourth, fifth, and sixth form the *second* or *thousands'* period; the third three, the *millions'* period; the fourth three, the *billions'* period.

The table gives the designation of each of the first fourteen periods:

Period.	Designation.	Period.	Designation.
First.	Units.	Eighth.	Sextillions.
Second.	Thousands.	Ninth.	Septillions.
Third.	Millions.	Tenth.	Octillions.
Fourth.	Billions.	Eleventh.	Nonillions.
Fifth.	Trillions.	Twelfth.	Decillions.
Sixth.	Quadrillions.	Thirteenth.	Undecillions.
Seventh.	Quintillions.	Fourteenth.	Duodecillions.

Thus, the number 23,016,797,681 is read twenty-three billion, sixteen million, seven hundred and ninety-seven thousand, six hundred and eighty-one.

Note. The above is the usual system of numeration. In the English system, the *second six* places form the millions' period, the third six the billions' period, etc. Thus, according to the English system, the number 57,608,351,000,000 would be read fifty-seven billion, six hundred and eight thousand three hundred and fifty-one million.

EXAMPLES.

9. Read the following numbers:

1.	2705618.	**6.**	144710325046728.
2.	6520741869.	**7.**	9080600713256.
3.	101294705.	**8.**	280115769001342.
4.	78220615437.	**9.**	2940073860045.
5.	35400986.	**10.**	48520010964700.

Write the following numbers in figures:

11. One million, three hundred and twenty-five thousand, seven hundred and twenty-six.

12. Five billion, seven hundred and eighty thousand, two hundred and five.

13. Seventy-nine million, one hundred and sixty thousand, and four.

14. Sixty-five billion, eight hundred and three million, one hundred and eighty-nine thousand, four hundred and fifty.

15. Three hundred and fifty-six million, eighty-one thousand, six hundred and twelve.

16. Two hundred and thirty-five billion, nine hundred and twenty-seven.

17. Eighty-five billion, two hundred and seventy-seven million, six thousand, one hundred.

18. Four trillion, one hundred and sixty billion, twenty-seven million, one hundred and sixteen thousand, and eighty-three.

19. Seven quadrillion, eight hundred and twenty-five trillion, four hundred and sixy-three million, four hundred and forty-five.

20. Nine hundred quintillion, five hundred and twenty billion, seventy thousand, three hundred and fourteen.

THE ROMAN SYSTEM OF NOTATION.

10. In the **Roman System**, the numbers *one, five, ten, fifty, one hundred, five hundred,* and *one thousand* are represented by the letters I, V, X, L, C, D, and M, respectively.

Numbers other than the above are represented by writing side by side two or more of the above letters.

When thus expressed, if a letter is written *after* another letter of the same or of greater value, the *sum* of their values is represented; if a letter is written *before* another letter of greater value, the *difference* of their values is represented.

The following table shows the methods usually employed for representing numbers up to five thousand:

ROMAN.	ARABIC.	ROMAN.	ARABIC.	ROMAN.	ARABIC.
I	1	XV	15	XC	90
II	2	XVI	16	C	100
III	3	XVII	17	CI	101
IV	4	XVIII	18	CC	200
V	5	XIX	19	CCC	300
VI	6	XX	20	CCCC	400
VII	7	XXI	21	D	500
VIII	8	XXII	22	DC	600
IX	9	XXX	30	DCC	700
X	10	XL	40	DCCC	800
XI	11	L	50	DCCCC	900
XII	12	LX	60	M	1000
XIII	13	LXX	70	MD	1500
XIV	14	LXXX	80	MM	2000

Note. The Roman Method is now rarely used except for numbering chapters of books, hours on clock-dials, etc.

II. ADDITION.

11. To **Add** two whole numbers is to *count upwards* from either of the numbers as many units as there are in the other.

Thus, to add 3 and 5, we count upwards from 3 *five* units as follows: 4, 5, 6, 7, 8; the result is 8.

In like manner, we may add three or more numbers.

Thus, to add 7, 4, and 8, we first count upwards from 7 four units, and then count upwards 8 units from the result. The answer is 19.

Note. The *order* in which the numbers are taken is immaterial.

12. The result of addition is called the **Sum.**

13. The symbol +, read *"plus"* or *"and,"* signifies addition.

14. The symbol = is read *"equals," "is equal to,"* or *"are."*

Thus, $3 + 5 = 8$ is read "three and five are eight."

15. 1. Find the sum of 396 and 842.

$$\begin{array}{r} 396 \\ 842 \\ \hline 1238 \end{array}, \textit{ Ans.}$$

396 is the same as 3 hundreds, 9 tens, and 6 units, and 842 the same as 8 hundreds, 4 tens, and 2 units.

We write the numbers so that units, tens, and hundreds shall be in the same vertical columns.

The sum of 2 units and 6 units is 8 units.

We then write 8 under the column of units.

The sum of 4 tens and 9 tens is 13 tens, or 1 hundred and 3 tens.

We then write 3 under the column of tens, and *carry* the 1 hundred mentally to the column of hundreds.

The sum of 1 hundred, 8 hundreds, and 3 hundreds is 12 hundreds, or 1 thousand and 2 hundreds.

We then write 2 under the column of hundreds, and 1 in the thousands' place of the answer.

Then the required result is 1 thousand, 2 hundreds, 3 tens, and 8 units, or 1238.

2. Add 546, 97, 384, and 780.

```
 546
  97
 384
 780
----
1807, Ans.
```

It is customary in practice to name *results* only when adding columns; thus, in Ex. 2, we say "4, 11, 17"; write the 7, and carry 1.

Then, "1, 9, 17, 26, 30"; write the 0 and carry 3.

Then, "3, 10, 13, 18."

From the above examples, we derive the following

RULE.

Write the numbers so that units, tens, hundreds, etc., shall be in the same vertical columns.

Add the digits in the units' column.

If the result is less than 10, *write it under the column of units; but if it is just* 10, *or more than* 10, *write the units of the sum under the column of units, and carry the tens mentally to the next column to the left.*

Proceed in a similar manner with each of the remaining columns, and write under the last column its entire sum.

Note. The work may be *proved* by performing the example a second time; adding the columns from top to bottom, instead of from bottom to top.

Another method of proof is to separate the numbers into two parts by a horizontal line. Adding the numbers above the line, then the numbers below, and then these two sums, the result should agree with that previously obtained.

16. In practice, computers frequently add two columns at once; thus, in the following example,

```
 3527
 8448
 1759
 2872
-----
16606
```

we should say 72, 131, 179, 206; write 06, and carry 2; then, 2, 30, 47, 131, 166.

17. *Only quantities of the same kind can be added.*

Thus, the sum of 7 *books* and 8 *books* is 15 *books;* but it is not possible to add 7 *books* and 8 *miles.*

EXAMPLES.

18. Add the following:

1.	2.	3.	4.	5.
1789	5403	4529	7854	1827
6543	786	7992	6215	4329
2177	9230	467	9448	25070
915	1157	8920	4007	6118
6783	898	3508	651	2522
325	7526	2463	5869	3909

6.	7.	8.	9.	10.
79856	45340	43765	59864	77167
35117	10087	89140	86723	63489
32949	76322	35174	48213	68791
18817	36450	64385	54876	74153
85622	78809	79160	83538	84375
16304	39713	23099	71766	97561

III. SUBTRACTION.

19. To **Subtract** one whole number from another is to *count downwards* from the second number as many units as there are in the first.

Thus, to subtract 5 from 8, we count downwards from 8 *five* units, as follows: 7, 6, 5, 4, 3; the result is 3.

20. The number to be subtracted is called the **Subtrahend.**

The number from which the subtrahend is to be subtracted is called the **Minuend.**

The result is called the **Remainder** or **Difference.**

21. The symbol $-$, read "*minus*" or "*less,*" signifies subtraction.

Thus, $8 - 5 = 3$ is read "eight less five are three."

22. It is evident from Art. 19 that, if we *count upwards* from the Remainder as many units as there are in the Subtrahend, we shall obtain the Minuend.

That is, the Minuend is the *sum* of the Subtrahend and Remainder.

23. 1. Subtract 483 from 758.

$$\begin{array}{r} 758 \\ 483 \\ \hline 275 \end{array}\text{, } \mathit{Ans.}$$

758 is the same as 7 hundreds, 5 tens, and 8 units, and 483 the same as 4 hundreds, 8 tens, and 3 units.

We write the subtrahend under the minuend so that units, tens, and hundreds shall be in the same vertical columns.

3 units from 8 units leave 5 units.

We cannot take 8 tens from 5 tens; but we can take 1 hundred, or 10 tens, from the 7 hundreds of the minuend, leaving 6 hundreds; and adding the 10 tens to the 5 tens, we have 15 tens; then 8 tens from 15 tens leave 7 tens.

Finally, 4 hundreds from 6 hundreds leave 2 hundreds.

Then the required result is 2 hundreds, 7 tens, and 5 units, or 275.

Now instead of *taking* 1 hundred from the 7 hundreds of the minuend, in the above example, we may get the same result as follows:

Adding 1 hundred to the 4 hundreds of the subtrahend, we have 5 hundreds.

Then, 5 hundreds from 7 hundreds leave 2 hundreds.

This second method is far preferable to the first.

From the second method of the above example, we derive the following

RULE.

Write the subtrahend under the minuend, so that units, tens, hundreds, etc., shall be in the same vertical columns.

Subtract the right-hand digit of the subtrahend from the digit above it, and write the result under the column of units.

If the right-hand digit of the subtrahend is greater than the digit above it, increase the latter by 10 *before subtracting; and add* 1 *mentally to the digit in the tens' place of the subtrahend.*

Proceed in a similar manner with each of the remaining digits of the subtrahend in order.

Note 1. If the minuend has more places than the subtrahend, we may make the number of places in the latter the same as in the former by mentally supplying ciphers in the missing places.

2\. Subtract 3728 from 571000.

$$\begin{array}{r} 571000 \\ \underline{3728} \\ 567272 \end{array}, \text{ Ans.}$$

In this case we say, "8 from 10 leaves 2; 3 from 10 leaves 7; 8 from 10 leaves 2; 4 from 11 leaves 7; 1 from 7 leaves 6."

Note 2. Since the minuend is the sum of the subtrahend and remainder (Art. 22), the work may be proved by adding the subtrahend to the remainder; the result should equal the minuend.

24. *Only quantities of the same kind can be subtracted.*

Thus, 16 *books* less 7 *books* are 9 *books;* but it is not possible to subtract 7 *books* from 16 *miles*.

EXAMPLES.

25. Subtract the following:

1.	2.	3.	4.	5.
6372	5034	8000	9037	48609
2177	786	1256	3409	9085

6.	7.	8.	9.	10.
75816	40709	58000	88713	64751
38912	36090	49374	73536	10968

PARENTHESES.

26. A **Parenthesis,** ( ), signifies that the numbers enclosed by it are to be taken *collectively.*

Thus, $17-(8+4)$ signifies that 8 and 4 are to be added together, and their sum subtracted from 17.

The **Vinculum,** ——, has the same force as a parenthesis.

Thus, $\overline{33-11}+\overline{19-5}$ signifies that 11 is to be subtracted from 33, then 5 from 19, and the second result added to the first.

EXAMPLES.

Find the values of the following:

1. $32-(13+7)$.
2. $\overline{50-17}+\overline{35-16}$.
3. $(45+18)-(22+9)$.
4. $122-(97-69)$.
5. $\overline{171-119}+\overline{137-88}$.
6. $(926-265)-(284+198)$.
7. $(823-486)-\overline{752-515}$.
8. $(132-74)+\overline{115+97}-(183-66)$.
9. $1088-\overline{905-323-479}-\overline{741-262}$.

Note. *Brackets,* [ ], and *Braces,* { }, have the same force as parentheses.

IV. MULTIPLICATION.

27. To **Multiply** one whole number by another is to take the first number as many times as there are units in the second.

Thus, to multiply 3 by 5, we take 3 five times, as follows: $3 + 3 + 3 + 3 + 3$; the result is 15.

28. The number taken is called the **Multiplicand.**

The number which shows how many times the multiplicand is taken, is called the **Multiplier.**

The result of multiplication is called the **Product.**

29. The symbol $\times$, read *"times,"* signifies multiplication.

Thus, $3 \times 5 = 15$ is read "three times five are fifteen."

30. To multiply 5 by 3, we take 5 three times, as follows: $5 + 5 + 5$; the result is 15.

That is, 3×5 is equal to 5×3.

It is evident from this that, in finding the product of two numbers, either may be regarded as the multiplicand, and the other as the multiplier.

31. To multiply together three or more numbers, we multiply the first number by the second, the product by the third number, and so on until all have been taken.

It is evident, as in Art. 30, that the *order* in which the numbers are multiplied is immaterial.

32. If any number of *things of the same kind* be multiplied by a whole number, the product will be things of the same kind as the multiplicand.

Thus, 6 times 7 *books* are 42 *books.*

But it is not possible to take 6 *books* times 7 books, nor to multiply 6 by 7 books.

33. The products of the numbers from 1 to 12 inclusive, taken two and two, are given in the following table.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

The arrangement of the table is as follows:

To find the product of 7 and 9, look in the left-hand vertical column for 7; then the product required will be found in the corresponding horizontal line in the column headed 9; the result found is 63.

34. **1.** Multiply 462 by 7.

Multiplicand, 462
Multiplier, 7
Product, 3234, *Ans.*

462 is the same as 4 hundreds, 6 tens, and 2 units.

We write the multiplier under the units' figure of the multiplicand.

7 times 2 units are 14 units, or 1 ten and 4 units.

We then write 4 under the column of units.

7 times 6 tens are 42 tens; adding to this the 1 ten reserved from the 14 units, we have 43 tens, or 4 hundreds and 3 tens.

We then write 3 under the column of tens.

7 times 4 hundreds are 28 hundreds; adding to this the 4 hundreds reserved from the 43 tens, we have 32 hundreds, or 3 thousands and 2 hundreds.

Writing 2 under the column of hundreds, and 3 in the thousands' place of the product, the required result is 3234.

It is customary to use the following words only in explaining the above process:

7 times 2 are 14; write the 4, and "carry" 1; 7 times 6 are 42, and 1 are 43; write the 3, and carry 4; 7 times 4 are 28, and 4 are 32.

2. Multiply 743 by 685.

Multiplicand,	743
Multiplier,	685
1st partial product,	3715
2d partial product,	59440
3d partial product,	445800
Product,	508955, *Ans.*

We multiply 743 first by 5 units, then by 8 tens, and finally by 6 hundreds, and add the partial products.

743 times 5 units are 3715 units.

743 times 8 tens are 5944 tens, or 59440.

743 times 6 hundreds are 4458 hundreds, or 445800.

Adding 3715, 59440, and 445800, the required result is 508955.

It is customary to arrange the written work as follows:

```
   743
   685
  ----
  3715
 5944
4458
------
508955, Ans.
```

From the above example, we derive the following

RULE.

Write the multiplier under the multiplicand, so that units, tens, hundreds, etc., shall be in the same vertical columns.

Multiply the multiplicand by the digit in the units' place of the multiplier, and write the result under the multiplier so that its right-hand figure shall be under the units' figure of the multiplier.

Multiply the multiplicand by the digit in the tens' place of the multiplier, and write the result under the first partial product so that its right-hand figure shall be under the tens' figure of the multiplier.

Proceed in a similar manner with each of the remaining digits of the multiplier, and add the partial products.

Note. The work may be *proved* by interchanging the multiplicand and multiplier.

If any digit of the multiplier is 0, the corresponding partial product is 0, and is not expressed in the work.

3. Multiply 1725 by 309.

```
  1725
   309
 15525
5175
533025, Ans.
```

We say in this case: 9 times 1725 are 15525, which we write so that its right-hand figure shall be under the 9 of the multiplier; then, 3 times 1725 are 5175, which we write so that its right-hand figure shall be under the 3 of the multiplier.

35. To Multiply by 10, 100, 1000, Etc.

To multiply a whole number by 10, 100, 1000, etc., we annex to the multiplicand as many ciphers as there are in the multiplier.

Example. Multiply 356 by 1000.

Annexing *three* ciphers to the multiplicand, we have

$$356 \times 1000 = 356000, \textit{Ans.}$$

36. To Multiply by any Number of Tens, Hundreds, Etc.

Any number of ciphers at the right of the multiplier may be omitted during the operation of multiplication, and annexed to the result.

1. Multiply 2734 by 2600.

```
 2734
   2600
16404
5468
7108400, Ans.
```

We say in this case: 6 times 2734 are 16404; 2 times 2734 are 5468; adding, and annexing *two* ciphers to the result, the product is 7108400.

In like manner, ciphers at the right of the multiplicand may be omitted during the operation.

2. Multiply 63000 by 580.

```
63000
 580
 504
315
36540000, Ans.
```

We say in this case: 8 times 63 are 504; 5 times 63 are 315; adding, and annexing *four* ciphers to the result, the product is 36540000.

37. When two numbers are to be multiplied together, circumstances will often determine which to take as the multiplier.

In general, the number having the least number of places should be taken as the multiplier; thus, to multiply 852 and 27, we should take 27 as the multiplier.

If one of the numbers has two or more digits alike, it may be easier to take it as the multiplier; thus, to multiply 666 and 329, it would be easier to take the former number as the multiplier.

Again, if one of the numbers has *ciphers* or *ones* for digits, it may be shorter to take it as the multiplier; thus, to multiply 394 and 2001, it would be shorter to take the latter number as the multiplier.

38. Short Methods in Multiplication.

To multiply by 99, 999, etc, we may proceed as follows:

Example. Multiply 1652 by 999.

```
1652000
   1652
1650348, Ans.
```

Since 999 is 1000 – 1, we multiply 1652 by 1000, and then by 1, and subtract the second result from the first.

In like manner, we may multiply by 98, 97, 998, 997, or by any number a little less than 100, 1000, 10000, etc.

The same artifice may be employed when the multiplier is a little less than 200, 300, 2000, 3000, etc.

Thus, to multiply 867 by 698, we multiply it by 700, and then by 2, and subtract the second result from the first.

EXAMPLES.

39. Multiply the following:

1. 873 by 956.
2. 2600 by 3950.
3. 487 by 8009.
4. 4067 by 997.
5. 3476 by 625.
6. 6872 by 599.
7. 60507 by 3784.
8. 15063 by 9874.
9. 54189 by 7998.
10. 7677 by 4912.
11. 2946 by 5335.
12. 82821 by 7269.
13. 93247 by 2461.
14. 35895 by 6927.

15. Find the value of $(17+15)\times(19-3)$.

Note. This signifies that 17 and 15 are to be added together, then 3 subtracted from 19, and then the first result multiplied by the second. (Compare Art. 26.)

Find the values of the following:

16. $(38-15)\times 26$.

17. $(13+21)\times(32+7)$.

18. $(77+43)\times(56-19)$.

19. $(92-36)\times(88-29)$.

20. $103\times(186-115+137)$.

21. $463-\overline{(35+29)\times 5}+148$.

22. $(391-\overline{274-89})\times(96-37)$.

23. $(127-98)\times(101+66)-(103-79)\times(86-47)$.

24. $(856-\overline{614-477})\times(982-\overline{378+249})$.

25. $(387-\overline{35+123})\times(458-\overline{129-75+48})+(713\times 294)$.

V. DIVISION.

40. To **Divide** one whole number by another is to find a number which, when multiplied by the second number, will produce the first.

Thus, to divide 15 by 5 is to find a number which when multiplied by 5, will produce 15.

The number is 3; hence, 15 divided by 5 are 3.

We also say, "5 is *contained* in 15 three times."

41. The number which is divided is called the **Dividend.**

The number by which the dividend is divided is called the **Divisor.**

The result is called the **Quotient.**

42. It is evident that the Dividend is the product of the Divisor and Quotient.

43. The symbol $\div$, read "*divided by,*" signifies division.

Thus, $15 \div 5 = 3$ is read "fifteen divided by five are three."

44. If one number does not exactly contain another, its excess above the next smaller number that does exactly contain the second number is called the **Remainder.**

Thus, 5 is contained in 17 three times, with a remainder 2.

45. 1. Divide 852 by 3.

Divisor, 3)852, Dividend.
Quotient, 284, *Ans.*

We write the divisor at the left of the dividend, with a) between them.

3 is contained in 8 hundreds 2 hundreds times, with a remainder of 2 hundreds, or 20 tens.

We then write 2 under the hundreds' figure of the dividend.

20 tens and 5 tens are 25 tens; 3 is contained in 25 tens 8 tens times, with a remainder of 1 ten, or 10 units.

We then write 8 under the tens' figure of the dividend.

10 units and 2 units are 12 units; 3 is contained in 12 units 4 times.

We then write 4 under the units' figure of the dividend.

The required result is 284.

The following words are used in explaining the above process:

3 in 8 twice, with 2 to carry; 3 in 25 eight times, with 1 to carry; 3 in 12 four times.

2. Divide 22236 by 68.

First Process.

```
68)22236
   327, Ans.
```

68 is contained in 222 three times, with a remainder of 18; 68 is contained in 183 twice, with a remainder of 47; 68 is contained in 476 seven times. The required result is 327.

In order to avoid the labor of calculating the remainders *mentally*, it is customary to arrange the written work as follows:

Second Process.

```
68)22236(327, Ans.
   204
   ---
    183
    136
    ---
     476
     476
     ---
```

We say, 68 is contained in 222 three times, with a remainder.

Multiplying 68 by 3, the product is 204, which we write under the 222; subtracting 204 from 222, the remainder is 18; annex to this the next dividend figure, 3.

68 is contained in 183 twice, with a remainder; multiplying 68 by 2, the product is 136, which we write under the 183; subtracting 136 from 183, the remainder is 47; annex to this the last dividend figure, 6.

68 is contained in 476 seven times; multiplying 68 by 7 the product is 476, which we write under the 476; subtracting, there is no remainder.

Hence, the required result is 327.

It will be seen that the second process is essentially the same as the first; the only difference being that, in the first, certain operations are performed *mentally*, which are written out in full in the second.

Note 1. The operation is called *Short Division* when the remainders and partial products are obtained mentally, as in Ex. 1, and the first process of Ex. 2; and *Long Division* when they are written out in full, as in the second process of Ex. 2.

The method of Short Division should always be used when the divisor is 12 or less.

From the second process of Ex. 2, we derive the following

RULE FOR LONG DIVISION.

Write the divisor at the left of the dividend.

Take, at the left of the dividend, the smallest number of digits that will form a number equal to or greater than the divisor.

Divide this number by the divisor, and write the quotient as the first digit of the quotient; subtract from the number the product of the divisor by the first digit of the quotient, and annex to the remainder the next figure of the dividend.

Divide this partial dividend by the divisor, and proceed as before; continuing the process until all the figures of the dividend have been taken.

Note 2. If any partial dividend is less than the divisor, write 0 for the corresponding digit of the quotient, and annex to the partial dividend the next figure of the dividend.

Note 3. If, on making trial of any number as a digit of the quotient, its product by the divisor is greater than the preceding partial dividend, the number tried is too great, and one less must be substituted for it.

If any remainder is equal to or greater than the divisor, the digit of the quotient last obtained is too small, and one greater must be substituted for it.

3. Divide 80791 by 386.

```
       (209, Quotient.
386)80791
    772
    ----
     3591
     3474
     ----
      117, Remainder.
```

In this case, the smallest number of digits at the left of the dividend that will form a number greater than the divisor, is three.

386 is contained in 807 twice, with a remainder; 2 times 386 is 772, which, subtracted from 807, leaves 35; annex to this the next dividend figure, 9.

Since 359 is less than the divisor, we write 0 as the second digit of the quotient, and annex to 359 the last dividend figure, 1.

386 is contained in 3591 nine times, with a remainder; 9 times 386 is 3474, which, subtracted from 3591, leaves 117.

Hence, the quotient is 209, and the remainder 117.

Note 4. The work may be *proved* by multiplying the divisor and quotient, and adding the remainder, if any, to the product; the result should equal the dividend.

46. To Divide by 10, 100, 1000, Etc.

To divide a whole number by 10, 100, 1000, etc., we cut off from the right of the dividend as many digits as there are ciphers in the divisor.

The result will be the quotient, and the digits cut off will form the remainder.

1. Divide 360000 by 1000.

Cutting off *three* digits from the right of the dividend, we have

$$360000 \div 1000 = 360, \textit{ Ans.}$$

2. Divide 7298 by 100.

Cutting off *two* digits from the right of the dividend, we have

$$7298 \div 100 = 72, \text{ with a remainder of } 98, \textit{ Ans.}$$

47. To Divide by Any Number of Tens, Hundreds, Etc.

It is evident that, if both dividend and divisor be divided by the *same number*, the value of the quotient is not changed; for the new divisor is contained in the new dividend just as many times as the old divisor is contained in the old dividend.

It follows from the above that we may *remove the same number of ciphers* from the right of both the dividend and divisor, and find the quotient of the resulting numbers; for this is the same as dividing both dividend and divisor by the same number.

1. Divide 400400 by 7700.

```
77)4004(52, Ans.
    385
    ---
     154
     154
     ---
```

In this case, we remove *two* ciphers from the right of both the dividend and divisor, and divide 4004 by 77.

2. Divide 3358617 by 65000.

First Process.

```
65000)3358617(51, Quotient.
      325000
      ------
       108617
        65000
        -----
        43617, Remainder.
```

The process may be shortened by omitting the ciphers at the right of the divisor, and the *same number of digits* at the right of the dividend, finding the quotient of the resulting numbers, and annexing to the remainder the digits omitted from the right of the dividend.

The written work will then stand as follows:

Second Process.

```
65)3358|617(51, Quotient.
   325
   ---
    108
     65
     --
     43617, Remainder.
```

From the above example, we derive the following

RULE.

If the divisor has, and the dividend has not, ciphers at its right, cut off the ciphers from the right of the divisor, and the same number of digits from the right of the dividend.

Divide the resulting numbers, and annex to the remainder the digits omitted from the right of the dividend.

48. If the dividend has, and the divisor has not, ciphers at its right, we proceed as follows:

Example. Divide 476000 by 56.

```
56)476000(8500, Ans.
   448
   ---
    280
    280
    ---
```

In this case, 56 is contained in 4760 85 times; annexing to this the two ciphers remaining at the right of the dividend, the quotient is 8500.

49. If any number of *things of the same kind* be divided by a whole number, the quotient will be things of the same kind as the dividend.

Thus, 42 *books* divided by 7 are 6 *books.*

Again, if any number of things of the same kind be divided by another number of things *of the same kind as the dividend,* the quotient will be a *number.*

Thus, 42 *books* divided by 7 *books* are 6.

EXAMPLES.

50. Divide the following:

1. 488304 by 12.
2. 517668 by 964.
3. 63102024 by 6008.
4. 24884574 by 49082.
5. 68515100 by 69700.
6. 3545000 by 587.
7. 83057629 by 10000.
8. 53803998 by 10629.
9. 58161020 by 1357.
10. 41675206 by 492000.
11. 23510372 by 73700.
12. 20301129888 by 25376.
13. 499107840 by 53760.
14. 626104565 by 7247.
15. Find the value of $(59-11)\div(25-17)$.

Note. This signifies that 11 is to be taken from 59, then 17 from 25, and the first result divided by the second. (Compare Art. 26.)

Find the values of the following:

16. $(\overline{78-12}\div 11)\times(15-7)$.
17. $(\overline{13\times 5}+26)\div 7$.
18. $81-(\overline{108\div 9)+43}$.
19. $(148-\overline{15+31})\div 34$.
20. $(322-106)\div(48-21)$.
21. $(132\div 6)\times(143\div 13)$.
22. $(297+279)\div(6\times 8)$.
23. $(33\times 57)-(442\div 17)$.
24. $(8567-\overline{12073-7988})\div(8051\div 97)$.

VI. FACTORING.

51. One whole number is said to be *divisible* by another when the first number can be divided by the second without a remainder.

In the above case, the first number is said to be a **Multiple** of the second, and the second a **Factor** of the first.

Thus, 15 is a multiple of 5, and 5 is a factor of 15.

52. An **Even Number** is one that is divisible by 2; as 2, 4, 6, 8, etc.

An **Odd Number** is one that is not divisible by 2; as 1, 3, 5, 7, etc.

53. A **Prime Number** is one that is divisible only by itself and 1; as 1, 2, 3, 5, 7, etc.

A **Composite Number** is one that is divisible by other numbers than itself and 1; as 4, 6, 8, 9, 10, etc.

54. The **Prime Factors** of a number are the prime numbers which, when multiplied together, will produce the given number.

Thus, the prime factors of 12 are 2, 2, and 3.

Note. It is usual to exclude 1 in giving the prime factors of a number.

To **Factor** a number is to find its prime factors.

55. If a number be multiplied by itself any number of times, the result is called a *power* of the first number.

An *exponent* is a number written at the right of, and above another, to indicate what power of the latter number is to be taken; thus,

2^2, read "2 *square*" or "2 *to the 2d power*," denotes 2×2;
5^3, read "5 *cube*" or "5 *to the 3d power*," denotes $5 \times 5 \times 5$;
7^4, read "7 *fourth*" or "7 *to the 4th power*," denotes $7 \times 7 \times 7 \times 7$; and so on.

56. The following principles will be found of great use in factoring numbers:

Any number of tens is divisible by 2; hence,

I. *Any number is divisible by* 2 *if its last digit is* 0 *or an even number.*

Thus, 738 is divisible by 2, because 8 is an even number.

Any number of hundreds is divisible by 4; hence,

II. *Any number is divisible by* 4 *if the number formed by its last two digits is divisible by* 4.

Thus, 3568 is divisible by 4, because 68 is divisible by 4.

Any number of thousands is divisible by 8; hence,

III. *Any number is divisible by* 8 *if the number formed by its last three digits is divisible by* 8.

Thus, 47352 is divisible by 8, because 352 is divisible by 8.

Any number of tens is divisible by 5; hence,

IV. *Any number is divisible by* 5 *if its last digit is* 0 *or* 5.

Thus, 120 and 8295 are divisible by 5.

Any number of tens is divisible by 10; hence,

V. *Any number is divisible by* 10 *if its last digit is* 0.

Thus, 3790 is divisible by 10.

VI. *Any number is divisible by* 3 *if the sum of its digits is divisible by* 3.

Thus, 582 is divisible by 3, because the sum of its digits, 15, is divisible by 3.

VII. *Any number is divisible by* 6 *if its last digit is* 0 *or an even number, and the sum of its digits is divisible by* 3.

This follows from I and VI.

VIII. *Any number is divisible by* 9 *if the sum of its digits is divisible by* 9.

Thus, 864 is divisible by 9, because the sum of its digits, 18, is divisible by 9.

IX. *Any number is divisible by* 11 *if the sum of the digits in the odd places is equal to the sum of the digits in the even places, or differs from it by a number divisible by* 11.

Thus, 4785 is divisible by 11, because the sum of the digits in the first and third places, 12, is equal to the sum of the digits in the second and fourth places.

Again, 39182 is divisible by 11, because the sum of the digits in the first, third, and fifth places, 6, differs from the sum of the digits in the second and fourth places, 17, by 11; a number divisible by 11.

Note. Principles VI, VIII, and IX may be proved as follows:

Proof of VI and VIII.

Any number of 10's is equal to the same number of 9's, plus the same number of units; any number of 100's is equal to the same number of 99's, plus the same number of units; etc.

Thus, 783 is equal to 7 99's plus 7 units, 8 9's plus 8 units, and 3 units.

But the sum of 7 99's and 8 9's is divisible by both 3 and 9.

Hence, 783 is divisible by 3 or 9 if the sum of 7 units, 8 units, and 3 units is divisible by 3 or 9, respectively; that is, if the *sum of its digits* is divisible by 3 or 9, respectively.

Similar considerations hold with respect to any number.

Proof of IX.

Any number of 10's is equal to the same number of 11's, *minus* the same number of units; any number of 100's is equal to the same number of 99's *plus* the same number of units; any number of 1000's is equal to the same number of 1001's, *minus* the same number of units; etc.

Thus, 4829 is equal to 4 1001's minus 4 units, 8 99's plus 8 units, 2 11's minus 2 units, and 9 units.

But the sum of 4 1001's, 8 99's, and 2 11's, is divisible by 11.

Hence, 4829 is divisible by 11 if the sum of 8 units and 9 units, minus the sum of 4 units and 2 units, is divisible by 11; that is, if the difference between the sum of the digits in the odd places and the sum of the digits in the even places is divisible by 11.

Similar considerations hold with respect to any number.

57. 1. Find the prime factors of 51480.

$$\begin{array}{r|l} 2^3 & 51480 \\ \hline 5 & 6435 \\ \hline 3^2 & 1287 \\ \hline 11 & 143 \\ \hline & 13 \end{array}$$

51480 is divisible by 8, or 2^3, because the number formed by its last three digits is divisible by 8 (Art. 56, III).

Dividing 51480 by 8, the quotient is 6435.

6435 is divisible by 5, because its last digit is 5.

Dividing 6435 by 5, the quotient is 1287.

1287 is divisible by 9, or 3^2, because the sum of its digits, 18, is divisible by 9 (Art. 56, VIII).

Dividing 1287 by 9, the quotient is 143.

143 is divisible by 11, because the sum of the digits in the first and third places is equal to the digit in the second place (Art. 56, IX).

Dividing 143 by 11, the quotient is 13, a prime number.

Then, $51480 = 2^3 \times 3^2 \times 5 \times 11 \times 13$, *Ans.*

From the above example, we derive the following

RULE.

Divide the number by any one of its factors; then divide the quotient by any one of its factors; and so on, continuing the process until the quotient is a prime number.

The several divisors and the last quotient are the factors required.

Note 1. In determining the prime factors of a number, divisors should be tried in the following order:

$2, (2^2, 2^3);\ 5;\ 3, (3^2);\ 11;$ then, 7, 13, 17, 19, 23, 29, etc.

2. Prove that 373 is a prime number.

Since the last digit is 3, neither 2 nor 5 is a factor.

Since the sum of the digits is 13, 3 is not a factor.

Since the sum of the digits in the first and third places differs from the digit in the second place by 1, 11 is not a factor.

Trying in order the prime numbers 7, 13, 17, and 19, we find that neither of them is contained in 373.

There is no need of trying 23, nor any greater prime number; for if one factor of 373 could be 23, the other would have to be 23, or some greater prime number; for it has already been proved that no prime number less than 23 is a factor.

But $23 \times 23 = 529$, a number greater than 373; and hence 23 cannot be a factor.

Therefore 373 is a prime number.

Note 2. In trying prime numbers as divisors, the process need not be continued after the square of the prime number next greater than the last one tried exceeds the given number.

Thus, in Ex. 2, no prime number greater than 19 need be tried, because the square of the next greater prime, 23, is greater than 373.

The following table of squares of prime numbers will be found of use:

No.	Sq.	No.	Sq.	No.	Sq.	No.	Sq.	No.	Sq.
13	169	29	841	43	1849	61	3721	79	6241
17	289	31	961	47	2209	67	4489	83	6889
19	361	37	1369	53	2809	71	5041	89	7921
23	529	41	1681	59	3481	73	5329	97	9409

58. The following table gives, for convenient reference, a list of the prime numbers from 1 to 997, inclusive:

1	41	101	167	239	313	397	467	569	643	733	823	911
2	43	103	173	241	317	401	479	571	647	739	827	919
3	47	107	179	251	331	409	487	577	653	743	829	929
5	53	109	181	257	337	419	491	587	659	751	839	937
7	59	113	191	263	347	421	499	593	661	757	853	941
11	61	127	193	269	349	431	503	599	673	761	857	947
13	67	131	197	271	353	433	509	601	677	769	859	953
17	71	137	199	277	359	439	521	607	683	773	863	967
19	73	139	211	281	367	443	523	613	691	787	877	971
23	79	149	223	283	373	449	541	617	701	797	881	977
29	83	151	227	293	379	457	547	619	709	809	883	983
31	89	157	229	307	383	461	557	631	719	811	887	991
37	97	163	233	311	389	463	563	641	727	821	907	997

EXAMPLES.

59. Find the prime factors of the following:

1. 684.	**5.** 3003.	**9.** 1729.	**13.** 53625.
2. 686.	**6.** 4459.	**10.** 8395.	**14.** 48204.
3. 2520.	**7.** 1331.	**11.** 24108.	**15.** 32292.
4. 4305.	**8.** 8085.	**12.** 19635.	**16.** 68364.

Prove that each of the following numbers is prime:

17. 1019.
18. 1193.
19. 1367.
20. 1531.
21. 1787.
22. 2081.
23. 2203.
24. 2999.

60. Verification of Addition, Subtraction, Multiplication, and Division by Casting out Nines.

The excess of any number above the next less multiple of 9 is called its **Excess of Nines.**

Thus, since 3527 is equal to 391×9, plus 8, its excess of nines is 8.

It was shown in the proof of VIII (Art. 56, Note) that any number is equal to a multiple of 9, plus the sum of its digits.

Thus, 3527 is equal to a multiple of 9, plus 17; or since $17 = 9 + 8$, it is equal to a multiple of 9, plus 8.

It follows from the above that the excess of nines of any number may be found by *subtracting from the sum of its digits the next less multiple of* 9.

Thus, since the sum of the digits of the number 4619 is 20, its excess of nines is $20 - 18$, or 2.

Addition.

$$\begin{array}{rrr} & 3527 \cdots & 8 \\ & \underline{4619} \cdots & \underline{2} \\ 1 \cdots & 8146 & 10 \cdots 1 \end{array}$$

The excess of nines of 3527 is 8, and of 4619 is 2.

Then since 3527 is equal to a multiple of 9, plus 8, and 4619 to a multiple of 9, plus 2, their *sum* is equal to a multiple of 9, plus 8, plus 2; or since $10 = 9 + 1$, it is equal to a multiple of 9, plus 1.

But the excess of nines of 8146 is 1; that is, 8146 is equal to a multiple of 9, plus 1.

This agrees with the statement made in the preceding paragraph with respect to the sum of 3527 and 4619.

Then, to verify the result of addition, we place to the right of each of the numbers to be added its excess of nines.

Adding these excesses, we place to the right of their sum its excess of nines.

If this equals the excess of nines of the result, the work may be considered to be correct.

Subtraction.

$$
\begin{array}{rrr}
 & 6782 \cdots & 5 \\
 & \underline{2946} \cdots & \underline{3} \\
2 \cdots & 3836 & 2
\end{array}
\qquad\qquad
\begin{array}{rrrr}
 & 2235 \cdots & 3 \cdots & 12 \\
 & \underline{1167} & \cdots & \underline{6} \\
6 \cdots & 1068 & & 6
\end{array}
$$

To the right of the minuend and subtrahend we place their excess of nines.

We then subtract the excess of the subtrahend from that of the minuend, increasing the latter by 9 if it is less than the excess of the subtrahend.

If the remainder equals the excess of nines of the result, the work may be considered to be correct.

Multiplication.

$$
\begin{array}{rrr}
 & 498 \cdots & 3 \\
 & \underline{376} \cdots & \underline{7} \\
 & 2988 & 21 \cdots 3 \\
 & 3486 & \\
 & \underline{1494} & \\
3 \cdots & 187248 &
\end{array}
$$

Since 498 is equal to a multiple of 9, plus 3, and 376 to a multiple of 9, plus 7, their product is equal to the result obtained by multiplying a multiple of 9 plus 3, first by a multiple of 9, and afterwards by 7, and adding the results.

It is evident from this that the product is equal to a multiple of 9, plus 3 times 7; or since $21 = 18 + 3$, it is equal to a multiple of 9, plus 3.

To the right of each of the numbers to be multiplied we place its excess of nines.

Multiplying the excess of the multiplicand by that of the multiplier, we place to the right of the product its excess of nines.

If this equals the excess of nines of the result, the work may be considered to be correct.

Division.

$$
\begin{array}{rl}
 & (768 \cdots 3 \\
5 \cdots 329) & 252784 \cdots 1 \\
 & \underline{2303} \\
 & \;\;2248 \\
 & \;\;\underline{1974} \\
 & \;\;\;\;2744 \\
 & \;\;\;\;\underline{2632} \\
 & \;\;\;\;\;\;112 \cdots 4
\end{array}
\qquad\qquad
\begin{array}{l}
3 \\
\underline{5} \\
15 \\
\underline{\;4} \\
19 \cdots 1
\end{array}
$$

Since the dividend is equal to the product of the divisor and quotient, plus the remainder, we proceed as follows:

Multiply the excess of nines of the quotient by that of the divisor.

Add to the product the excess of nines of the remainder, and place to the right of the sum its excess of nines.

If this equals the excess of nines of the dividend, the work may be considered to be correct.

Note. The above methods are not always to be depended upon as tests of the accuracy of operations.

Suppose, for instance, that, in the illustrative example under Addition, we had taken the sum of 2 and 1 as 4, and the sum of 5 and 6 as 10:

$$
\begin{array}{r}
3527 \\
\underline{4619} \\
8056
\end{array}
$$

The excess of nines in the sum will still be 1, and yet the work is not performed correctly.

A balance of errors like this is, however, unlikely to occur.

61. Casting out Elevens.

The excess of any number above the next less multiple of 11 is called its **Excess of Elevens.**

Thus, since 3527 is equal to 320×11, plus 7, its excess of elevens is 7.

It was shown in the proof of IX (Art. 56, Note) that any number is equal to a multiple of 11, plus the sum of the digits in the odd places, minus the sum of the digits in the even places.

Thus, 3527 is equal to a multiple of 11, plus 12, minus 5; or, to a multiple of 11, plus 7.

It follows from the above that the excess of elevens of any number may be found by *subtracting the sum of the digits in the even places from the sum of the digits in the odd places, the latter being increased by* 11, *or a multiple of* 11, *if necessary.*

Thus, for the number 9484, the sum of the digits in the odd places is 8, and the sum of the digits in the even places is 17.

Increasing the former by 11, we have 19.

Then the excess of elevens is $19 - 17$, or 2.

The methods for verifying Addition, Subtraction, Multiplication, and Division by casting out elevens are precisely similar in theory and practice to the methods by casting out nines.

VII. GREATEST COMMON DIVISOR.

62. A **Common Divisor,** or **Common Factor,** of two or more whole numbers is a number that will divide each of them without a remainder.

Thus, 3 is a common divisor of 18, 24, and 30.

63. The **Greatest Common Divisor** (G. C. D.) of two or more whole numbers is the greatest number that will divide each of them without a remainder.

Thus, 6 is the greatest common divisor of 18, 24, and 30.

64. Two numbers are said to be *prime to each other* when they have no common divisor except 1.

Thus, 8 and 9 are prime to each other.

65. In determining the G. C. D. of numbers, we may distinguish two cases:

66. Case I. *When the numbers can be readily factored as in* Art. 57.

1. Find the G. C. D. of 144, 264, and 540.

$$\begin{aligned} 144 &= 2^4 \times 3^2 \\ 264 &= 2^3 \times 3 \times 11 \\ 540 &= \underline{2^2 \times 3^3 \times 5} \\ \text{G. C. D.} &= 2^2 \times 3 \\ &= 12, \textit{ Ans.} \end{aligned}$$

Factoring each number by the method of Art. 57, it is evident that the greatest number that will exactly divide 144, 264, and 540, is $2^2 \times 3$.

Hence, the required G. C. D. is 12.

From the above example we derive tho following

RULE.

Factor each of the numbers.

Take every prime number which is a common divisor of all the given numbers, the least number of times that it occurs in any one of the numbers.

The product of these numbers will be the G. C. D. required.

Note. If any prime number is a common divisor of all the given numbers, its exponent in the G. C. D. will be the *lowest* exponent with which it occurs in any one of the numbers.

Thus, in Ex. 1, we have in the given numbers 2^4, 2^3, and 2^2 respectively, and in the G. C. D., 2^2.

If one of the numbers is exactly contained in another, the latter need not be considered in the operation of finding the G. C. D.; for since every factor of the first number is also a factor of the second, the result is not affected by omitting the second number from the process.

Thus, in finding the G. C. D. of 28, 49, 140, and 196, it would be sufficient to find the G. C. D. of 28 and 49.

EXAMPLES.

Find the G. C. D. of:

2. 165 and 210.

3. 288 and 648.

4. 306 and 476.

5. 36, 144, and 234.

6. 128, 192, and 384.

7. 675 and 1125.

8. 105, 385, and 455.

9. 96, 108, 132, and 156.

10. 240, 336, and 480.

11. 81, 117, 126, and 135.

12. 104, 182, and 351.

13. 180, 264, and 378.

14. 16, 52, 160, 224, and 260.

15. 320, 640, and 1008.

16. 390, 910, and 1365.

17. 360, 750, and 2700.

18. 432, 1944, and 2592.

19. 525, 3375, and 7425.

20. 1540, 5005, and 6545.

21. 6804, 7056, and 8232.

22. A farmer has three pieces of timber whose lengths are 63, 84, and 105 feet, respectively. What is the length of the longest logs, all of the same length, that can be cut from them?

23. Two schools, containing 480 and 672 pupils, respectively, are divided into classes, each containing the same number of pupils. What is the greatest number of pupils that each class can contain, and how many classes of this size are there in each school?

24. Three rooms are 168, 196, and 224 inches wide, respectively. What is the width of the widest carpeting that is contained exactly in each room?

25. How many quarts are there in the largest receptacle that will exactly measure the contents of three jars, holding 216, 288, and 312 quarts, respectively?

26. I have three fields containing 392, 504, and 616 square rods, respectively. Find the size of the largest house-lots, all of the same size, into which the fields can be divided.

27. The sides of a field are 110, 154, 198, and 264 feet, respectively. What is the length of the longest fence-rail that is contained exactly in each side?

67. CASE II. *When the numbers cannot be readily factored as in* Art. 57.

1. Find the G. C. D. of 221 and 493.

Dividing the greater number by the less, we have

$$\begin{array}{r} 221)493(2, \text{ Quotient.} \\ \underline{442}\phantom{(2, \text{ Quotient.}} \\ 51, \text{ Remainder.} \end{array}$$

Now whatever factors occur in 221 must also occur in twice 221, or 442.

Hence, any factors which are common to 221 and 493 must occur in the result obtained by subtracting 442 from 493; that is, they must occur in 51.

Again, any factors which are common to 221 and 51 must occur in the result obtained by adding twice 221 to 51; that is, they must occur in 493.

Then, since every factor common to 221 and 493 occurs in 51, and every factor common to 221 and 51 occurs in 493, it follows that 221, 493, and 51 have the *same common factors.*

Hence, the G. C. D. of 221 and 493 must be the same as the G. C. D. of 221 and 51.

That is, *the G. C. D. of any two numbers is the same as the G. C. D. of the less number, and the remainder obtained by dividing the greater number by the less.*

Dividing the divisor, 221, by the remainder, 51, we have

$$\begin{array}{l} 51)221(4 \\ \quad\ \ \underline{204} \\ \quad\ \ \ \ 17, \text{ Remainder.} \end{array}$$

Then, by the principle just stated, the G. C. D. of 221 and 51 is the same as the G. C. D. of 51 and 17.

Dividing the divisor, 51, by the remainder, 17, we have

$$\begin{array}{l} 17)51(3 \\ \quad\ \ \underline{51} \end{array}$$

That is, 17 is the G. C. D. of 51 and 17.

Then, 17 is also the G. C. D. of 221 and 51, and is consequently the G. C. D. of 493 and 221.

From the above example, we derive the following

RULE.

Divide the greater number by the less.

If there be a remainder, divide the divisor by it; and continue thus to make the remainder the divisor, and the preceding divisor the dividend, until there is no remainder.

The last divisor is the G. C. D. required.

2. Find the G. C. D. of 377 and 667.

$$\begin{array}{l} 377)667(1 \\ \qquad\ \underline{377} \\ \qquad\ 290)377(1 \\ \qquad\qquad\quad \underline{290} \\ \qquad\qquad\quad\ 87)290(3 \\ \qquad\qquad\qquad\quad \underline{261} \\ \qquad\qquad\qquad\quad\ 29)87(3 \\ \qquad\qquad\qquad\qquad\quad \underline{87} \end{array}$$

Then, 29 is the G. C. D. required, *Ans.*

EXAMPLES.

Find the G. C. D. of:

3. 559 and 817.
4. 391 and 598.
5. 589 and 899.
6. 703 and 893.
7. 533 and 1271.
8. 731 and 1247.
9. 3658 and 4602.
10. 3703 and 6923.
11. 1591 and 2183.
12. 5605 and 6785.
13. 6059 and 7446.
14. 5312 and 10043.
15. 2291 and 3713.
16. 10057 and 11659.

68. The G. C. D. of *three* numbers which cannot be readily factored by the method of Art. 57, may be found as follows:

Let A, B, and C represent the numbers.

Let G represent the G. C. D. of A and B; then every common factor of G and C is also a common factor of A, B, and C.

But every common factor of A and B exactly divides G.

Whence, every common divisor of A, B, and C is also a common divisor of G and C.

Therefore, the *greatest* common divisor of A, B, and C is the same as the *greatest* common divisor of G and C.

Hence, *to find the G. C. D. of three numbers, find the G. C. D. of two of them, and then of this result and the third number.*

We proceed in a similar manner to find the G. C. D. of four or more numbers.

1. Find the G. C. D. of 741, 1653, and 7163.

We first find the G. C. D. of 741 and 1653, which is 57.
We then find the G. C. D. of 57 and 7163, which is 19, *Ans.*

EXAMPLES.

Find the G. C. D. of:

2. 663, 741, and 4199.
3. 442, 782, and 5083.
4. 969, 1653, and 9367.
5. 5083, 5681, and 7429.

VIII. LEAST COMMON MULTIPLE.

69. A **Common Multiple** of two or more whole numbers is a number that will exactly contain each of them.

Thus, 72 is a common multiple of 6, 9, and 12.

70. The **Least Common Multiple** (L. C. M.) of two or more whole numbers is the smallest number that will exactly contain each of them.

Thus, 36 is the least common multiple of 6, 9, and 12.

71. In determining the L. C. M. of numbers, we may distinguish two cases:

72. CASE I. *When the numbers can be readily factored.*

Example. Find the L. C. M. of 40, 84, and 144.

$$\begin{aligned} 40 &= 2^3 \times 5 \\ 84 &= 2^2 \times 3 \times 7 \\ 144 &= 2^4 \times 3^2 \\ \hline \text{L. C. M.} &= 2^4 \times 3^2 \times 5 \times 7 \\ &= 5040, \textit{ Ans.} \end{aligned}$$

Factoring each of the numbers by the method of Art. 57, it is evident that the smallest number that will exactly contain 40, 84, and 144, is $2^4 \times 3^2 \times 5 \times 7$.

Hence, the required L. C. M. is 5040.

From the above example, we derive the following

RULE.

Factor each of the numbers.

Take every prime number, which is a factor of any one of the given numbers, the greatest number of times that it occurs in any one of the numbers.

The product of these numbers will be the L. C. M. required.

Note. If any prime number is a factor of any one of the given numbers, its exponent in the L. C. M. will be the *greatest* exponent with which it occurs in any one of the numbers.

Thus, in the above example, we have in the given numbers 2^3, 2^2, and 2^4, respectively, and in the L. C. M., 2^4.

73. Second Method.

The following rule will be found preferable to that of Art. 72 in the solution of examples:

Arrange the numbers in a horizontal line.

If two or more of the numbers have a common prime factor, divide them by it, and write the quotients, together with the undivided numbers, in the next line.

Continue in this way until a line is obtained in which the numbers have no common factor.

The product of the divisors and the numbers in the last line will be the L. C. M. required.

1. Find the L. C. M. of 24, 60, and 105.

$$\begin{array}{r|rrr} 2 & 24 & 60 & 105 \\ \hline 2 & 12 & 30 & 105 \\ \hline 3 & 6 & 15 & 105 \\ \hline 5 & 2 & 5 & 35 \\ \hline & 2 & 1 & 7 \end{array}$$

L. C. M. $=2\times2\times3\times5\times2\times7$
$=840$, *Ans.*

Dividing 24 and 60 by the common prime factor 2, the second line becomes 12, 30, 105.

Dividing 12 and 30 by the common prime factor 2, the third line becomes 6, 15, 105.

Dividing 6, 15, and 105 by the common prime factor 3, the fourth line becomes 2, 5, 35.

Dividing 5 and 35 by the common prime factor 5, the fifth line becomes 2, 1, 7.

Since the numbers 2 and 7 have no common prime factor, the required L. C. M. is the product of the divisors, 2, 2, 3, 5, and the numbers in the last line, 2, 7; the result is 840.

It is evident that, in the above process, every prime number which is a factor of any one of the given numbers, is taken the greatest number of times that it occurs in any one of the numbers.

Hence, the result is the L. C. M. of the given numbers.

If one of the given numbers exactly divides another, the former need not be considered in the operation of finding the L. C. M.; for since every factor of the first number is also a factor of the second, the result is not affected by omitting the first number from the process.

Thus, in finding the L. C. M. of 15, 26, 78, and 90, it would be sufficient to find the L. C. M. of 78 and 90.

Note. If two numbers are *prime to each other* (Art. 64), their product is their L. C. M.

EXAMPLES.

Find the L. C. M. of:

2. 4, 6, 9, and 10.

3. 28 and 63.

4. 24, 112, and 160.

5. 108 and 144.

6. 110 and 165.

7. 231 and 770.

8. 18, 38, 54, and 57.

9. 20, 75, 180, and 300.

10. 176 and 264.

11. 32, 88, and 121.

12. 87, 116, and 192.

13. 144, 216, and 324.

14. 28, 49, 147, and 196.

15. 24, 42, 72, 84, and 112.

16. 15, 35, and 77.

17. 36, 104, and 351.

18. 115, 138, 230, and 345.

19. 33, 44, 55, and 132.

20. 288, 324, 432, and 648.

21. 189, 243, and 405.

22. 98, 126, 140, and 168.

23. 52, 81, 117, and 120.

24. 119, 136, 252, and 280.

25. 315, 1350, and 1500.

26. How many quarts are there in the smallest vessel whose contents can be exactly measured by measures containing 10, 15, and 18 quarts, respectively?

27. Two horse-cars make round trips in 48 and 60 minutes, respectively. If they set out at the same time, after how many minutes will they meet again at the starting-point?

28. What is the smallest sum of money with which I can purchase cows at $ 45 each, oxen at $ 54 each, or horses at $ 72 each?

29. Three men, A, B, and C, can walk around a race-course in 9, 12, and 14 minutes, respectively. If they all set out at the same time, after how many minutes will they all meet at the starting-point, and how many times will each have been around the course?

74. CASE II. *When the numbers cannot be readily factored.*

1. Find the L. C. M. of 221 and 247.

If we divide 221 by the *greatest common divisor* of 221 and 247, the quotient will be the product of those factors of 221 *which are not found in* 247.

Then, if we multiply this quotient by 247, the product will be divisible by both 221 and 247; and it is evidently the smallest number that is divisible by both of them.

That is, the product is the L. C. M. of 221 and 247.

221)247(1	13)221(17	247
221	13	17
26)221(8	91	1729
208	91	247
G. C. D.=13)26(2		L. C. M.=4199, *Ans.*
26		

We first find the G. C. D of 221 and 247 by the rule of Art. 67; the result is 13.

Dividing 221 by 13, the quotient is 17.

Multiplying 247 by 17, the product is 4199.

Then, the required L. C. M. is 4199.

From the above example, we derive the following

RULE.

Find the G. C. D. of the given numbers.

Divide one of the numbers by their G. C. D., and multiply the quotient by the other number.

EXAMPLES.

Find the L. C. M. of:

2. 289 and 323.

3. 629 and 703.

4. 551 and 589.

5. 667 and 713.

6. 841 and 899.

7. 299 and 529.

8. 361 and 437.

9. 391 and 493.

10. 403 and 961.

11. 533 and 1189.

12. 1403 and 1817.

13. 6649 and 7957.

75. The L. C. M. of *three* numbers which cannot be readily factored, may be found as follows:

Let A, B, and C represent the numbers.

Let M represent the L. C. M. of A and B; then, every common multiple of M and C is also a common multiple of A, B, and C.

But every common multiple of A and B exactly contains M.

Whence, every common multiple of A, B, and C is also a common multiple of M and C.

Therefore, the *least* common multiple of A, B, and C is the same as the *least* common multiple of M and C.

Hence, *to find the L. C. M. of three numbers, find the L. C. M. of two of them, and then of this result and the third number.*

We proceed in a similar manner to find the L. C. M. of four or more numbers.

1. Find the L. C. M. of 713, 1081, and 1395.

We first find the L. C. M. of 713 and 1081, which is 33511.
We then find the L. C. M. of 1395 and 33511, which is 1507995, *Ans.*

EXAMPLES.

Find the L. C. M. of:

2. 1271, 1674, and 1968. **3.** 1505, 1591, and 2109.

IX. FRACTIONS.

76. If unity is divided into 4 equal parts, and 3 parts are taken, the result is expressed by $\frac{3}{4}$; read *"three-fourths."*

If unity is divided into any number of equal parts, and any number of parts are taken, the result is called a **Fraction.**

77. The **Denominator** of a fraction is the number which shows into how many equal parts unity is divided, and the **Numerator** is the number which shows how many parts are taken.

Thus, in the fraction $\frac{3}{4}$, the denominator is 4, and the numerator is 3.

The numerator and denominator are called the **Terms** of the fraction.

78. A fraction is usually expressed by writing the numerator above, and the denominator below, a horizontal line; and when thus expressed, it is called a **Common Fraction.**

79. A **Mixed Number** is the sum of a whole number and a fraction.

Thus, $5 + \frac{3}{4}$, or, as it is usually written, $5\frac{3}{4}$, is a mixed number.

80. Let each of the lines AB, BC, and CD, in the following figure, represent one unit; then AD will represent 3 units.

Let AB be divided into 4 equal parts; AE, EF, FG, and GB; then the fraction $\frac{3}{4}$ will be represented by AG.

Now it is evident that, if AD be divided into 4 equal parts, one of these parts will be AG.

Hence, the fraction $\frac{3}{4}$ represents the result obtained by dividing 3 units by 4.

And in general, any fraction is an expression of *division;* the numerator answering to the dividend, and the denominator to the divisor.

81. It follows from Art. 80 that an *integer* may be expressed in a fractional form by writing 1 for a denominator.

Thus, 3 is the same as $\frac{3}{1}$.

82. A **Proper Fraction** is one whose numerator is less than its denominator; as $\frac{3}{4}$.

An **Improper Fraction** is one whose numerator is equal to or greater than its denominator; as $\frac{5}{5}$, or $\frac{10}{7}$.

REDUCTION OF FRACTIONS.

83. To Reduce an Improper Fraction to a Whole or Mixed Number.

1. Reduce $\frac{54}{9}$ to a whole number.

Since a fraction is an expression of division (Art. 80),

$$\tfrac{54}{9} = 54 \div 9 = 6, \textit{ Ans.}$$

2. Reduce $\frac{290}{23}$ to a mixed number.

Since 290 is equal to the sum of 276 and 14, we have

$$\tfrac{290}{23} = \tfrac{276}{23} + \tfrac{14}{23} = 12 + \tfrac{14}{23}, \text{ or } 12\tfrac{14}{23}, \textit{ Ans.}$$

It is customary to perform the work as follows:

$$\begin{array}{l} 23)290(12\tfrac{14}{23}, \textit{ Ans.} \\ \quad\ \ \underline{23} \\ \quad\ \ \ \ 60 \\ \quad\ \ \ \ \underline{46} \\ \quad\ \ \ \ 14, \text{ Remainder.} \end{array}$$

From the above examples, we derive the following

RULE.

Divide the numerator by the denominator.

If there is a remainder, write it over the divisor, and add the fraction thus formed to the quotient.

EXAMPLES.

Reduce each of the following to a whole or mixed number:

3. $\frac{45}{13}$.	7. $\frac{2057}{58}$.	11. $\frac{3187}{45}$.	15. $\frac{12369}{85}$.
4. $\frac{279}{31}$.	8. $\frac{2169}{76}$.	12. $\frac{6027}{92}$.	16. $\frac{12412}{151}$.
5. $\frac{597}{40}$.	9. $\frac{2806}{61}$.	13. $\frac{6783}{74}$.	17. $\frac{18591}{262}$.
6. $\frac{1049}{22}$.	10. $\frac{2858}{29}$.	14. $\frac{10465}{123}$.	18. $\frac{20403}{380}$.

84. To Reduce a Whole Number to a Fraction having a given Denominator.

1. Reduce 5 to sevenths.

Since 1 is equal to 7 sevenths, 5 is equal to 5 times 7 sevenths, or 35 sevenths; whence, $5 = \frac{35}{7}$, *Ans.*

From the above example, we derive the following

RULE.

To reduce a whole number to a fraction having a given denominator, multiply the whole number by the denominator, and write the result as the numerator of the required fraction.

EXAMPLES.

2. Reduce 6 to 8ths.
3. Reduce 13 to 6ths.
4. Reduce 11 to 9ths.
5. Reduce 16 to 12ths.
6. Reduce 22 to 18ths.
7. Reduce 19 to 15ths.
8. Reduce 31 to 24ths.
9. Reduce 48 to 37ths.

85. To Reduce a Mixed Number to an Improper Fraction.

1. Reduce $9\frac{7}{8}$ to an improper fraction.

Since 9 is equal to 72 eighths, $9\frac{7}{8}$ is equal to the sum of 72 eighths and 7 eighths, which is 79 eighths; whence, $9\frac{7}{8} = \frac{79}{8}$, *Ans.*

From the above example, we derive the following

RULE.

Multiply the whole number by the denominator of the fraction; add to the product the numerator of the fraction, and write the result over the given denominator.

EXAMPLES.

Reduce each of the following to an improper fraction:

2. $8\frac{10}{11}$.	6. $7\frac{23}{48}$.	10. $54\frac{13}{28}$.	14. $74\frac{17}{66}$.
3. $10\frac{7}{9}$.	7. $9\frac{45}{73}$.	11. $29\frac{31}{64}$.	15. $96\frac{50}{57}$.
4. $11\frac{7}{10}$.	8. $40\frac{7}{15}$.	12. $58\frac{35}{36}$.	16. $127\frac{21}{82}$.
5. $13\frac{1}{12}$.	9. $79\frac{33}{50}$.	13. $37\frac{43}{75}$.	17. $156\frac{74}{93}$.

86. To Reduce a Fraction to its Lowest Terms.

A fraction is said to be in its **Lowest Terms** when its numerator and denominator have no common factor.

87. Let the line AG, in the following figure, represent one unit; and let it be divided into 6 equal parts, AB, BC, CD, DE, EF, and FG.

Then the fraction $\frac{4}{6}$ will be represented by AE.

But since the divisions AC, CE, and EG are all equal, the line AE also represents the fraction $\frac{2}{3}$.

Hence, the fraction $\frac{4}{6}$ is equal to $\frac{2}{3}$.

Now the fraction $\frac{2}{3}$ may be obtained from $\frac{4}{6}$ by dividing both numerator and denominator by 2.

Hence, if both numerator and denominator of $\frac{4}{6}$ be divided by 2, the value of the fraction is not changed.

And in general, *if both numerator and denominator of any fraction be divided by the same number, the value of the fraction is not changed.*

88. In reducing fractions to their lowest terms, we may distinguish two cases:

89. Case I. *When the numerator and denominator can be readily factored.*

Since both numerator and denominator can be divided by the same number without changing the value of the fraction (Art. 87), we have the following

RULE.

Divide both numerator and denominator by any common factor.

The greater the common divisor used, the more rapid will be the process.

1. Reduce $\frac{198}{252}$ to its lowest terms.

$\frac{198}{252} = \frac{99}{126} = \frac{33}{42} = \frac{11}{14}$, *Ans.*

Dividing both terms of $\frac{198}{252}$ by 2, the result is $\frac{99}{126}$.
Dividing both terms of $\frac{99}{126}$ by 3, the result is $\frac{33}{42}$.
Dividing both terms of $\frac{33}{42}$ by 3, the result is $\frac{11}{14}$.

If all the factors of the *numerator* be removed by division, 1 remains to form a numerator.

If all the factors of the *denominator* be removed, the result is a *whole number*, this being a case of exact division.

EXAMPLES.

Reduce each of the following to its lowest terms:

2. $\frac{90}{135}$.	**6.** $\frac{209}{330}$.	**10.** $\frac{616}{1056}$.	**14.** $\frac{2940}{8085}$.
3. $\frac{22}{132}$.	**7.** $\frac{288}{2304}$.	**11.** $\frac{660}{1638}$.	**15.** $\frac{2700}{1125}$.
4. $\frac{126}{18}$.	**8.** $\frac{308}{176}$.	**12.** $\frac{1936}{2662}$.	**16.** $\frac{2592}{5832}$.
5. $\frac{539}{154}$.	**9.** $\frac{1155}{550}$.	**13.** $\frac{2688}{2142}$.	**17.** $\frac{6615}{2646}$.

90. Cancellation.

Cancellation is the process of dividing both numerator and denominator by *striking out their common factors.*

It is useful in cases when either the numerator or denominator is expressed in the form of a product.

1. Reduce $\frac{21 \times 12 \times 15}{20 \times 14 \times 36}$ to its lowest terms.

$$\frac{\overset{3}{\cancel{\overset{3}{\cancel{21}}}} \times \cancel{12} \times \overset{3}{\cancel{15}}}{\underset{4}{\cancel{20}} \times \underset{2}{\cancel{14}} \times \underset{\cancel{3}}{\cancel{36}}} = \frac{3}{4 \times 2} = \frac{3}{8}, \quad \textit{Ans.}$$

Cancelling 7 from 21 and 14, we write 3 above 21, and 2 below 14.

Cancelling 5 from 15 and 20, we write 3 above 15, and 4 below 20.

Cancelling 12 from 12 and 36, we write 3 below 36.
We then cancel the 3 above 21 with the 3 below 36.

The result is $\frac{3}{4 \times 2}$, or $\frac{3}{8}$.

EXAMPLES.

Reduce each of the following to its lowest terms:

2. $\frac{49 \times 88}{55 \times 112}$.

3. $\frac{46 \times 63}{7 \times 23 \times 30}$.

4. $\frac{20 \times 108}{18 \times 28 \times 36}$.

5. $\frac{16 \times 95 \times 96}{114 \times 128}$.

6. $\frac{21 \times 39 \times 55}{20 \times 26 \times 33}$.

7. $\frac{8 \times 12 \times 14 \times 15}{10 \times 16 \times 18 \times 21}$.

8. $\frac{34 \times 38 \times 39}{57 \times 85 \times 91}$.

9. $\frac{27 \times 77 \times 105}{135 \times 165}$.

10. $\frac{21 \times 26 \times 52}{39 \times 56 \times 117}$.

11. $\frac{54 \times 84 \times 270}{50 \times 162 \times 196}$.

91. Case II. *When the numerator and denominator cannot be readily factored.*

Since the G. C. D. of the numerator and denominator is the greatest number that will exactly divide each of them, we have the following

RULE.

Divide both numerator and denominator by their greatest common divisor.

1. Reduce $\frac{247}{323}$ to its lowest terms.

$$\begin{array}{l}
247)323(1\\
\quad\ \ \underline{247}\\
\qquad 76)247(3\\
\qquad\quad\ \ \underline{228}\\
\text{G. C. D.} = 19)76(4\\
\qquad\qquad\quad\ \ \underline{76}
\end{array}$$

$$\begin{array}{ll}
19)247(13 & 19)323(17\\
\ \ \underline{19} & \ \ \underline{19}\\
\ \ \ 57 & \ \ 133\\
\ \ \ \underline{57} & \ \ \underline{133}
\end{array}$$

$\frac{13}{17}$, *Ans.*

We find, by the rule of Art. 87, that the G. C. D. of 247 and 323 is 19.

Dividing 247 by 19, the quotient is 13.

Dividing 323 by 19, the quotient is 17.

Then the required result is $\frac{13}{17}$.

EXAMPLES.

Reduce each of the following to its lowest terms:

2. $\frac{119}{289}$. 4. $\frac{527}{899}$. 6. $\frac{1073}{1147}$. 8. $\frac{1147}{703}$. 10. $\frac{5068}{8071}$.

3. $\frac{437}{299}$. 5. $\frac{667}{377}$. 7. $\frac{2231}{2599}$. 9. $\frac{6818}{8458}$. 11. $\frac{3431}{5969}$.

92. To Reduce Fractions to their Least Common Denominator.

Fractions are said to have a **Common Denominator** when they all have the same denominator.

To reduce fractions to their **Least Common Denominator** (L. C. D.) is to express them as equivalent fractions, having for their common denominator the *least common multiple of the given denominators.*

93. It was shown in Art. 87 that the fraction $\frac{2}{3}$ is equal to $\frac{4}{6}$.

But the fraction $\frac{4}{6}$ may be obtained from $\frac{2}{3}$ by multiplying both numerator and denominator by 2.

Hence, *if both numerator and denominator of any fraction be multiplied by the same number, the value of the fraction is not changed.*

94. 1. Reduce $\frac{5}{6}$, $\frac{7}{10}$, and $\frac{11}{15}$ to their least common denominator.

By Art. 73, the L. C. M. of 6, 10, and 15 is 30.

Now, by Art. 93, both terms of a fraction may be multiplied by the same number without changing the value of the fraction.

Multiplying both terms of $\frac{5}{6}$ by 5, both terms of $\frac{7}{10}$ by 3, and both terms of $\frac{11}{15}$ by 2, the given fractions become

$\frac{25}{30}$, $\frac{21}{30}$, and $\frac{22}{30}$, *Ans.*

It will be observed that the terms of each fraction are multiplied by a number which is obtained by dividing the least common denominator by its own denominator; hence the following

RULE.

Find the L. C. M. of the given denominators.

Divide this by each denominator separately, multiply the respective numerators by the quotients, and write the results over the common denominator.

If the given denominators are *prime to each other* (Art. 64), the least common denominator is *the product of all the denominators;* and each numerator is multiplied by all the denominators except its own.

2. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to their least common denominator.

The L. C. D. is $3 \times 4 \times 5$, or 60.

Multiplying each numerator by all the denominators except its own, the fractions become

$\frac{40}{60}$, $\frac{45}{60}$, and $\frac{48}{60}$, *Ans.*

EXAMPLES.

Reduce to their least common denominator:

3. $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{6}{7}$.

4. $\frac{4}{5}$, $\frac{2}{9}$, and $\frac{14}{15}$.

5. $\frac{5}{6}$, $\frac{6}{7}$, and $\frac{8}{9}$.

6. $\frac{31}{24}$, $\frac{21}{32}$, and $\frac{25}{48}$.

7. $\frac{3}{10}$, $\frac{13}{14}$, and $\frac{22}{35}$.

8. $\frac{7}{12}$, $\frac{9}{20}$, and $\frac{19}{30}$.

9. $\frac{11}{12}$, $\frac{15}{28}$, and $\frac{25}{42}$.

10. $\frac{5}{24}$, $\frac{23}{80}$, $\frac{17}{40}$, and $\frac{37}{60}$.

11. $\frac{9}{10}$, $\frac{22}{15}$, $\frac{19}{18}$, and $\frac{47}{90}$.

12. $\frac{7}{15}$, $\frac{11}{30}$, $\frac{21}{50}$, and $\frac{28}{75}$.

13. $\frac{9}{10}$, $\frac{5}{12}$, $\frac{15}{14}$, $\frac{16}{21}$, and $\frac{23}{35}$.

14. $\frac{35}{18}$, $\frac{21}{25}$, $\frac{19}{45}$, $\frac{13}{60}$, and $\frac{8}{75}$.

The relative magnitude of fractions may be determined by reducing them, if necessary, to their least common denominator.

15. Which of the fractions, $\frac{1}{3}$ and $\frac{3}{8}$, is the greater?

We have, $\frac{1}{3}=\frac{8}{24}$, and $\frac{3}{8}=\frac{9}{24}$.

It is evident from this that $\frac{3}{8}$ is greater than $\frac{1}{3}$.

Arrange in order of magnitude:

16. $\frac{5}{11}$ and $\frac{8}{17}$.

17. $\frac{7}{10}$, $\frac{3}{5}$, and $\frac{2}{3}$.

18. $\frac{5}{7}$, $\frac{13}{21}$, and $\frac{23}{35}$.

19. $\frac{3}{16}$, $\frac{2}{9}$, and $\frac{7}{36}$.

95. To reduce a fraction to an equivalent fraction having any required denominator, divide the required denominator by the given denominator, and multiply both terms of the given fraction by the result.

1. Reduce $\frac{11}{15}$ to 165ths.

Dividing 165 by 15, the quotient is 11.

Multiplying both terms of $\frac{11}{15}$ by 11, the result is $\frac{121}{165}$, *Ans.*

EXAMPLES.

2. Reduce $\frac{8}{13}$ to 78ths.

3. Reduce $\frac{5}{18}$ to 126ths.

4. Reduce $\frac{15}{28}$ to 224ths.

5. Reduce $\frac{17}{25}$ to 375ths.

6. Reduce $\frac{23}{36}$ to 504ths.

7. Reduce $\frac{13}{24}$ to 576ths.

ADDITION OF FRACTIONS.

96. 1. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.

The L. C. M. of 3, 4, and 6 is 12.

Reducing each fraction to 12ths, we have

$$\frac{2}{3}+\frac{3}{4}+\frac{5}{6}=\frac{8}{12}+\frac{9}{12}+\frac{10}{12}=\frac{27}{12}=\frac{9}{4},\ \textit{Ans.}$$

From the above example, we derive the following

RULE.

To add two or more fractions, reduce them, if necessary, to their least common denominator.

Add the numerators of the resulting fractions, and write the result over the common denominator.

The final result should be reduced to its *lowest terms.*

To add two or more *mixed* numbers, first add the whole numbers, and then the fractions, and then find the sum of these results.

2. Find the sum of $3\frac{1}{6}$, $1\frac{3}{10}$, 5, and $2\frac{11}{15}$.

$3+1+5+2=11.$

$\frac{1}{6}+\frac{3}{10}+\frac{11}{15}=\frac{5}{30}+\frac{9}{30}+\frac{22}{30}$
$=\frac{36}{30}=\frac{6}{5}=1\frac{1}{5}.$

$11+1\frac{1}{5}=12\frac{1}{5}$, *Ans.*

The sum of the whole numbers 3, 1, 5, and 2, is 11.

The sum of the fractions $\frac{1}{6}$, $\frac{3}{10}$, and $\frac{11}{15}$, is $\frac{6}{5}$, or $1\frac{1}{5}$.

Then the sum of 11 and $1\frac{1}{5}$ is $12\frac{1}{5}$.

EXAMPLES.

Find the values of the following:

3. $\frac{7}{8}+\frac{11}{13}$.
4. $\frac{2}{15}+\frac{5}{21}$.
5. $1\frac{11}{20}+2\frac{17}{60}$.
6. $5\frac{11}{12}+3\frac{16}{21}$.
7. $4\frac{4}{15}+7\frac{5}{16}$.
8. $\frac{3}{4}+\frac{2}{9}+\frac{5}{18}$.
9. $\frac{2}{3}+\frac{7}{12}+\frac{8}{15}$.
10. $\frac{1}{15}+\frac{7}{33}+\frac{14}{55}$.
11. $\frac{9}{10}+\frac{15}{22}+\frac{23}{35}$.
12. $1\frac{2}{3}+1\frac{5}{6}+1\frac{8}{9}$.
13. $1\frac{1}{3}+2\frac{3}{5}+3\frac{9}{10}$.
14. $7\frac{2}{3}+3\frac{5}{6}+5\frac{5}{7}$. 78
15. $\frac{2}{3}+\frac{3}{4}+\frac{4}{7}+\frac{7}{12}$.
16. $\frac{5}{12}+\frac{6}{18}+\frac{4}{27}+\frac{7}{36}$. 130
17. $\frac{9}{14}+\frac{7}{15}+\frac{10}{21}+\frac{13}{35}$. 168
18. $\frac{1}{3}+\frac{3}{8}+\frac{7}{24}+\frac{5}{27}$.
19. $6\frac{11}{18}+4\frac{23}{30}+8\frac{31}{45}$. 360
20. $4\frac{4}{15}+9\frac{17}{24}+13\frac{9}{40}$.
21. $11\frac{1}{6}+21\frac{29}{50}+14\frac{44}{75}$. 473
22. $4+3\frac{5}{9}+2\frac{5}{18}+1\frac{5}{27}$. 11 1/54
23. $2+6\frac{2}{3}+8\frac{5}{6}+4\frac{6}{13}$. 21 25/26
24. $3\frac{1}{3}+5\frac{3}{8}+\frac{7}{24}+1\frac{7}{32}$. 10 7/32
25. $6\frac{1}{3}+5\frac{1}{6}+4\frac{5}{8}+3\frac{3}{10}$.
26. $3\frac{1}{4}+9\frac{1}{6}+7\frac{3}{8}+5\frac{4}{21}$.
27. $2\frac{3}{25}+4\frac{7}{36}+6\frac{4}{45}+8\frac{14}{75}$.
28. $\frac{1}{4}+\frac{2}{5}+\frac{5}{6}+\frac{3}{8}+\frac{7}{9}$.
29. $\frac{3}{14}+\frac{9}{16}+\frac{1}{21}+\frac{7}{24}+\frac{5}{28}$. 205/144

336.

30. $7\frac{3}{7} + 9\frac{5}{14} + 2\frac{8}{21} + 12\frac{31}{84} + 3\frac{13}{35}$.

31. $13\frac{1}{4} + 16\frac{2}{5} + 19\frac{3}{8} + 22\frac{7}{10} + 25\frac{13}{20}$.

32. $34\frac{1}{6} + 17\frac{2}{9} + 28\frac{3}{10} + 40\frac{4}{15} + 52\frac{5}{18}$. 172 $\frac{7}{30}$

SUBTRACTION OF FRACTIONS.

97. 1. Subtract $\frac{9}{14}$ from $\frac{5}{6}$.

Reducing each fraction to 42ds, we have

$$\frac{5}{6} - \frac{9}{14} = \frac{35}{42} - \frac{27}{42} = \frac{8}{42} = \frac{4}{21}, \ \textit{Ans.}$$

From the above example, we derive the following

RULE.

To subtract one fraction from another, reduce them, if necessary, to their least common denominator.

Subtract the numerator of the subtrahend from that of the minuend, and write the result over the common denominator.

The final result should be reduced to its lowest terms.

To subtract one mixed number from another, first subtract the integers, and then the fractions, and then find the sum of these results.

2. Subtract $3\frac{1}{6}$ from $5\frac{5}{9}$.

$5 - 3 = 2.$

$\frac{5}{9} - \frac{1}{6} = \frac{10}{18} - \frac{3}{18} = \frac{7}{18}.$

$2 + \frac{7}{18} = 2\frac{7}{18}$, *Ans.*

Subtracting 3 from 5, the result is 2. Subtracting $\frac{1}{6}$ from $\frac{5}{9}$, the result is $\frac{7}{18}$. Then the sum of 2 and $\frac{7}{18}$ is $2\frac{7}{18}$.

If the fractional part of the subtrahend exceeds the fractional part of the minuend, increase the latter by 1, subtracting 1 from the integral part of the minuend to compensate.

3. Subtract $3\frac{5}{9}$ from $5\frac{1}{6}$.

$5\frac{1}{6} = 4\frac{7}{6}.$

$4 - 3 = 1.$

$\frac{7}{6} - \frac{5}{9} = \frac{21}{18} - \frac{10}{18} = \frac{11}{18}.$

$1\frac{11}{18}$, *Ans.*

Since $\frac{5}{9}$ is greater than $\frac{1}{6}$, we subtract 1 from 5, leaving 4, and then add $\frac{6}{6}$ to the $\frac{1}{6}$, giving $\frac{7}{6}$; thus, $5\frac{1}{6}$ is the same as $4\frac{7}{6}$.

4. Subtract $2\frac{5}{8}$ from 7.

$$7-2\frac{5}{8}=6\frac{8}{8}-2\frac{5}{8}=4\frac{3}{8},\ \textit{Ans.}$$

EXAMPLES.

Find the values of the following:

5. $\frac{5}{12}-\frac{3}{20}$.
6. $\frac{7}{12}-\frac{4}{21}$.
7. $\frac{7}{18}-\frac{2}{15}$.
8. $2-\frac{8}{17}$.
9. $5-3\frac{18}{81}$.
10. $\frac{23}{42}-\frac{7}{18}$.
11. $9-\frac{15}{23}$.
12. $11-7\frac{25}{59}$.
13. $\frac{9}{20}-\frac{4}{45}$.
14. $\frac{7}{6}-\frac{29}{39}$.
15. $\frac{21}{40}-\frac{5}{24}$.
16. $\frac{40}{63}-\frac{11}{18}$.
17. $\frac{29}{35}-\frac{13}{20}$.
18. $8\frac{7}{10}-2\frac{3}{14}$.
19. $7\frac{11}{42}-6\frac{7}{12}$.
20. $1\frac{7}{15}-\frac{24}{65}$.
21. $12\frac{14}{15}-5\frac{11}{35}$.
22. $14\frac{23}{45}-10\frac{19}{27}$.
23. $18\frac{7}{30}-8\frac{11}{24}$.
24. $16\frac{19}{48}-9\frac{21}{32}$.
25. $23\frac{10}{21}-17\frac{14}{15}$.
26. $12\frac{10}{27}-\frac{17}{18}$.
27. $33\frac{8}{21}-25\frac{7}{33}$.
28. $27\frac{5}{28}-19\frac{11}{20}$.
29. $17\frac{16}{39}-4\frac{18}{65}$.
30. $24\frac{7}{15}-15\frac{10}{33}$.
31. $31\frac{4}{35}-18\frac{17}{63}$.

In finding the value of a series of fractions connected by plus and minus signs, it is better to add all those fractions which are preceded by minus signs, and subtract their sum from the sum of the other fractions.

32. Find the value of $4\frac{19}{24}-3\frac{7}{12}+2\frac{15}{16}-1\frac{11}{48}$.

$$\begin{aligned} 4\tfrac{19}{24}+2\tfrac{15}{16}&=6\tfrac{38+45}{48}=6\tfrac{83}{48}\\ 3\tfrac{7}{12}+1\tfrac{11}{48}&=4\tfrac{28+11}{48}=\underline{4\tfrac{39}{48}}\\ &\qquad\qquad 2\tfrac{44}{48}=2\tfrac{11}{12},\ \textit{Ans.}\end{aligned}$$

The sum of the fractions $4\frac{19}{24}$ and $2\frac{15}{16}$ is $6\frac{83}{48}$.
The sum of the fractions $3\frac{7}{12}$ and $1\frac{11}{48}$ is $4\frac{39}{48}$.
Subtracting $4\frac{39}{48}$ from $6\frac{83}{48}$, the result is $2\frac{44}{48}$.

Find the values of the following:

33. $5\frac{7}{8}-3\frac{1}{6}-2\frac{7}{12}$.
34. $4\frac{2}{3}-1\frac{1}{2}+8\frac{5}{6}-7\frac{3}{4}$.
35. $9\frac{2}{5}+2\frac{1}{3}-5\frac{7}{15}-3\frac{5}{9}$.
36. $7\frac{5}{12}-1\frac{1}{4}-1\frac{1}{6}-1\frac{3}{5}$.
37. $9\frac{11}{12}-3\frac{15}{32}+7\frac{5}{6}+\frac{5}{16}-5\frac{3}{8}$.

38. $25\frac{1}{12} + 20\frac{3}{7} - 17\frac{11}{28} - 13\frac{8}{21} - 8\frac{9}{14}$.

39. $52\frac{5}{8} - 15\frac{7}{12} - 3\frac{2}{9} - 26\frac{13}{28} + 9\frac{23}{24}$.

40. $12\frac{19}{30} - 6\frac{8}{15} - 3\frac{33}{50} + 7\frac{41}{75} - \frac{11}{25} + 8\frac{9}{10}$.

41. $32\frac{35}{72} + 17\frac{5}{18} - 18\frac{9}{16} - 14\frac{7}{24} + 20\frac{17}{48} - 10\frac{13}{36}$.

42. $21\frac{13}{14} - 11\frac{7}{15} - 5\frac{33}{70} + 58\frac{37}{42} - 12\frac{22}{35} + \frac{67}{105} - 19\frac{16}{21}$.

MULTIPLICATION OF FRACTIONS.

98. To Multiply a Fraction by a Whole Number.

Let the line AD, in the following figure, represent one unit; and let it be divided into 8 equal parts.

Then the fraction $\frac{3}{8}$ will be represented by AB, and $\frac{6}{8}$ by AC.

But AC is twice AB; hence, the fraction $\frac{6}{8}$ is twice $\frac{3}{8}$.

Now $\frac{6}{8}$ may be obtained from $\frac{3}{8}$ by multiplying its numerator by 2; hence, *if the numerator of a fraction be multiplied by any number, the fraction is multiplied by that number.*

Again, since $\frac{6}{8}$ is equal to $\frac{3}{4}$, the fraction $\frac{3}{4}$ is twice $\frac{3}{8}$.

But $\frac{3}{4}$ may be obtained from $\frac{3}{8}$ by dividing its denominator by 2; hence, *if the denominator of a fraction be divided by any number, the fraction is multiplied by that number.*

99. We derive from Art. 98 the following rule for multiplying a fraction by a whole number:

If possible, divide the denominator by the whole number; otherwise, multiply the numerator by the whole number.

1. Multiply $\frac{3}{10}$ by 5.

Dividing the denominator by 5, we have

$$\frac{3}{10} \times 5 = \frac{3}{2}, \ \textit{Ans.}$$

2. Multiply $\frac{5}{9}$ by 4.

Multiplying the numerator by 4, we have

$$\frac{5}{9} \times 4 = \frac{20}{9}, \text{ Ans.}$$

Common factors in the whole number and the denominator of the fraction should be cancelled (Art. 90) before performing the multiplication.

3. Multiply $\frac{13}{27}$ by 18.

$$\frac{13}{\underset{3}{\cancel{27}}} \times \overset{2}{\cancel{18}} = \frac{13}{3} \times 2 = \frac{26}{3}, \text{ Ans.}$$

In this case, we cancel 9 from 18 and 27.

Note. To multiply a whole number by a fraction is the same as multiplying the fraction by the whole number.

Thus, $5 \times \frac{4}{9}$ is the same as $\frac{4}{9} \times 5$.

To multiply a *mixed number* by an integer, multiply the whole number and the fraction separately, and then find the sum of these results.

4. Multiply $3\frac{11}{16}$ by 12.

$$3 \times 12 = 36.$$

$$\frac{11}{\underset{4}{\cancel{16}}} \times \overset{3}{\cancel{12}} = \frac{33}{4} = 8\tfrac{1}{4}.$$

$$36 + 8\tfrac{1}{4} = 44\tfrac{1}{4}, \text{ Ans.}$$

Multiplying 3 by 12, the product is 36.

Multiplying $\frac{11}{16}$ by 12, the product is $\frac{33}{4}$, or $8\frac{1}{4}$.

Then the sum of 36 and $8\frac{1}{4}$ is $44\frac{1}{4}$.

EXAMPLES.

Find the values of the following:

5. $\frac{23}{45} \times 9.$

6. $\frac{6}{11} \times 8.$

7. $\frac{17}{28} \times 42.$

8. $75 \times \frac{14}{45}.$

9. $117 \times \frac{10}{13}.$

10. $\frac{11}{60} \times 80.$

11. $66 \times \frac{15}{44}.$

12. $\frac{19}{96} \times 72.$

13. $4\frac{5}{24} \times 3.$

14. $8\frac{2}{7} \times 6.$

15. $9 \times 7\frac{18}{108}.$

16. $14\frac{6}{11} \times 33.$

17. $15\frac{3}{10} \times 18.$

18. $35 \times 17\frac{9}{14}.$

19. $25 \times 22\frac{7}{15}.$

20. $16\frac{7}{8} \times 64.$

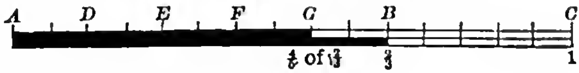
21. $80 \times 18\frac{11}{12}.$

22. $30\frac{43}{49} \times 84.$

100. To Multiply a Fraction by a Fraction.

To multiply $\frac{2}{3}$ by $\frac{4}{5}$ is to *take* $\frac{4}{5}$ *of* $\frac{2}{3}$; that is, *we divide* $\frac{2}{3}$ *into 5 equal parts, and take 4 of them.*

Let the line AC, in the following figure, represent one unit; and let it be divided into 15 equal parts.



Then AB will represent $\frac{10}{15}$, or $\frac{2}{3}$.

Now since AB is divided into 5 equal parts, AD, DE, EF, FG, and GB, AG will represent $\frac{4}{5}$ of $\frac{2}{3}$.

But AG also represents $\frac{8}{15}$.

Hence, $\frac{4}{5}$ of $\frac{2}{3}$, or $\frac{2}{3} \times \frac{4}{5}$, is equal to $\frac{8}{15}$.

101. We derive from Art. 100 the following rule for multiplying one fraction by another:

Multiply the numerators together for the numerator of the product, and the denominators for its denominator.

Common factors in the numerators and denominators should be cancelled before performing the multiplication.

1. Multiply $\frac{3}{5}$ by $\frac{4}{7}$.

$$\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}, \textit{ Ans.}$$

To multiply any number of fractions, we multiply their numerators together for the numerator of the product, and their denominators for its denominator.

2. Find the value of $\frac{6}{7} \times \frac{5}{9}$ of $\frac{11}{15}$.

$$\frac{\cancelto{2}{6}}{7} \times \frac{\cancel{5}}{\cancelto{3}{9}} \times \frac{11}{\cancelto{3}{15}} = \frac{22}{63}, \textit{ Ans.}$$

In this case, we cancel 3 from 6 and 9, and then 5 from 5 and 15.

Mixed numbers should be reduced to a fractional form (Art. 85) before applying the rule.

3. Find the value of $\frac{25}{48}$ of $1\frac{7}{20} \times 2\frac{2}{15} \times 9$.

$\frac{25}{48}$ of $1\frac{7}{20} \times 2\frac{2}{15} \times 9$

$$=\frac{\overset{\overset{}{5}}{\cancel{25}}}{\underset{\underset{}{3}}{\cancel{48}}}\times\frac{\overset{9}{\cancel{27}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{20}}}\times\frac{\overset{\cancel{2}}{\cancel{32}}}{\underset{\cancel{3}}{\cancel{15}}}\times\overset{3}{\cancel{9}}=\frac{27}{2},\ \textit{Ans.}$$

By Art. 85, $1\frac{7}{20}$ is equal to $\frac{27}{20}$, and $2\frac{2}{15}$ to $\frac{32}{15}$.

We first cancel 16 from 48 and 32; then 5 from 25 and 15; then 5 from 5 and 20; then 3 from 3 and 27; then 2 from 2 and 4; and finally 3 from 3 and 9.

EXAMPLES.

Find the values of the following:

4. $\frac{5}{11} \times \frac{7}{4}$.
5. $\frac{33}{68} \times 2\frac{7}{22}$.
6. $1\frac{12}{25} \times 2\frac{16}{37}$.
7. $\frac{15}{38}$ of $2\frac{17}{20}$.
8. $\frac{36}{175}$ of $\frac{100}{63}$.
9. $3\frac{47}{66} \times 1\frac{13}{35}$.
10. $\frac{18}{25} \times \frac{20}{27} \times \frac{45}{64}$.
11. $\frac{21}{25} \times \frac{55}{98}$ of $5\frac{5}{6}$.
12. $\frac{10}{27} \times \frac{12}{25} \times 5\frac{5}{8}$.
13. $\frac{16}{27}$ of $4\frac{1}{20} \times 2\frac{29}{48}$.
14. $\frac{45}{119}$ of $\frac{84}{65}$ of $\frac{68}{27}$.
15. $1\frac{4}{21} \times 2\frac{8}{45} \times 1\frac{1}{35}$.
16. $\frac{7}{8} \times \frac{6}{49} \times \frac{10}{63} \times 28$.
17. $\frac{14}{27}$ of $\frac{9}{70}$ of $26 \times 4\frac{2}{13}$.
18. $\frac{9}{20} \times \frac{15}{56} \times \frac{49}{63}$ of $\frac{14}{225}$.
19. $\frac{11}{12}$ of $\frac{22}{27}$ of $\frac{21}{44}$ of $\frac{36}{121}$.
20. $\frac{77}{125} \times \frac{5}{11} \times \frac{121}{343} \times 35$.
21. $1\frac{21}{44} \times 1\frac{23}{91} \times 2\frac{22}{57} \times 1\frac{26}{51}$.
22. $\frac{9}{49} \times \frac{28}{135} \times 20 \times 2\frac{4}{25} \times 10\frac{15}{16}$.
23. $\frac{15}{22}$ of $\frac{33}{400}$ of $2\frac{16}{27} \times 24 \times 3\frac{3}{14}$.

DIVISION OF FRACTIONS.

102. To Divide a Fraction by a Whole Number.

It is evident from the above figure that the fraction $\frac{3}{8}$ is equal to the fraction $\frac{6}{8}$ divided by 2.

But $\frac{3}{8}$ may be obtained from $\frac{6}{8}$ by dividing its numerator by 2.

Hence, *if the numerator of a fraction be divided by any number, the fraction is divided by that number.*

Again, the fraction $\frac{3}{8}$ is equal to $\frac{3}{4}$ divided by 2.

But $\frac{3}{8}$ may be obtained from $\frac{3}{4}$ by multiplying its denominator by 2; hence, *if the denominator of a fraction be multiplied by any number, the fraction is divided by that number.*

103. We derive from Art. 102 the following rule for dividing a fraction by a whole number:

If possible, divide the numerator by the whole number; otherwise, multiply the denominator by the whole number.

1. Divide $\frac{6}{7}$ by 3.

Dividing the numerator by 3, we have

$$\tfrac{6}{7} \div 3 = \tfrac{2}{7}, \text{ Ans.}$$

2. Divide $\frac{4}{3}$ by 5.

Multiplying the denominator by 5, we have

$$\tfrac{4}{3} \div 5 = \tfrac{4}{15}, \text{ Ans.}$$

To divide a *mixed number* by an integer, the dividend should first be reduced to a fractional form (Art. 85).

3. Divide $6\frac{2}{9}$ by 8.

By Art. 85, $6\frac{2}{9} = \frac{56}{9}$; and $\frac{56}{9} \div 8 = \frac{7}{9}$, *Ans.*

If the integral part of the mixed number is equal to or greater than the divisor, it is better to proceed as follows:

4. Divide $86\frac{3}{5}$ by 12.

$$12\underline{)86\tfrac{3}{5}}$$
$$7\tfrac{13}{60}, \text{ Ans.}$$

12 is contained in $86\frac{3}{5}$ seven times, with a remainder of $2\frac{3}{5}$, or $\frac{13}{5}$.

Dividing $\frac{13}{5}$ by 12, the quotient is $\frac{13}{60}$.

EXAMPLES.

Find the values of the following:

5. $\frac{42}{13} \div 7$.	**8.** $\frac{173}{20} \div 9$.	**11.** $2\frac{11}{12} \div 6$.	**14.** $40\frac{3}{4} \div 11$.
6. $\frac{19}{10} \div 5$.	**9.** $1\frac{9}{25} \div 2$.	**12.** $16\frac{4}{5} \div 7$.	**15.** $9\frac{11}{17} \div 4$.
7. $\frac{132}{43} \div 12$.	**10.** $8\frac{2}{11} \div 10$.	**13.** $10\frac{1}{8} \div 3$.	**16.** $29\frac{5}{7} \div 8$.

104. To Divide a Whole Number or a Fraction by a Fraction.

1. Divide 3 by $\frac{5}{7}$.

We have, $3 \div \frac{5}{7} = \frac{21}{7} \div \frac{5}{7}$.

But the quotient of 21 *sevenths* divided by 5 *sevenths* is the same as the quotient of 21 divided by 5, which is $\frac{21}{5}$.

Therefore, $3 \div \frac{5}{7} = \frac{21}{5}$, *Ans.*

We observe, in the above example, that the quotient may be obtained by *multiplying* 3 *by* $\frac{7}{5}$, *which is the fraction* $\frac{5}{7}$ *inverted;* whence the following

RULE.

To divide a whole number or a fraction by a fraction, invert the divisor, and proceed as in multiplication.

2. Divide $\frac{9}{11}$ by $\frac{8}{9}$.

$$\frac{9}{11} \div \frac{8}{9} = \frac{9}{11} \times \frac{9}{8} = \frac{81}{88}, \text{ Ans.}$$

If the numerator and denominator of the divisor are exactly contained in the numerator and denominator of the dividend, we divide the numerator of the dividend by that of the divisor for the numerator of the quotient, and the denominator of the dividend by that of the divisor for the denominator of the quotient.

3. Divide $\frac{35}{44}$ by $\frac{7}{11}$.

Since $35 \div 7 = 5$, and $44 \div 11 = 4$, we have

$$\frac{35}{44} \div \frac{7}{11} = \frac{5}{4}, \text{ Ans.}$$

If the divisor is an integer, it must be written in a fractional form (Art. 81) before applying the rule.

4. Divide $\frac{36}{25}$ by 63.

$$\frac{36}{25} \div \frac{63}{1} = \frac{\overset{4}{\cancel{36}}}{25} \times \frac{1}{\underset{7}{\cancel{63}}}$$

$$= \frac{4}{175}, \text{ Ans.}$$

By Art. 81, 63 may be written in the form $\frac{63}{1}$; which, when inverted, becomes $\frac{1}{63}$. We cancel 9 from 36 and 63.

If either the dividend or divisor is a mixed number, it must be expressed in a fractional form before applying the rule.

5. Divide 21 by $3\frac{11}{15}$.

$$21 \div 3\frac{11}{15} = 21 \div \frac{56}{15}$$
$$= \overset{3}{\cancel{21}} \times \frac{15}{\underset{8}{\cancel{56}}} = \frac{45}{8},\ \textit{Ans.}$$

$3\frac{11}{15}$ is the same as $\frac{56}{15}$.
We cancel 7 from 21 and 56.

EXAMPLES.

Find the values of the following:

6. $\frac{25}{12} \div \frac{35}{18}$.
7. $27 \div \frac{13}{8}$.
8. $\frac{75}{22} \div 100$.
9. $45 \div 6\frac{9}{11}$.
10. $\frac{29}{34} \div 13\frac{7}{24}$.
11. $3\frac{1}{15} \div 2\frac{19}{25}$.
12. $2\frac{1}{16} \div 12$.
13. $1\frac{19}{35} \div \frac{90}{7}$.
14. $\frac{11}{9} \div 2\frac{3}{7}$.
15. $\frac{63}{52} \div \frac{9}{13}$.
16. $28 \div \frac{42}{11}$.
17. $3\frac{11}{15} \div 96$.
18. $11\frac{4}{9} \div 7\frac{12}{13}$
19. $3\frac{21}{32} \div \frac{81}{44}$.
20. $\frac{110}{21} \div 88$.
21. $85 \div 1\frac{22}{29}$.
22. $18\frac{1}{12} \div 4\frac{10}{13}$.
23. $\frac{36}{85} \div \frac{78}{17}$.
24. $\frac{480}{19} \div 51\frac{1}{5}$.
25. $1\frac{13}{98} \div \frac{74}{161}$.
26. $1\frac{47}{88} \div 1\frac{57}{132}$.
27. $\frac{65}{186} \div 1\frac{19}{31}$.
28. $\frac{161}{33} \div \frac{115}{44}$.
29. $2\frac{15}{119} \div 2\frac{14}{85}$.
30. $\frac{40}{21} \div (\frac{4}{3} - \frac{7}{9})$.
31. $\frac{49}{25} \div (\frac{9}{8} \times \frac{28}{15})$.
32. $(\frac{1}{5} \text{ of } \frac{7}{12}) \times (\frac{2}{3} \div \frac{4}{9})$.
33. $(\frac{5}{18} + \frac{7}{24}) \div \frac{82}{81}$.
34. $(\frac{2}{3} + \frac{3}{4}) \div (\frac{5}{6} - \frac{4}{5})$.
35. $(\frac{2}{3} \div \frac{8}{5}) \div (\frac{7}{4} \div 2\frac{7}{10})$.
36. $(\frac{25}{14} \text{ of } \frac{21}{10}) \times (\frac{49}{30} \div \frac{35}{12})$.
37. $(\frac{7}{6} - \frac{5}{8}) \div (\frac{11}{9} + \frac{7}{12})$.
38. $(\frac{3}{5} \text{ of } 1\frac{3}{4} \times \frac{12}{35}) \div \frac{36}{65}$.
39. $(1\frac{3}{8} \div 4\frac{7}{12}) + (\frac{17}{27} \div 2\frac{4}{15})$.

COMPLEX FRACTIONS.

105. A **Complex Fraction** is one having a fraction in its numerator or denominator, or in both.

A fraction both of whose terms are integers is called a **Simple Fraction**.

A complex fraction may be regarded as a case in division of fractions; and it may be reduced to a simple fraction by the rule of Art. 104.

1. Reduce $\frac{\frac{5}{6}}{\frac{10}{9}}$ to a simple fraction.

$\frac{\frac{5}{6}}{\frac{10}{9}}$ is the same as $\frac{5}{6} \div \frac{10}{9}$; *inverting the denominator*, we have

$$\frac{\frac{5}{6}}{\frac{10}{9}} = \frac{5}{\underset{2}{\cancel{6}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{10}}} = \frac{3}{4}, \textit{ Ans.}$$

Another method is to *multiply both numerator and denominator by the least common multiple of their denominators.*

Thus Ex. 1 may be solved as follows:

The L.C.M. of 6 and 9 is 18; then multiplying both numerator and denominator by 18 (Art. 93), we have

$$\frac{\frac{5}{6}}{\frac{10}{9}} = \frac{\frac{5}{6} \times 18}{\frac{10}{9} \times 18} = \frac{15}{20} = \frac{3}{4}, \textit{ Ans.}$$

2. Simplify $\frac{2\frac{5}{29}}{42}$.

$$\frac{2\frac{5}{29}}{42} = \frac{\frac{63}{29}}{\frac{42}{1}} = \frac{\overset{3}{\cancel{63}}}{29} \times \frac{1}{\underset{2}{\cancel{42}}} = \frac{3}{58}, \textit{ Ans.}$$

3. Simplify $\frac{3\frac{1}{3} + 1\frac{7}{8}}{5\frac{5}{6} - 3\frac{3}{4}}$.

The L. C. M. of the denominators 3, 8, 6, and 4, is 24. Multiplying each term of the fraction by 24, we have

$$\frac{80 + 45}{140 - 90} = \frac{125}{50} = \frac{5}{2}, \textit{ Ans.}$$

EXAMPLES.

Simplify the following:

4. $\dfrac{\frac{14}{25}}{\frac{21}{20}}$.
7. $\dfrac{56}{3\frac{8}{9}}$.
10. $\dfrac{51}{\frac{84}{11}}$.
13. $\dfrac{1\frac{13}{15}}{2\frac{16}{27}}$.

5. $\dfrac{2\frac{2}{5}}{4\frac{1}{4}}$.
8. $\dfrac{\frac{25}{18}}{\frac{55}{12}}$.
11. $\dfrac{11\frac{4}{7}}{6\frac{3}{13}}$.
14. $\dfrac{\frac{32}{63}}{\frac{80}{147}}$.

6. $\dfrac{\frac{39}{8}}{52}$.
9. $\dfrac{1\frac{2}{33}}{2\frac{5}{22}}$.
12. $\dfrac{\frac{56}{57}}{\frac{75}{76}}$.
15. $\dfrac{\frac{66}{185}}{2\frac{17}{74}}$.

16. $\dfrac{\frac{5}{2}+\frac{11}{3}}{\frac{31}{4}+\frac{26}{5}}$.
17. $\dfrac{5\frac{3}{4}-1\frac{1}{6}}{4\frac{2}{3}-3\frac{5}{8}}$.
18. $\dfrac{2\frac{1}{15}+1\frac{5}{6}}{8-5\frac{2}{5}}$.

19. $\dfrac{\frac{4}{7} \text{ of } \frac{8}{9}}{\frac{25}{21} \text{ of } 2\frac{2}{7}}$.
22. $\dfrac{6\frac{1}{2}-4\frac{1}{8}+2\frac{4}{5}}{7\frac{3}{4}+3\frac{9}{10}-8\frac{1}{5}}$.

20. $\dfrac{1\frac{3}{14}}{2\frac{8}{11}} \div \dfrac{4\frac{8}{15}}{10\frac{1}{2}}$.
23. $\dfrac{3\frac{17}{24}+2\frac{3}{4}+5\frac{5}{12}}{10\frac{9}{16}-1\frac{5}{6}-3\frac{5}{8}}$.

21. $\dfrac{\frac{3}{5} \text{ of } \frac{8}{7}-\frac{2}{15} \text{ of } \frac{10}{7}}{\frac{8}{9} \text{ of } \frac{3}{10}+\frac{24}{25} \text{ of } \frac{15}{14}}$.
24. $\dfrac{\frac{9}{20}-\frac{13}{30}+\frac{29}{60}-\frac{5}{12}}{\frac{11}{24}+\frac{27}{40}-\frac{3}{10}-\frac{7}{15}}$.

25. $\dfrac{\frac{5}{18} \text{ of } 1\frac{7}{25}+\frac{8}{9} \text{ of } 1\frac{5}{12}}{\frac{4}{9} \text{ of } 2\frac{1}{10}-\frac{5}{6} \text{ of } \frac{8}{15}}$.

106. The **Reciprocal** of a number is 1 divided by that number.

Thus, the reciprocal of 5 is $\frac{1}{5}$.

The reciprocal of $\frac{7}{8}$ is $\dfrac{1}{\frac{7}{8}}$, or $\frac{8}{7}$.

That is, *the reciprocal of a fraction is the fraction inverted.*

107. To find what Fraction one Number is of another.

1. What fraction of 21 is 14?

Since 1 is $\frac{1}{21}$ of 21, 14 is 14 times $\frac{1}{21}$ of 21, or $\frac{14}{21}$ of 21.
Result, $\frac{14}{21}$, or $\frac{2}{3}$.

From the above example, we derive the following

RULE.

Make the first number the denominator, and the second the numerator, of a fraction.

EXAMPLES.

What fraction of:

2. 36 is 27?
3. 49 is 70?
4. $\frac{27}{20}$ is $\frac{12}{25}$?
5. $4\frac{1}{2}$ is $3\frac{3}{4}$?
6. $\frac{12}{35}$ is $\frac{16}{21}$?
7. $6\frac{1}{4}$ is $11\frac{2}{3}$?
8. $2\frac{4}{7}$ is 24?
9. 40 is $5\frac{5}{11}$?
10. $\frac{34}{57}$ is $\frac{51}{38}$?
11. $4\frac{8}{11}$ is $7\frac{3}{7}$?
12. $2\frac{8}{45}$ is $2\frac{1}{10}$?
13. 90 is $4\frac{2}{13}$?
14. $\frac{11}{20}$ is $1\frac{47}{52}$?
15. $13\frac{10}{11}$ is 68?
16. $1\frac{26}{135}$ is $1\frac{34}{81}$?
17. $\frac{5}{4}+\frac{8}{9}$ is $\frac{5}{2}+\frac{10}{3}$?
18. $9+4\frac{1}{3}$ is $11-3\frac{1}{2}$?
19. $5\frac{3}{4}-2\frac{5}{6}$ is $4\frac{7}{12}+1\frac{2}{3}$?
20. $6\frac{5}{8}-2\frac{2}{3}$ is $5\frac{3}{4}-4\frac{1}{6}$?
21. $\frac{3}{5}$ of $\frac{7}{8}$ is $\frac{4}{5}\times 1\frac{5}{9}$?
22. $1\frac{2}{3}\times 2\frac{2}{9}$ is $\frac{4}{5}$ of $3\frac{1}{3}$?
23. $6\frac{1}{4}+3\frac{2}{3}$ is $5\frac{1}{12}-2\frac{1}{6}$?
24. $\frac{13}{17}$ of $2\frac{2}{11}$ is $\frac{6}{11}$ of $1\frac{1}{7}$?
25. $\frac{5}{6}+\frac{9}{10}-\frac{11}{12}$ is $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}$?
26. $3\frac{5}{8}-1\frac{2}{9}+5\frac{47}{72}$ is $5\frac{7}{12}+2\frac{1}{6}-3\frac{13}{18}$?

108. To find a Number when one of its Fractional Parts is given.

1. 7 is $\frac{8}{9}$ of what number?

If 7 is $\frac{8}{9}$, one-ninth of the required number will be $\frac{1}{8}$ of 7, or $\frac{7}{8}$. Then the required number is 9 times $\frac{7}{8}$, or $\frac{63}{8}$, *Ans.*

It is evident from the above that the required result may be obtained by *multiplying the first number by the second number inverted.*

2. $2\frac{1}{10}$ is $\frac{9}{4}$ of what number?

$$2\frac{1}{10}\times\frac{4}{9}=\frac{\overset{7}{\cancel{21}}}{\underset{5}{\cancel{10}}}\times\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{9}}}=\frac{14}{15},\ Ans.$$

EXAMPLES.

3. 8 is $\frac{12}{7}$ of what number?
4. 28 is $\frac{35}{11}$ of what number?

5. $\frac{32}{9}$ is $\frac{40}{18}$ of what number?

6. $3\frac{3}{10}$ is $\frac{22}{25}$ of what number?

7. $\frac{20}{27}$ is $\frac{5}{18}$ of what number?

8. $1\frac{5}{21}$ is $\frac{65}{14}$ of what number?

9. $\frac{57}{92}$ is $\frac{38}{69}$ of what number?

10. $\frac{7}{5}$ of $\frac{11}{25}$ is $\frac{44}{75}$ of what number?

GREATEST COMMON DIVISOR OF FRACTIONS.

109. The *Greatest Common Divisor* of two or more fractions is the greatest fraction that is contained in each of them an *integral* number of times.

In order that one fraction may be contained in another an integral number of times, its numerator must be a *divisor* of the numerator, and its denominator a *multiple* of the denominator, of the second fraction.

Thus, $\frac{2}{9}$ is contained an integral number of times in $\frac{4}{3}$, since 2 is a divisor of 4, and 9 a multiple of 3.

Now, the greater the numerator of a fraction, and the smaller its denominator, the greater is the value of the fraction.

Hence, *the greatest common divisor of two or more fractions is the greatest common divisor of their numerators, divided by the least common multiple of their denominators.*

1. Find the G. C. D. of $\frac{24}{5}$, $\frac{16}{15}$, and $\frac{40}{9}$.

The G. C. D. of 24, 16, and 40, is 8.
The L. C. M. of 5, 15, and 9, is 45.
Then, the required G. C. D. is $\frac{8}{45}$, *Ans.*

EXAMPLES.

Find the G. C. D. of:

2. $\frac{4}{15}$, $\frac{16}{25}$, and $\frac{20}{3}$.

3. $4\frac{1}{8}$, $4\frac{7}{12}$, and $6\frac{3}{16}$.

4. $\frac{19}{5}$, $\frac{57}{10}$, and $\frac{38}{15}$.

5. $\frac{7}{26}$, $\frac{21}{52}$, and $2\frac{9}{13}$.

6. $33\frac{3}{4}$, $94\frac{1}{2}$, and $37\frac{1}{8}$.

7. $\frac{8}{9}$, $\frac{16}{21}$, $4\frac{4}{5}$, and $13\frac{5}{7}$.

8. $73\frac{7}{11}$, $12\frac{48}{77}$, and $39\frac{42}{55}$.

9. $\frac{84}{25}$, $\frac{56}{45}$, $\frac{112}{99}$, and $\frac{98}{75}$.

LEAST COMMON MULTIPLE OF FRACTIONS.

110. The *Least Common Multiple* of two or more fractions is the smallest number that will contain each of them an *integral* number of times.

In order that one fraction may contain another an integral number of times, its numerator must be a *multiple* of the numerator, and its denominator a *divisor* of the denominator, of the second fraction.

Thus, $\frac{4}{3}$ contains $\frac{2}{9}$ an integral number of times, since 4 is a multiple of 2, and 3 a divisor of 9.

Now, the smaller the numerator of a fraction, and the greater its denominator, the smaller is the value of the fraction.

Hence, *the least common multiple of two or more fractions is the least common multiple of their numerators, divided by the greatest common divisor of their denominators.*

1. Find the L. C. M. of $\frac{3}{14}$, $\frac{4}{21}$, and $\frac{9}{35}$.

The L. C. M. of 3, 4, and 9, is 36.
The G. C. D. of 14, 21, and 35, is 7.
Then, the required L. C. M. is $\frac{36}{7}$, *Ans.*

EXAMPLES.

Find the L. C. M. of:

2. $\frac{3}{4}$, $\frac{11}{16}$, and $\frac{33}{20}$.
3. $2\frac{2}{5}$, $1\frac{7}{9}$, and $3\frac{3}{7}$.
4. $\frac{3}{14}$, $\frac{6}{7}$, and $\frac{9}{35}$.
5. $\frac{7}{26}$, $\frac{21}{52}$, and $2\frac{9}{13}$.
6. $2\frac{1}{22}$, $\frac{9}{55}$, and $\frac{75}{77}$.
7. $2\frac{5}{8}$, $3\frac{15}{16}$, and $3\frac{27}{40}$.
8. $\frac{5}{32}$, $\frac{7}{48}$, $\frac{9}{64}$, and $\frac{3}{80}$.
9. $\frac{72}{245}$, $\frac{40}{451}$, $\frac{16}{147}$, and $\frac{90}{343}$.

MISCELLANEOUS EXAMPLES.

111. 1. Reduce $\frac{22751}{578}$ to a mixed number.

2. Reduce 28 to a fraction having 137 for a denominator.

3. Multiply $\frac{8}{25}$ by 18.

4. Multiply $4\frac{37}{69}$ by 23.

5. Divide $4\frac{2}{7}$ by 11.

6. Divide $1\frac{16}{41}$ by 19.

7. Reduce $123\frac{35}{78}$ to an improper fraction.
8. Arrange in order of magnitude $\frac{37}{80}$, $\frac{43}{86}$, and $\frac{53}{42}$.
9. Reduce $\frac{29}{48}$ to 768ths.
10. Divide $33\frac{3}{13}$ by 12.
11. Divide $37\frac{1}{27}$ by $44\frac{4}{9}$.
12. $\frac{7}{37}$ of $\frac{11}{5}$ is $\frac{55}{74}$ of what number?
13. Reduce $\frac{21168}{24696}$ to its lowest terms.
14. Find the value of $3\frac{1}{7} \times 5\frac{1}{5} \times 7\frac{1}{2} \times 9\frac{1}{3}$.
15. Reduce $\frac{11201}{6331}$ to its lowest terms.
16. Add together $2\frac{7}{72}$, $5\frac{11}{42}$, $8\frac{13}{56}$, and $11\frac{16}{63}$.
17. Find the value of $(2-\frac{67}{91}) \times (2-\frac{67}{92})$.
18. Subtract $\frac{79}{315}$ from $\frac{203}{525}$.
19. Find the value of $51 \div (3-\frac{1}{23})$.
20. Find the value of $\frac{5}{8}-\frac{6}{5}+\frac{7}{4}-\frac{8}{7}+\frac{5}{6}$.
21. Divide $2\frac{2}{119}$ by $1\frac{7}{17}$.
22. Add together $\frac{17}{27}$, $\frac{43}{82}$, $\frac{19}{16}$, and $\frac{121}{168}$.
23. What fraction of $\frac{5}{17}$ of $2\frac{1}{9}$ is $2\frac{1}{85}$?
24. Reduce $\frac{5207}{7421}$ to its lowest terms.
25. Subtract $13\frac{16}{39}$ from $23\frac{18}{65}$.
26. Multiply together $\frac{85}{91}$, $\frac{39}{80}$, $2\frac{9}{34}$, and $2\frac{2}{35}$.
27. Find the value of $(3-\frac{64}{75}) \div (1-\frac{89}{135})$.
28. Add together $5\frac{3}{4}$, $6\frac{4}{5}$, $7\frac{5}{6}$, $8\frac{6}{7}$, and $9\frac{7}{8}$.
29. Arrange in order of magnitude $\frac{61}{72}$, $\frac{73}{84}$, $\frac{85}{96}$.
30. Multiply together $\frac{80}{343}$, $2\frac{38}{125}$, $\frac{175}{432}$, and $1\frac{29}{216}$.
31. Find the G. C. D. of $2\frac{28}{63}$, $4\frac{2}{27}$, $5\frac{17}{45}$, and $9\frac{3}{7}$.
32. Find the L. C. M. of $\frac{9}{14}$, $\frac{15}{28}$, $\frac{11}{42}$, $\frac{33}{70}$, and $\frac{55}{84}$.
33. Simplify $\dfrac{2-\frac{17}{32}}{1+\frac{17}{128}}$.
34. Find the value of $(3\frac{1}{3}+4\frac{4}{5})-(2\frac{2}{7}+1\frac{7}{10})$.
35. Simplify $\dfrac{539 \times 637 \times 66}{343 \times 231 \times 363}$.

36. Simplify $\dfrac{5\frac{1}{3}-3\frac{1}{4}+4\frac{3}{5}-2\frac{5}{6}}{8\frac{1}{3}+3\frac{3}{4}-1\frac{4}{5}-7\frac{1}{6}}$.

37. Simplify $(\frac{5}{2}\text{ of }2\frac{1}{6})+(\frac{2}{5}\text{ of }8\frac{1}{9})-(\frac{5}{6}\text{ of }2\frac{1}{3})-(\frac{5}{4}\text{ of }1\frac{5}{6})$.

38. Simplify $\frac{18}{8}\text{ of }\frac{10}{17}-\dfrac{5\frac{2}{3}-3\frac{1}{2}}{4\frac{1}{4}+2\frac{5}{6}}+\frac{9}{10}\text{ of }3\frac{5}{6}$.

39. Simplify $\dfrac{(1\frac{7}{10}+1\frac{2}{13})+(3\frac{9}{16}\times 1\frac{1}{19})}{(3\frac{5}{6}\div 2\frac{5}{9})-(4\frac{4}{9}\div 10\frac{5}{6})}$.

40. Simplify $\dfrac{3\frac{1}{2}-2\frac{2}{3}}{4\frac{1}{4}-3\frac{1}{3}}+(\frac{4}{9}\text{ of }3\frac{3}{5})-(1\frac{7}{20}\div\frac{18}{25})-\frac{21}{40}$.

41. Simplify $\dfrac{(4\frac{7}{12}\times 1\frac{11}{40})-(3\frac{9}{14}\div 5\frac{1}{7})}{(7\frac{2}{9}+6\frac{7}{8})\div(8\frac{7}{12}-5\frac{5}{18})}$.

42. Simplify $\dfrac{(\frac{22}{21}\times\frac{123}{40})+(\frac{22}{15}\times\frac{27}{14})}{(\frac{25}{18}\div\frac{20}{63})-(\frac{15}{16}\div\frac{9}{20})}$.

PROBLEMS.

112. 1. If a man can do $\frac{5}{9}$ of a piece of work in $2\frac{1}{12}$ hours, in how many hours can he do the whole?

If he can do five-ninths of the work in $\frac{25}{12}$ hours, he can do one-ninth in one-fifth of $\frac{25}{12}$ hours; and he can do nine-ninths, or the whole, in 9 times $\frac{1}{5}$, or $\frac{9}{5}$ of $\frac{25}{12}$ hours.

$$\frac{\overset{3}{\cancel{9}}}{\cancel{5}}\times\frac{\overset{5}{\cancel{25}}}{\underset{4}{\cancel{12}}}=\frac{15}{4}=3\tfrac{3}{4}\text{ hours, } \textit{Ans.}$$

2. A tank can be filled by one pipe in 8 minutes, and by another in 12 minutes. How many minutes will it take to fill the tank, if both pipes are opened?

The first pipe in one minute will fill $\frac{1}{8}$ of the tank, and the second in one minute will fill $\frac{1}{12}$ of the tank.

Then both together will fill $\frac{1}{8}+\frac{1}{12}$, or $\frac{5}{24}$ of the tank in one minute.

Then it will take as many minutes for both pipes together to fill the tank as $\frac{5}{24}$ is contained times in $\frac{24}{24}$; that is $\frac{24}{5}$, or $4\frac{4}{5}$ minutes, *Ans.*

3. If $4\frac{2}{3}$ tons of coal is worth $\$31\frac{1}{2}$, how much is $7\frac{1}{9}$ tons worth?

If $\frac{14}{3}$ tons is worth $\frac{63}{2}$ dollars, one ton is worth as many dollars as $\frac{14}{3}$ is contained times in $\frac{63}{2}$.

$$\frac{63}{2} \div \frac{14}{3} = \frac{\overset{9}{\cancel{63}}}{2} \times \frac{3}{\underset{2}{\cancel{14}}} = \frac{27}{4}.$$

Then, if one ton is worth $\frac{27}{4}$ dollars, $\frac{64}{9}$ tons will be worth $\frac{64}{9}$ times $\frac{27}{4}$ dollars.

$$\frac{\overset{16}{\cancel{64}}}{\cancel{9}} \times \frac{\overset{3}{\cancel{27}}}{\cancel{4}} = \$48, \textit{ Ans.}$$

4. A can do a piece of work in 12 days, B can do the same work in 14 days, and C in 21 days. How many days will it take all of them together to do the work?

5. A tank can be emptied by one pipe in $9\frac{3}{4}$ minutes, and by another in $10\frac{2}{5}$ minutes. How many minutes will it take to empty the tank if both pipes are opened?

6. A man walked 63 miles. He performed the first half of his journey at the rate of $4\frac{1}{3}$ miles an hour, and the last half at the rate of $5\frac{1}{4}$ miles an hour. How many hours did it take him?

7. If $\frac{5}{12}$ of a ton of hay is worth $\$8\frac{1}{2}$, how much is 10 tons worth?

8. A man having lost $\frac{1}{9}$ of his money, and then spent $\frac{5}{18}$ of the remainder, found that he had \$112 left. How much had he at first?

9. How many pecks of apples, at $25\frac{1}{3}$ cents a peck, must be given for $12\frac{4}{5}$ pounds of sugar, at $4\frac{3}{4}$ cents a pound?

10. I sold a house and lot for \$3125, which was $\frac{25}{31}$ of what they cost me. How much did I lose by the operation?

11. The circumference of the hind-wheel of a carriage is $9\frac{3}{7}$ feet, and of the fore-wheel $8\frac{4}{5}$ feet. How many times does each wheel turn in travelling 5280 feet?

12. A merchant who owned $\frac{3}{4}$ of a ship, sold $\frac{5}{6}$ of his share for \$ 15625. What was the value of the whole ship at the same rate?

13. A man sold a horse and carriage for \$ 624, receiving $\frac{3}{5}$ as much for the horse as for the carriage. What did he receive for each?

14. If a horse travels $7\frac{5}{9}$ miles an hour, how long will it take him to travel $20\frac{2}{3}$ miles?

15. If $3\frac{4}{15}$ is $\frac{16}{25}$ of a certain number, what is $\frac{24}{35}$ of the same number?

16. What number must be multiplied by $3\frac{27}{35}$, so that the product may be $20\frac{4}{5}$?

17. A man spent $\frac{2}{3}$ of his money, and then received \$105, when he found that he had $\frac{5}{6}$ of his original amount. How much had he at first?

18. My income is \$ $8\frac{3}{8}$ a week, and my expenses are \$ $5\frac{7}{12}$ a week. How many weeks will it take me to save \$ $100\frac{1}{2}$?

19. A bale of cloth contains 75 pieces, each piece containing $23\frac{2}{5}$ yards. What is the whole worth at \$ $1\frac{4}{9}$ a yard?

20. What number is that $\frac{13}{18}$ of which exceeds $\frac{5}{24}$ of it by 111?

21. If $\frac{7}{15}$ of a piece of land is worth \$ $604\frac{4}{5}$, how much is $\frac{11}{24}$ of it worth?

22. A dealer has $58\frac{5}{6}$ tons of coal in his yard. On each of six successive days he puts in $9\frac{7}{8}$ tons, and sells on each day $5\frac{3}{4}$ tons. How many tons has he in his yard at the end of the sixth day?

23. If a rod 2 feet long casts a shadow $\frac{2}{3}$ of a foot long at 12 o'clock, how high is a flag-pole which casts a shadow $35\frac{2}{3}$ feet long at the same time?

24. If $3\frac{3}{4}$ pounds of sugar cost 18 cents, how much will $6\frac{2}{3}$ pounds cost?

25. If a town pays \$480 for the support of 14 paupers for 15 weeks, how much should it pay for the support of 25 paupers for 21 weeks?

26. A, B, and C found a purse containing money. A took $\frac{1}{4}$ of the money; B then took $\frac{2}{3}$ of what remained, and C $\frac{1}{2}$ the remainder, which was \$$10\frac{7}{8}$. How much money did the purse contain?

27. In a certain school, $\frac{7}{20}$ of the pupils are in the fourth class, $\frac{4}{15}$ in the third class, $\frac{7}{30}$ in the second class, and the remainder, 27, in the first class. How many pupils are there in each class?

28. I have three fields containing, respectively, $5\frac{5}{9}$ acres, $4\frac{17}{27}$ acres, and $11\frac{1}{9}$ acres. Find the size of the largest house lots, all of the same size, into which the fields can be divided.

29. If $8\frac{3}{4}$ tons of coal can be bought for \$$37\frac{3}{8}$, how many tons can be bought for \$$22\frac{3}{5}$?

30. If a man can walk $26\frac{3}{5}$ miles in $6\frac{1}{3}$ hours, how far can he walk in $8\frac{13}{14}$ hours?

31. A merchant sold goods for \$451, and gained $\frac{3}{8}$ of what they cost him. How much did he gain by the operation?

32. If $20\frac{5}{8}$ acres of land cost \$8000, how much will $13\frac{7}{8}$ acres cost?

33. If a man can do a piece of work in $7\frac{1}{2}$ days, working $11\frac{2}{3}$ hours a day, how many days will it take him working $9\frac{1}{3}$ hours a day?

34. A tank has two pipes. One fills it at the rate of $13\frac{1}{2}$ gallons an hour, and the other discharges the contents at the rate of $5\frac{2}{3}$ gallons an hour. If the tank holds $18\frac{4}{5}$ gallons, how many hours will it take to fill it?

35. A can mow a field in 5 days, and A and B together can mow it in $3\frac{1}{3}$ days. How many days will it take B alone to mow the field?

36. I have \$ 39 in the bank. If my income is \$ $7\frac{3}{8}$ a week, and my expenses \$ $9\frac{1}{6}$ a week, how many weeks will my fund last me?

37. If a man can do a piece of work in $1\frac{34}{81}$ days, what part of it can he do in $1\frac{26}{135}$ days?

38. If the dividend is $\frac{3}{4}$ of $21\frac{2}{3}$, and the quotient $\frac{2}{3}$ of $6\frac{1}{2}$, what is the divisor?

39. If a man can do a piece of work in $5\frac{2}{5}$ days, working $8\frac{1}{3}$ hours a day, how long will it take him working $9\frac{3}{8}$ hours a day?

40. A dealer bought a number of bales of silk, each containing 135 yards, at \$ $1\frac{2}{3}$ a yard, and sold it at \$ $2\frac{1}{5}$ a yard, gaining \$ 792 by the transaction. How many bales did he buy?

41. Two pendulums beat once in $\frac{33}{35}$ of a second, and once in $\frac{88}{91}$ of a second, respectively. If at any time the beats occur together, after how many seconds will they again occur together?

42. The product of three numbers is $1\frac{23}{40}$; if two of them are $1\frac{13}{20}$ and $2\frac{3}{16}$, what is the third?

43. A can do a piece of work in 15 hours, B in 20 hours, and C in 30 hours. B and C worked alone for 5 hours, when A joined them. How many hours will it take all of them together to finish the work?

44. If a man travels $3\frac{1}{8}$ miles an hour, and $9\frac{1}{15}$ hours a day, how many days will it take him to travel $906\frac{2}{3}$ miles?

45. A body falls $16\frac{1}{12}$ feet the first second, and in each succeeding second $32\frac{1}{6}$ feet more than in the next preceding. How far does it fall in 5 seconds?

46. Three men, A, B, and C, can walk around a circular race-course in $8\frac{1}{6}$, $7\frac{7}{9}$, and $6\frac{8}{15}$ minutes, respectively. If they all set out together, after how many minutes will they all meet at the starting-point, and how many times will each have gone around the course?

47. A leaves Boston at a certain time, and travels at the rate of $3\frac{1}{2}$ miles an hour. After hè has been gone $2\frac{5}{8}$ hours, B sets out to overtake him, and travels at the rate of $4\frac{2}{3}$ miles an hour. How far apart are A and B $5\frac{3}{4}$ hours after B sets out?

48. A can reap a field in 9 days, working 8 hours a day; B can reap the same field in 8 days, working $7\frac{1}{2}$ hours a day. How long will it take both together to reap the field, working 9 hours a day?

49. The sides of a field are $23\frac{5}{6}$ rods, $60\frac{2}{3}$ rods, $23\frac{1}{9}$ rods, and $58\frac{1}{2}$ rods, respectively. What is the length of the longest pole that will be contained exactly in each side?

50. Multiply $\frac{1}{3}$ of $\frac{3}{4}$ of $9\frac{1}{6}$ by one-half of itself, and divide the product by $\frac{77}{192}$.

51. A can do a piece of work in 9 hours; A and B together can do it in 6 hours, and B and C together can do it in 4 hours. How many hours will it take A and C together to do the work?

52. What number is that $\frac{5}{12}$ of $\frac{32}{15}$ of which exceeds $\frac{3}{8}$ of $\frac{10}{9}$ of it by $2\frac{25}{81}$?

53. A pole stands $\frac{2}{7}$ in the mud, $\frac{7}{16}$ in the water, and the remainder, $12\frac{2}{5}$ feet, above water. Find the length of the pole.

54. A sum of money was divided between A, B, C, and D, in such a way that A received $\frac{5}{12}$, B $\frac{2}{15}$, C $\frac{4}{21}$, and D the remainder, which was \$$65\frac{2}{5}$. What was the sum divided, and how much did each receive?

55. If 17 horses consume $8\frac{1}{3}$ bushels of oats in $3\frac{3}{4}$ days, how many bushels will 12 horses consume in $6\frac{4}{5}$ days?

56. A can do a piece of work in 12 days, B in 14 days, C in 18 days, and D in 21 days. How long will it take all of them together to do the work, and what part of the work does each perform?

X. DECIMALS.

113. A fraction whose denominator is a *power of ten* is usually expressed by placing a *point* at the right of the numerator, and then moving it *to the left* as many places as there are ciphers in the denominator.

When thus expressed, the fraction is called a **Decimal Fraction**, or simply a **Decimal**.

The point is called a **Decimal Point**.

114. Consider, for example, the fraction $\frac{2305}{1000}$.

In this case there are *three* ciphers in the denominator.

Placing a point at the right of the 2305, and then moving it three places to the left, we have

$$\frac{2305}{1000} = 2.305$$

If the number of digits in the numerator is less than the number of ciphers in the denominator, ciphers may be written in the places to the left of the first digit of the numerator.

Thus, consider the fraction $\frac{16}{10000}$.

Placing a point at the right of the 16, moving it four places to the left, and writing two ciphers at the left of the first digit, we have

$$\frac{16}{10000} = .0016$$

115. The figure immediately to the right of the decimal point is said to be in the *first decimal place;* the next one to the right in the *second decimal place;* etc.

The following table gives the signification of each of the first six decimal places:

1st; *tenths.*	4th; *ten-thousandths.*
2d; *hundredths.*	5th; *hundred-thousandths.*
3d; *thousandths.*	6th; *millionths.*

116. To *read* a decimal, first read the number to the left of the decimal point, if any; then the number to the right of the point, regarded as an integer, followed by the name of the right-hand decimal place.

Thus, 2.305 is read "two, and three hundred and five thousandths."

.0016 is read "sixteen ten-thousandths."

In order to avoid ambiguity, it is better to make a pauso at the decimal point, and another before pronouncing the name of the right-hand decimal place.

EXAMPLES.

117. Read the following:

1. .5.	**6.** .0039.	**11.** 2574.00009.
2. .17.	**7.** 8.028.	**12.** .0004859.
3. 15.3.	**8.** 24.0071.	**13.** 863.108642.
4. .461.	**9.** .84072.	**14.** 5.9085495.
5. 90.06.	**10.** .689313.	**15.** :00003287.

Write the following as decimals:

16. Forty-nine hundredths.

17. Fifty-two, and four tenths.

18. One hundred and fifty-eight thousandths.

19. Nine, and thirteen ten-thousandths.

20. Thirty-seven, and two hundred-thousandths.

21. Fifty-nine thousand threo hundred and ninety-eight millionths.

22. Eight hundred and thirty-two, and forty thousand one hundred and two hundred-thousandths.

23. Twenty-six, and eight hundred and five thousand three hundred and three millionths.

24. Seven thousand four hundred and twenty-five ten-millionths.

TO REDUCE A DECIMAL TO A COMMON FRACTION.

118. A decimal may be expressed in the form of a common fraction by writing the decimal without its decimal point for a numerator, and for a denominator 1, followed by as many ciphers as there are places to the right of the decimal point.

Thus, $11.28 = \frac{1128}{100} = \frac{282}{25}$;

$.0523 = \frac{523}{10000}$; etc.

EXAMPLES.

Express as common fractions in their lowest terms:

1. 2.8.	**6.** 8.512.	**11.** .0376.	**16.** .01375.
2. .005.	**7.** 30.75.	**12.** .0096.	**17.** 4.4375.
3. 75.44.	**8.** .1975.	**13.** 3.0875.	**18.** .15625.
4. .684.	**9.** 68.461.	**14.** .08309.	**19.** .008128.
5. 1.85.	**10.** .025.	**15.** .00128.	**20.** 2.109375.

ADDITION OF DECIMALS.

119. 1. Add 7.89, 31.4, and .086.

```
 7.89
31.4
  .086
-------
39.376, Ans.
```

We write the numbers so that their decimal points shall be in the same vertical column.

The sum of 8 hundredths and 9 hundredths is 17 hundredths, or 1 tenth and 7 hundredths.

The sum of 1 tenth, 4 tenths, and 8 tenths is 13 tenths, or 1 unit and 3 tenths.

The sum of 1 unit, 1 unit, and 7 units, is 9 units.

Then the required result is 3 tens, 9 units, 3 tenths, 7 hundredths, and 6 thousandths, or 39.376.

EXAMPLES.

Add the following:

2. 25.5, .00076, 1.7862, and .084.

3. 2.601, .9693, 35.08, and .00745.

4. 165, .94468, .0051, and 59.226.
5. .4085, 8.62, .03947, and 2.139.
6. 5.0902, .00007, .637, and .014961.
7. .39665, 9.9, 72.1508, and .004052.
8. .000616, 93.38967, .0562, and 807.74.
9. .06212, 35.49, 87.56, 4920.04, and 297.868.

SUBTRACTION OF DECIMALS.

120. **1.** Subtract 89.725 from 162.0738.

$$\begin{array}{r} 162.0738 \\ \underline{89.725} \\ 72.3488, \textit{ Ans.} \end{array}$$

We write the numbers so that their decimal points shall be in the same vertical column.

5 thousandths from 13 thousandths leave 8 thousandths.

3 hundredths from 7 hundredths leave 4 hundredths.

7 tenths from 10 tenths leave 3 tenths.

10 units from 12 units leave 2 units.

9 tens from 16 tens leave 7 tens.

Then the required result is 7 tens, 2 units, 3 tenths, 4 hundredths, 8 thousandths, and 8 ten-thousandths, or 72.3488.

If the subtrahend has more places than the minuend, we may make the number of places in the latter the same as in the former by mentally supplying ciphers in the missing places.

2. Subtract .008504 from .0162.

$$\begin{array}{l} .0162 \\ \underline{.008504} \\ .007696, \textit{ Ans.} \end{array}$$

EXAMPLES.

Subtract the following:

3. .4169 from 5.2705.
4. .0726 from .32933.
5. .318 from 1.
6. .00986 from .0204.
7. 2.08429 from 11.352.
8. .005341 from .0091291.
9. .0623907 from 10.
10. .08194812 from 2.52866.
11. 48.6007 from 830.352.
12. .0002584 from .07683.

MULTIPLICATION OF DECIMALS.

121. 1. Multiply 30.84 by 2.516.

Writing the decimals as common fractions, we have

$$30.84 \times 2.516 = \frac{3084}{100} \times \frac{2516}{1000} = \frac{3084 \times 2516}{100000}.$$

$$\begin{array}{r} 3084 \\ \underline{2516} \\ 18504 \\ 3084 \\ 15420 \\ \underline{6168} \\ 7759344 \end{array}$$

Then, $30.84 \times 2.516 = \frac{7759344}{100000} = 77.59344$, *Ans.*

It is customary to arrange the work as follows:

$$\begin{array}{r} 30.84 \\ \underline{2.516} \\ 18504 \\ 3084 \\ 15\,420 \\ \underline{61\,68} \\ 77.59344 \end{array}$$

It will be observed that the number of decimal places in the result is the sum of the number of decimal places in the multiplicand and the number of decimal places in the multiplier; hence the following

RULE.

Multiply the numbers as if they were integers, and point off as many decimal places in the result as the sum of the number of decimal places in the multiplicand and multiplier.

If the number of digits in the product is not sufficient for this purpose, ciphers may be written in the places to the left of its first digit.

2. Multiply .764 by .0108.

$$\begin{array}{r} .764 \\ \underline{.0108} \\ 6112 \\ \underline{764} \\ .0082512, \end{array}$$ *Ans.*

In this case, we point off *seven* decimal places in the product, writing *two* ciphers at the left of the first digit.

EXAMPLES.

Multiply the following:

3. 8.27 by 29.3.
4. .0966 by .561.
5. .00708 by .0365.
6. .6581 by 9.7.
7. .05648 by .082.
8. 1.821 by 34.5.
9. 45.66 by .00207.
10. .5114 by .4053.
11. .068022 by .16.
12. .4486 by 5.83.
13. 18.052 by .75.
14. 21.96 by 4.78.
15. .07819 by 63.05.
16. .009256 by .08219.

122. To Multiply a Decimal by 10, 100, 1000, Etc.

To multiply a decimal by 10, 100, etc., we move its decimal point one, two, etc., places to the *right*.

Or in general, to multiply a decimal by 1 followed by any number of ciphers, we move its decimal point to the right as many places as there are ciphers in the multiplier.

Example. Multiply 87.35 by 10000.

Moving the decimal point *four* places to the right, we have

$$87.35 \times 10000 = 873500, \textit{ Ans.}$$

123. To Multiply a Decimal by .1, .01, .001, Etc.

To multiply a decimal by .1, .01, .001, etc., we move its decimal point one, two, three, etc., places to the *left*.

Or in general, to multiply a decimal by .1, or by 1 preceded by any number of ciphers and then a decimal point, we move its decimal point as many places to the left as there are places in the multiplier.

Example. Multiply 6.294 by .001.

Moving the decimal point *three* places to the left, we have

$$6.294 \times .001 = .006294, \textit{ Ans.}$$

124. To Multiply a Decimal by Any Number of Tens, Hundreds, Etc.

Any number of ciphers at the right of the multiplier may be omitted, if the decimal point of the multiplicand be moved to the right as many places as there are ciphers omitted.

Example. Multiply 32.851 by 5200.

$$\begin{array}{r} 3285.1 \\ 52 \\ \hline 6570\ 2 \\ 164255 \\ \hline 170825.2 \end{array}, \textit{ Ans.}$$

We move the decimal point of 32.851 *two* places to the right, and multiply 3285.1 by 52. The result is 170825.2.

In like manner, ciphers at the right of the multiplicand may be omitted, if the decimal point of the multiplier be moved to the right as many places as there are ciphers omitted.

EXAMPLES.

125. Multiply the following:

1. 85.2 by 10.
2. 377 by .01.
3. 4.14 by 300.
4. .00695 by 1000.
5. .000208 by 100.
6. .753 by 1620.
7. .1261 by .1.
8. 87900 by .0743.
9. 5839 by .0001.
10. 1.417 by 10000.
11. 368.8 by .00001.
12. .04854 by 89000.
13. 937620 by .001.
14. .060239 by 100000.
15. 14537000 by .0985.
16. 27.405 by 5240000.

DIVISION OF DECIMALS.

126. *Example.* Divide 4742.66 by 754.

Writing the decimal as a common fraction,

$$4742.66 \div 754 = \tfrac{474266}{100} \div 754.$$

$$\begin{array}{l}
754)474266(629 \\
\quad\;\; \underline{4524} \\
\quad\quad\; 2186 \\
\quad\quad\; \underline{1508} \\
\quad\quad\quad 6786 \\
\quad\quad\quad \underline{6786}
\end{array}$$

Then, $\tfrac{474266}{100} \div 754 = \tfrac{629}{100} = 6.29$, *Ans.*

It is customary to arrange the work as follows:

$$\begin{array}{l}
754)4742.66(6.29 \\
\quad\;\; \underline{4524} \\
\quad\quad\; 218\ 6 \\
\quad\quad\; \underline{150\ 8} \\
\quad\quad\quad 67\ 86 \\
\quad\quad\quad \underline{67\ 86}
\end{array}$$

It will be observed that the number of decimal places in the quotient is the same as the number of decimal places in the dividend.

127. If the divisor is not an integer, it may always be made so by *moving the decimal points of both dividend and divisor as many places to the right as there are decimal places in the divisor.*

1. Divide .0275918 by .7261.

$$\begin{array}{l}
\quad\quad\;\; (.038,\ \textit{Ans.} \\
7261)275.918 \\
\quad\quad\; \underline{217\ 83} \\
\quad\quad\quad 58\ 088 \\
\quad\quad\quad \underline{58\ 088}
\end{array}$$

In this case, we move the decimal points of both dividend and divisor *four* places to the right, and point off *three* places in the quotient.

It is convenient, in Long Division of Decimals, to write the quotient above the dividend in such a way that each of its digits shall be directly over the right-hand digit of the corresponding partial product.

Thus, in Ex. 1, the digit 3 of the quotient is directly over the right-hand digit of the first partial product, 21783, and the digit 8 is directly over the right-hand digit of the second partial product, 58088.

In this case, the decimal point of the quotient will always be *directly over the decimal point of the dividend.*

If the number of decimal places in the dividend is less than the number of decimal places in the divisor, ciphers may be written in the missing places.

2. Divide 318.68 by .257.

```
    (1240, Ans.
257)318680
    257
     616
     514
     1028
     1028
```

In this case, we move the decimal points of both dividend and divisor *three* places to the right, annexing one cipher to the dividend.

If the dividend is not exactly divisible by the divisor, it may sometimes be made so by annexing ciphers.

3. Divide 211.347 by 40.84.

```
        (5.175, Ans.
4084)21134.700
     20420
       714 7
       408 4
       306 30
       285 88
        20 420
        20 420
```

In this case, we annex two ciphers to the dividend to make it divisible by the divisor.

EXAMPLES.

Divide the following:

4. 4.361 by .7.
5. .11504 by .0004.
6. .005088 by .06.
7. .9588 by 9.4.
8. 284.24 by .038.
9. 19.3752 by 20.7.
10. 4.6292 by 2.84.
11. .492453 by .549.
12. .313048 by 87.2.
13. 29379.7 by .47.
14. .0201474 by .054.
15. .00113291 by .193.
16. 19.7635 by 8.41.
17. .31236 by .00685.
18. 46.290881 by .9107.
19. .487578 by .00665.
20. 6.618015 by 7.174.
21. 609.4429 by .001243.
22. 332.45 by .0488.
23. .35363808 by 89.28.

128. To Divide a Decimal by 10, 100, 1000, Etc.

To divide a decimal by 10, 100, etc., we move its decimal point one, two, etc., places to the *left*.

Or in general, to divide a decimal by 1 followed by any number of ciphers, we move its decimal point to the left as many places as there are ciphers in the divisor.

Example. Divide 87.35 by 10000.

Moving the decimal point *four* places to the left, we have

$$87.35 \div 10000 = .008735, \quad \textit{Ans.}$$

129. To Divide a Decimal by .1, .01, .001, Etc.

To divide a decimal by .1, .01, .001, etc., we move its decimal point one, two, three, etc., places to the *right*.

Or in general, to divide a decimal by .1, or by 1 preceded by any number of ciphers and then a decimal point, we move its decimal point as many places to the right as there are places in the divisor.

Example. Divide 6.294 by .0001.

Moving the decimal point *four* places to the right, we have

$$6.294 \div .0001 = 62940, \quad \textit{Ans.}$$

130. To Divide a Decimal by Any Number of Tens, Hundreds, Etc.

To divide a decimal by any number of tens, hundreds, etc., we omit the ciphers at the right of the divisor, and move the decimal point of the dividend as many places to the left as there are ciphers omitted.

Example. Divide 4716.28 by 62800.

```
    (.0751, Ans.
628)47.1628
    43 96
     3 202
     3 140
        628
        628
```

In this case, we move the decimal point of 4716.28 *two* places to the left, and divide the result by 628.

EXAMPLES.

131. Divide the following:

1. 542 by 100.
2. 2.99 by .01.
3. 630 by 5000.
4. .426 by 10.
5. 2697.5 by 8300.
6. .00724 by .1.
7. 6.8337 by 270.
8. .463 by .0001.
9. 7.815 by 1000.
10. .0008171 by .001.
11. 855.36 by 6480000.
12. 5156.8 by 10000.
13. .001807 by .00001.
14. 30.82 by 100000.

TO REDUCE A COMMON FRACTION TO A DECIMAL.

132. 1. Reduce $\frac{57}{125}$ to a decimal.

We have $\frac{57}{125} = 57 \div 125$; dividing 57 by 125, we obtain

```
    (.456, Ans.
125)57.000
    50 0
     7 00
     6 25
       750
       750
```

2. Reduce $\frac{2}{3}$ to a decimal.

$$\begin{array}{r} .6666\tfrac{2}{3},\ \textit{Ans.} \\ 3)\overline{2.0000} \\ \underline{1\,8} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

In this case, the division never terminates, no matter how far the operation may be carried.

Whenever the process has been carried as far as desired, the remainder may be written over the divisor, and the fraction thus formed added to the quotient.

In many numerical computations, only an *approximate* value of the quotient is required; in such a case, the fraction which expresses the remainder may be omitted provided that, if it is equal to or greater than $\frac{1}{2}$, the last digit of the quotient is increased by 1.

Thus, $.725\frac{1}{3}$ would be taken as .725, approximately; $.62\frac{1}{2}$ as .63; and $.6666\frac{2}{3}$ as .6667.

The result .6667 is said to be the *approximate value of* $\frac{2}{3}$ *to the nearest fourth decimal place*, or *to the nearest ten-thousandth.*

Note. It may sometimes happen that neither the expression of the remainder, nor the nearest approximate value is necessary; in such a case, the incompleteness of the quotient is denoted by a + sign.

Thus, .6666+ would be written in place of $.6666\frac{2}{3}$.

EXAMPLES.

Reduce the following to decimals:

3. $\frac{7}{8}$.	6. $\frac{183}{40}$.	9. $\frac{89}{625}$.	12. $\frac{31}{32}$.
4. $\frac{2}{25}$.	7. $\frac{341}{5000}$.	10. $\frac{5}{64}$.	13. $\frac{433}{1280}$.
5. $\frac{11}{16}$.	8. $\frac{147}{12500}$.	11. $\frac{21}{3125}$.	14. $\frac{73}{15625}$.

Find the approximate value of each of the following to the nearest fifth decimal place:

15. $\frac{4}{7}$.	17. $\frac{5}{18}$.	19. $\frac{253}{8100}$.	21. $\frac{160}{161}$.
16. $\frac{11}{12}$.	18. $\frac{23}{27}$.	20. $\frac{31}{480}$.	22. $\frac{357}{649}$.

CIRCULATING DECIMALS.

133. In expressing a common fraction as a decimal by the method of Art. 132, if the factors of the denominator are not all 2's and 5's, a single figure, or a set of figures, will be found to recur indefinitely in the quotient.

Thus, if 17 be divided by 54, the quotient is .3148148+, where the digits 148 recur indefinitely.

Such a decimal is called a **Circulating Decimal**, or simply a **Circulate**; and the digit, or set of digits, which is repeated is called the **Repetend**.

134. A circulate is usually expressed, when a single digit is repeated, by writing a dot over it; and when a set of digits is repeated, by writing dots over the first and last of the set.

Thus, .3148148+ is expressed $.3\dot{1}4\dot{8}$.

135. A **Pure Circulate** is one which has no digits except the ones which are repeated; as $.\dot{3}\dot{8}$.

A **Mixed Circulate** is one which has one or more digits preceding the ones which are repeated; as $4.3\dot{1}5\dot{7}$.

136. To Express a Common Fraction as a Circulate.

1. Express $\frac{17}{54}$ as a circulate.

$$
\begin{array}{r@{}l}
 & (.3\dot{1}4\dot{8},\ \textit{Ans.} \\
54) & 17.0 \\
 & 16\,2 \\ \hline
 & \ \ 80 \\
 & \ \ 54 \\ \hline
 & \ \ 260 \\
 & \ \ 216 \\ \hline
 & \ \ \ \ 440 \\
 & \ \ \ \ 432 \\ \hline
 & \ \ \ \ \ \ 80
\end{array}
$$

Dividing 17 by 54, the first four digits of the quotient are .3148.

At this point a remainder, 80, is obtained which is *the same as the first remainder.*

It is evident from this that the digits 148 will recur indefinitely in the quotient.

Then the required result is $.3\dot{1}4\dot{8}$.

In any case, the division must be carried out until a remainder is obtained which is *the same as the dividend, or some preceding remainder.*

EXAMPLES.

Express each of the following as a circulate:

2. $\frac{8}{11}$.	5. $\frac{28}{225}$.	8. $\frac{438}{1350}$.	11. $\frac{9}{1300}$.
3. $3\frac{4}{9}$.	6. $\frac{113}{165}$.	9. $\frac{125}{271}$.	12. $\frac{1387}{2160}$.
4. $\frac{10}{37}$.	7. $19\frac{73}{101}$.	10. $\frac{171}{202}$.	13. $7\frac{25}{126}$.

137. To Find the Common Fraction which will produce a Given Circulate.

1. What fraction will produce $.\dot{5}\dot{3}$?

Let F represent the fraction.

Then, one-hundred times F is equal to $53.\dot{5}\dot{3}$.

Therefore, one-hundred times F, minus F, is equal to $53.\dot{5}\dot{3}$, minus $.\dot{5}\dot{3}$, or 53.

That is, ninety-nine times F is equal to 53.

Whence, F is equal to 53 divided by 99, or $\frac{53}{99}$, *Ans.*

From the above example, we derive the following

RULE.

To find the common fraction which will produce a given pure circulate, divide the repetend by a number having for its digits as many nines as there are digits in the repetend.

Thus, the fraction which will produce $.\dot{3}5\dot{3}$ is $\frac{353}{999}$.

2. What fraction will produce $.5\dot{1}8\dot{5}$?

By the above rule, the fraction is

$$.5\frac{185}{999} = .5\frac{5}{27} = \frac{5\frac{5}{27}}{10} = \frac{140}{270} = \frac{14}{27}, \textit{ Ans.}$$

EXAMPLES.

Find the common fraction which will produce each of the following:

3. $.\dot{3}\dot{9}$.	**8.** $.3\dot{2}\dot{7}$.	**13.** $.5\dot{2}4\dot{3}$.	**18.** $.2\dot{7}12\dot{8}$.
4. $.4\dot{7}$.	**9.** $.\dot{0}94\dot{5}$.	**14.** $.\dot{9}48\dot{2}$.	**19.** $.513\dot{7}\dot{8}$.
5. $.\dot{8}\dot{1}$.	**10.** $48.57\dot{3}$.	**15.** $.20\dot{7}\dot{5}$.	**20.** $.24\dot{2}5\dot{9}$.
6. $.\dot{4}0\dot{7}$.	**11.** $.6\dot{0}8\dot{1}$.	**16.** $7.84\dot{5}\dot{6}$.	**21.** $5.072\dot{1}\dot{6}$.
7. $2.\dot{6}7\dot{5}$.	**12.** $.372\dot{6}$.	**17.** $.60\dot{4}0\dot{5}$.	**22.** $.3\dot{6}13\dot{8}$.

TO MULTIPLY OR DIVIDE A NUMBER BY AN ALIQUOT PART OF 10, 100, 1000, ETC.

138. An **Aliquot Part** of a number is a number that is exactly contained in it.

Thus, $3\frac{1}{3}$ is an aliquot part of 10, for it is contained in 10 three times.

139. 1. Multiply 5.736 by $16\frac{2}{3}$.

Since $16\frac{2}{3}$ is one-sixth of 100, we may multiply 5.736 by 100, and divide the result by 6.

Multiplying 5.736 by 100, the product is 573.6 (Art. 122).

Dividing 573.6 by 6, the quotient is 95.6, *Ans.*

2. Divide 1.056 by 125.

Since 125 is one-eighth of 1000, we may divide 1.056 by 1000, and multiply the result by 8.

Dividing 1.056 by 1000, the quotient is .001056 (Art. 128).

Multiplying .001056 by 8, the product is .008448, *Ans.*

EXAMPLES.

Multiply the following:

3. 72 by $3\frac{1}{3}$.	**7.** 2.46 by $166\frac{2}{3}$.
4. .84 by 25.	**8.** .00047 by 125.
5. .0393 by $6\frac{2}{3}$.	**9.** .976 by $6\frac{1}{4}$.
6. .00525 by $33\frac{1}{3}$.	**10.** 175.2 by $83\frac{1}{3}$.

Divide the following:

11. 35 by $2\frac{1}{2}$.
12. 10.1 by 50.
13. .76 by $333\frac{1}{3}$.
14. 257 by $16\frac{2}{3}$.
15. 413 by 250.
16. .98 by $12\frac{1}{2}$.
17. 6524 by $8\frac{1}{3}$.
18. 802.9 by $62\frac{1}{2}$.

MISCELLANEOUS EXAMPLES.

140. 1. Add .38752, 12.893, .008245, and 5.0169.

2. Subtract .000695367 from .00518224.

3. Express .03616 as a common fraction, and reduce the result to its lowest terms.

4. Multiply 5629070 by .000001.

5. Express $\frac{13}{80}$ and $\frac{29}{64}$ as decimals, and find their sum.

6. Divide 1.0642 by 78.25.

7. Express $\frac{1059}{3700}$ as a circulating decimal.

8. Express .25625 as a common fraction, and reduce the result to its lowest terms.

9. Multiply 84.175 by .0007302.

10. Divide .015271938 by .4521.

11. Multiply .0297 by $111\frac{1}{9}$.

12. Reduce $\frac{5}{128}$ to a decimal.

13. Multiply .00935 by 668000.

14. Divide 473.1 by .0166.

15. What common fraction will produce $.\dot{9}70\dot{2}$?

16. Divide .0603712 by .00000001.

17. Find the approximate value of $\frac{81}{575}$ to the nearest fifth decimal place.

18. Divide 6466.6 by 7450000.

19. Divide 85.29 by $3333\frac{1}{3}$.

20. Express $\frac{10}{81}$ as a circulating decimal.

21. Express $\frac{87}{320}$ and $\frac{121}{6400}$ as decimals, and subtract the second result from the first.

22. Divide .48868466 by .005407.

23. What common fraction will produce $.51\dot{2}9\dot{6}$?

24. Simplify $\dfrac{.0091 + .03\frac{1}{2}}{2\frac{3}{4} - 1.7}$.

25. Simplify $\dfrac{.05 - (.007 + .0043063)}{.9 - (.12 - .063)}$.

26. Simplify $\dfrac{.12}{.049} \times \dfrac{.28}{.0045} \times \dfrac{.0035}{.96}$.

27. Simplify $\dfrac{68}{21.5}\left(\dfrac{.0539}{.034} - \dfrac{.423}{.85}\right)$.

28. Simplify $\dfrac{.06 - .003}{.5 - .025} + \dfrac{.008 + .0004}{.03 + .0015}$.

29. Simplify $\dfrac{(.38 \times .00027) + (.057 \times .0036)}{1 - .9487}$.

30. Simplify $\dfrac{.\dot{7} + .\dot{9}\dot{8}}{.\dot{1}\dot{0} + .\dot{8}} - \dfrac{.7\dot{3} - .2\dot{6}\dot{9}}{.7\dot{1} - .4\dot{3}\dot{6}}$.

UNITED STATES MONEY.

141. **Money** is that which is used to measure *value.*

142. A **Denomination** is a *unit* of measure; as for example, a *dollar,* or a *cent.*

The denominations of United States Money are given in the following

TABLE.

10 mills (m.)	= 1 cent.	(c.)
10 cents	= 1 dime.	(d.)
10 dimes	= 1 dollar.	($)
10 dollars	= 1 eagle.	(e.)

143. The only denominations used in ordinary business transactions are *dollars* and *cents;* eagles and dimes are usually expressed in terms of dollars and cents, respectively, and mills as the fraction of a cent.

Thus, 5 eagles, 3 dollars, 7 dimes, 9 cents, and 2 mills, is the same as 53 dollars, and $79\frac{1}{5}$ cents.

Since a cent is one-hundredth of a dollar, any sum of money expressed in dollars and cents may be expressed as a decimal of a dollar by writing the number of cents in the hundredths' place.

Thus, 53 dollars and $79\frac{1}{5}$ cents may be expressed as $53.79\frac{1}{5}$ dollars, or $\$53.79\frac{1}{5}$.

144. A sum of money expressed as a decimal of a dollar may be expressed as a decimal of a dime by multiplying by 10; as a decimal of a cent by multiplying by 100; and as a decimal of a mill by multiplying by 1000.

Hence (Art. 122), a sum of money expressed as a decimal of a dollar may be expressed as a decimal of a dime by moving its decimal point one place to the right; as a decimal of a cent by moving the decimal point two places to the right; and as a decimal of a mill by moving the decimal point three places to the right.

Thus, $\$7.32 = 73.2$ dimes $= 732$ cents $= 7320$ mills.

145. A sum of money expressed as a decimal of a mill may be expressed as a decimal of a cent by moving its decimal point one place to the left; as a decimal of a dime by moving the decimal point two places to the left; and as a decimal of a dollar by moving the decimal point three places to the left.

Thus, 7852.1 mills $=$ 785.21 cents $=$ 78.521 dimes $=$ $\$7.8521$.

146. If two sums of money be expressed as decimals of the *same denomination*, they may be added or subtracted by the methods of Arts. 119 or 120; the result being a decimal of the same denomination.

They may also be divided by the method of Art. 127.

Again, a sum of money expressed as a decimal of any denomination may be multiplied or divided by the methods of Arts. 121 or 127; the result being a decimal of the same denomination.

1. Add together \$43.29, 1106.7 cents, 7480 mills, and 515 dimes.

Reducing each sum to the decimal of a dollar, by the rule of Art. 145, we have,

$$\begin{array}{r} \$43.29 \\ 11.067 \\ 7.48 \\ 51.5 \\ \hline \$113.337 \end{array}, \textit{ Ans.}$$

2. Divide .0118364 dimes by .932 mills.

Reducing the dimes to the decimal of a mill by moving the decimal point two places to the right, we have

```
.932)1.18364(1.27, Ans.
      932
      2516
      1864
       6524
       6524
```

EXAMPLES.

3. Express \$19.29 as a decimal of a mill.

4. Express .000382 eagles as a decimal of a cent.

5. Express 453.7 dimes as a decimal of an eagle.

6. Express 29 mills as a decimal of a dollar.

7. Find the sum of \$115.28, 6325.2 cents, 8.3 dimes, and 47101 mills.

8. Find the sum of .437 dimes, 1055.4 mills, \$7.2195, and 543.96 cents.

9. Subtract 3845 mills from $ 9.63.

10. Subtract 115.28 cents from 39.07 dimes.

11. Subtract $ 59.223 from 20091 cents.

12. Subtract $ .092183 from 489.56 mills, and express the result as a decimal of an eagle.

13. Multiply $ 320.16 by 100.

14. Multiply $ 95.78 by .0001.

15. Divide $ 187.25 by 10000.

16. Divide $ 42.56 by .001.

17. Multiply $ 73.29 by 380.

18. Multiply $ 53.08 by 72.9.

19. Multiply $ 216.273 by .414.

20. Multiply 302.8 mills by 967, and express the result as a decimal of a dime.

21. Divide $ 308.238 by 86.1.

22. Divide $ 210.60 by 5400.

23. Divide $ 669.90 by 2175 cents.

24. Divide .410652 mills by .0561 cents.

25. Divide 15.2513 dimes by $ .349.

26. Divide 23.39888 cents by 34.01 dimes.

PROBLEMS.

147. 1. What is the cost of 27 pounds of tea at $43\frac{1}{3}$ cents a pound?

$$\begin{array}{r} 27 \\ \underline{.43\frac{1}{3}} \\ 81 \\ 10\,8 \\ \underline{9} \\ \$\,11.70, \textit{ Ans.} \end{array}$$

We multiply 27 first by 3, then by 4, and finally by $\frac{1}{3}$, and add the results.

2. How many yards of cloth at $66\frac{2}{3}$ cents a yard can be bought for \$8.00?

$$\frac{800}{66\frac{2}{3}}=\frac{800}{\frac{200}{3}}=\overset{4}{\cancel{800}}\times\frac{3}{\cancel{200}}=12,\ \textit{Ans.}$$

\$8.00 is the same as 800 cents. Dividing 800 cents by $66\frac{2}{3}$ cents, the quotient is 12.

3. What is the cost of $3\frac{3}{4}$ tons of coal at \$5.76 a ton?

4. What is the cost of 147 yards of cloth at $16\frac{2}{3}$ cents a yard?

5. Find the sum of $\frac{11}{12}$ of \$5.67, and $\frac{3}{4}$ of \$8.75.

6. How many pounds of coffee, at $41\frac{2}{3}$ cents a pound, can be bought for \$121.25?

7. If $35\frac{7}{12}$ yards of silk cost \$59.78, how much will one yard cost?

8. What is the cost of 18 tons of coal at \$$5.83\frac{1}{3}$ a ton?

9. A grocer received on Monday \$135.25, on Tuesday \$84.40, on Wednesday \$106.65, on Thursday \$122.70, on Friday \$93.62, and on Saturday \$185.56. What were his total receipts for the week?

10. What is the cost of a barrel of sugar weighing 276 pounds, at \$$.04\frac{3}{8}$ a pound?

11. What is the cost of $17\frac{4}{5}$ acres of land at \$238.45 an acre?

12. If $12\frac{3}{8}$ cords of wood cost \$116.66, how much will one cord cost?

13. If bricks be sold at \$6.75 a thousand, how much will 8960 bricks cost?

14. How many barrels, each containing 32.75 gallons, can be filled from 1200 gallons of wine, and how many gallons will remain?

15. How many barrels of flour, at \$$7.35\frac{5}{6}$ a barrel, can be bought for \$$551.87\frac{1}{2}$?

16. If $13\frac{5}{6}$ yards of cloth can be bought for \$33.20, how much will $15\frac{5}{8}$ yards cost?

17. If 24 pounds of butter be given for 100.8 pounds of sugar, how many pounds should be given for 91 pounds of sugar?

18. A man having \$100, spent \$28.59, then received \$15.75, then spent \$48.98, and finally received \$37.37. How much money then had he?

19. A farmer sold 15 loads of wheat, each containing 8.75 bushels, for 92 cents a bushel. How much money did he receive?

20. If $5\frac{1}{3}$ tons of coal cost \$50, how much will $\frac{32}{45}$ of a ton cost?

21. The product of three numbers is 3.9701556. If two of them are 92.7 and 5.16, what is the third?

22. If $13\frac{2}{5}$ barrels of flour cost \$89.11, how much will $\frac{15}{19}$ of a barrel cost?

23. A, B, and C received \$17.40 for a piece of work. If A did $\frac{5}{12}$ of the work, B $\frac{4}{15}$, and C the remainder, how much money should each receive?

24. A man worked $25\frac{5}{6}$ days. He paid out $\frac{3}{8}$ of his earnings for board, and had \$12.40 left. Find his daily wages.

25. A and B start at the same time, from the same place, and walk in the same direction at the rates of 3.4798 and 4.1263 miles an hour, respectively. How far apart are they at the end of 7.255 hours?

26. A dealer sold goods for \$124.25, and gained $\frac{2}{5}$ of what they cost him. How much did they cost him?

27. Which is the greater, $\frac{7}{8}$ of \$16.12, or $\frac{5}{6}$ of \$17.67, and how much?

28. Find the cost of $18\frac{3}{5}$ feet of steel rod, at \$.35 a foot, and $24\frac{1}{6}$ feet at \$.456 a foot.

29. A merchant sold goods for \$87.95, and lost $\frac{2}{7}$ of what they cost him. How much did he lose by the operation?

30. If $15\frac{5}{8}$ yards of cloth can be bought for \$12.73, how many yards can be bought for \$18.76?

31. A man bought 7 car-loads of wheat, each car containing 165 bushels, at \$ .84$\frac{1}{3}$ a bushel. What was the cost?

32. If 5$\frac{3}{8}$ pounds of coffee can be bought for \$ 2.36$\frac{1}{2}$, how many pounds can be bought for \$ 6.54$\frac{1}{2}$?

33. If A can do a piece of work in 2.4 days, and B in 3.2 days, how long will it take both of them together to do the work?

34. Find the cost of 237 bales of silk, each containing 125$\frac{5}{8}$ yards, at \$ 1.26 a yard.

35. A man spent $\frac{2}{3}$ of his money for provisions, $\frac{7}{8}$ of the remainder for clothing, and had \$ 12.87 left. How much had he at first?

36. If .488 of a ton of coal be worth \$ 3.05, how much will 7.56 tons cost?

37. A man left $\frac{1}{2}$ of his estate to his wife, $\frac{5}{8}$ of the remainder to his son, and the rest to his daughter. The wife received \$ 546.75 more than the daughter. What did each receive?

38. A farmer sold 35 tubs of butter, each weighing 43$\frac{1}{5}$ pounds, at 21$\frac{1}{3}$ cents a pound. He bought 21 barrels of flour at \$ 6$\frac{3}{4}$ a barrel, and received the balance in cash. How much money did he receive?

39. Three men, A, B, and C, can do a piece of work in 18, 24, and 36 hours, respectively. How long will it take all of them together to do the work? If they receive \$ 11.25 for the work, how should the money be divided?

40. A merchant bought goods to the amount of \$ 228.60. He kept $\frac{2}{15}$ of them for his own use, and sold the remainder for $\frac{5}{12}$ more than they cost him. How much did he gain?

41. A grocer bought 10 pounds of tea at \$ .38$\frac{7}{8}$ a pound, 12 pounds at \$ .41$\frac{5}{8}$ a pound, and 15 pounds at \$ .433 a pound. He sold the whole at \$ .44$\frac{1}{4}$ a pound. How much did he gain?

42. A gentleman divided $ 22.05 between his two sons in such a way that the younger received $\frac{3}{4}$ as much as the elder. How much did each receive?

43. If a horse travels 8.3 hours a day, at the rate of 6.25 miles an hour, how many days will it take him to travel 830 miles?

44. Find the cost of 12.6 bales of silk, each containing 73.625 yards, at $ 1.20 a yard.

45. A tank has two pipes. One of them can fill it in 8.5 minutes, and the other can empty it in 12.75 minutes. How many minutes will it take to fill the tank, if both pipes are opened together?

46. A man invests $\frac{3}{8}$ of his property in real estate, $\frac{3}{10}$ of the remainder in railway shares, and the balance in city bonds. The amount invested in city bonds exceeds by $ 93.75 the amount invested in real estate. Find the amount of each kind of investment.

47. A gentleman left $\frac{1}{3}$ of his property to his wife, $\frac{1}{4}$ to his elder son, $\frac{1}{5}$ to his younger son, $\frac{1}{8}$ to his daughter, and the balance, $ 369.75, to a charitable institution. How much did each receive?

48. A, B, and C can do a piece of work in 6, 14, and 21 days, respectively. B and C worked alone for 5 days, when they were joined by A, and the work was completed by all of them together. If they received $ 54 for the work, how should the money be divided?

49. Three pipes can empty a tank in 2.25, 3.375, and 5.0625 minutes, respectively. How many minutes will it take to empty the tank if all the pipes are opened?

XI. MEASURES.

148. Measures of Length.

Measures of Length, or Linear Measures, are those used in measuring lengths or distances.

TABLE.

12	inches (in.)	= 1 foot.	(ft.)
3	feet	= 1 yard.	(yd.)
$5\frac{1}{2}$	yards	= 1 rod.	(rd.)
320	rods	= 1 mile.	(mi.)

It follows from the above that

$$1 \text{ mile} = 1760 \text{ yards} = 5280 \text{ feet.}$$

Surveyors use, in the measurement of land, a *chain* (ch.) whose length is 4 rods, divided into 100 *links* (li.) of 7.92 inches each.

80 chains are equal to one mile.

149. Measures of Area.

Measures of Area, Surface Measures, or Square Measures, are those used in measuring areas.

TABLE.

144	square inches (sq. in.)	= 1 square foot.	(sq. ft.)
9	square feet	= 1 square yard.	(sq. yd.)
$30\frac{1}{4}$	square yards	= 1 square rod.	(sq. rd.)
160	square rods	= 1 acre.	(A.)
640	acres	= 1 square mile.	(sq. mi.)

It follows from the above that

$$1 \text{ acre} = 43560 \text{ square feet.}$$

10 square chains are equal to one acre.

150. Measures of Volume.

Measures of Volume, or **Cubic Measures,** are those used in measuring volumes.

TABLE.

1728 cubic inches (cu. in.)	= 1 cubic foot.	(cu. ft.)
27 cubic feet	= 1 cubic yard.	(cu. yd.)

The following is used in measuring wood:

128 cubic feet = 1 cord. (cd.)

151. Measures of Capacity.

Liquid Measures are used in measuring liquids.

TABLE.

4 gills (gi.)	= 1 pint.	(pt.)
2 pints	= 1 quart.	(qt.)
4 quarts	= 1 gallon.	(gal.)

The following are less frequently used:

$31\frac{1}{2}$ gallons = 1 barrel.

63 gallons = 1 hogshead.

Dry Measures are used in measuring grain, vegetables, fruit, etc.

TABLE.

2 pints (pt.)	= 1 quart.	(qt.)
8 quarts	= 1 peck.	(pk.)
4 pecks	= 1 bushel.	(bu.)

The *quart liquid measure* contains $57\frac{3}{4}$ cubic inches, and the *quart dry measure* $67\frac{1}{5}$ cubic inches; the *gallon* contains 231 cubic inches, and the *bushel* 2150.42 cubic inches.

Apothecaries' Liquid Measures are used in compounding medicines.

TABLE.

60 minims (♏)	= 1	fluid dram.	(f ʒ)
8 fluid drams	= 1	fluid ounce.	(f ℥)
16 fluid ounces	= 1	pint.	(O.)

152. Measures of Weight.

Avoirdupois Weight is used in weighing all common articles.

TABLE.

16 drams (dr.)	= 1	ounce.	(oz.)
16 ounces	= 1	pound.	(lb.)
100 pounds	= 1	hundred-weight.	(cwt.)
20 hundred-weight	= 1	ton.	(T.)

The following are used at the United States Custom Houses, and in weighing iron and coal at the mines:

112 pounds	= 1	long hundred-weight.
2240 pounds	= 1	long ton.

Troy Weight is used in weighing gold, silver, and jewels.

TABLE.

24 grains (gr.)	= 1	pennyweight.	(pwt.)
20 pennyweights	= 1	ounce.	(oz.)
12 ounces	= 1	pound.	(lb.)

Apothecaries' Weight is used in compounding medicines.

TABLE.

20 grains (gr.)	= 1	scruple.	(℈)
3 scruples	= 1	dram.	(ʒ)
8 drams	= 1	ounce.	(℥)
12 ounces	= 1	pound.	(℔)

A *pound avoirdupois* contains 7000 grains, and a *pound troy* 5760 grains; the *pound, ounce,* and *grain,* in apothecaries' weight, have the same weight as in troy weight.

153. Measures of Time.

TABLE.

60 seconds (sec.)	= 1 minute.	(min.)
60 minutes	= 1 hour.	(h.)
24 hours	= 1 day.	(d.)
7 days	= 1 week.	(wk.)
365 days	= 1 common year.	(y.)
366 days	= 1 leap year.	

The *Calendar Months* are as follows:

January,	31	days.	July,	31	days.
February,	28 or 29	"	August,	31	"
March,	31	"	September,	30	"
April,	30	"	October,	31	"
May,	31	"	November,	30	"
June,	30	"	December,	31	"

The *Solar Year* is 365 days, 5 hours, 48 minutes, and 49.7 seconds, or very nearly $365\frac{1}{4}$ days.

If the number of any year is divisible by 4, the month of February has 29 days, and the year is called a *leap year;* but if the number of the year is divisible by 100, it is not a leap year, unless it is divisible by 400.

Thus, 1600 is a leap year, but not 1700.

154. English Money.

TABLE.

4 farthings (far.)	= 1 penny.	(*d.*)
12 pence	= 1 shilling.	(*s.*)
20 shillings	= 1 pound.	(£)

The following are also used:

1 florin	= 2 shillings.
1 crown	= 5 shillings.
1 guinea	= 21 shillings.

The English *coin* of the value of 20 shillings is called a *sovereign*.

155. Angular Measures.

Angular Measures are used in measuring angles and arcs of circles.

TABLE.

60 seconds (″)	= 1 minute.	(′)
60 minutes	= 1 degree.	(°)

The following are also used:

A *quadrant* is an arc of 90°.
A *circumference* is an arc of 360°.
A *right-angle* is an angle of 90°.

The length of a degree ($\frac{1}{360}$) of the earth's equator is about $69\frac{1}{6}$ miles.

156. Miscellaneous Tables.

NUMBERS.

12 units	= 1 dozen.
12 dozen	= 1 gross.
12 gross	= 1 great gross.
20 units	= 1 score.

PAPER.

24 sheets	= 1 quire.
20 quires	= 1 ream.
2 reams	= 1 bundle.
5 bundles	= 1 bale.

XII. DENOMINATE NUMBERS.

157. A **Denomination** is a unit of measure; as for example, a *mile*, a *pound*, or a *bushel*. (Compare Art. 142.)

158. Two or more denominations are said to be *of the same kind* when each can be expressed in terms of the others.

Thus, a *mile* and a *rod* are denominations of the same kind.

159. If two or more denominations are of the same kind, the product of the first by any number, plus the product of the second by any number, and so on, is called a **Compound Number.**

For example, 5 miles + 31 rods + 4 yards + 2 feet, or as it is usually expressed, 5 mi. 31 rd. 4 yd. 2 ft., is a compound number.

Note. The number by which the denomination is multiplied may be an integer, a mixed number, or a fraction.

Thus, 3 ft. $5\frac{1}{2}$ in. is a compound number.

In contradistinction, the product of a single denomination by any number is called a **Simple Number.**

For example, 5 mi. is a simple number.

160. Simple and Compound Numbers are called **Denominate Numbers.**

REDUCTION OF DENOMINATE NUMBERS.

161. Reduction Descending.

Reduction Descending is the process of expressing denominate numbers in terms of lower denominations.

1. Express £4 in pence.

£4	
20	Since £1 = 20*s.*, £4 = 4 × 20*s.*, or 80*s.*
80*s.*	Since 1*s.* = 12*d.*, 80*s.* = 80 × 12*d.*, or 960*d.*
12	Therefore, £4 = 960*d.*
960*d.*, *Ans.*	

2. Express 33 rd. 4 yd. 2 ft. in inches.

$$\begin{array}{r} 33 \text{ rd.} \\ \underline{5\frac{1}{2}} \\ 165 \\ \underline{16\frac{1}{2}} \\ 181\frac{1}{2} \text{ yd.} \\ \underline{4} \\ 185\frac{1}{2} \text{ yd.} \\ \underline{3} \\ 556\frac{1}{2} \text{ ft.} \\ \underline{2} \\ 558\frac{1}{2} \text{ ft.} \\ \underline{12} \\ 6702 \text{ in., } \textit{Ans.} \end{array}$$

Since 1 rd. = $5\frac{1}{2}$ yd., 33 rd. = 33 × $5\frac{1}{2}$ yd., or $181\frac{1}{2}$ yd.; then, $181\frac{1}{2}$ yd. + 4 yd. = $185\frac{1}{2}$ yd.

Since 1 yd. = 3 ft., $185\frac{1}{2}$ yd. = $185\frac{1}{2}$ × 3 ft., or $556\frac{1}{2}$ ft.; then, $556\frac{1}{2}$ ft. + 2 ft. = $558\frac{1}{2}$ ft.

Since 1 ft. = 12 in., $558\frac{1}{2}$ ft. = $558\frac{1}{2}$ × 12 in., or 6702 in.

Then, 33 rd. 4 yd. 2 ft. = 6702 in.

Note. To multiply 33 by $5\frac{1}{2}$, in Ex. 2, we multiply it by 5, and then by $\frac{1}{2}$, and add the second result to the first.

To multiply $185\frac{1}{2}$ by 3, we multiply 185 by 3, giving 555; we then multiply $\frac{1}{2}$ by 3, giving $\frac{3}{2}$, or $1\frac{1}{2}$; adding this to 555, the sum is $556\frac{1}{2}$.

3. Express $\frac{5}{6}$ sq. yd. in square inches.

$$\frac{5}{6} \text{ sq. yd.} = \frac{5}{6} \times 9 \text{ sq. ft.}$$

$$= \frac{5}{\cancel{6}} \times 9 \times \overset{24}{\cancel{144}} \text{ sq. in.} = 1080 \text{ sq. in., } \textit{Ans.}$$

4. Express 1.508 h. in seconds.

$$\begin{array}{r} 1.508 \text{ h.} \\ \underline{60} \\ 90.48 \text{ min.} \\ \underline{60} \\ 5428.8 \text{ sec., } \textit{Ans.} \end{array}$$

EXAMPLES.

5. Express 105 bu. in pints.

6. How many pennyweights are there in 27 lb.?

7. How many inches are there in 35 rd.?

8. Reduce 5 cd. to cubic inches.

9. Reduce 127 gal. 3 qt. to gills.
10. How many farthings are there in £7 15*s.* 9*d.*?
11. Express 1 T. 15 cwt. 81 lb. 14 oz. in drams.
12. Express 78 lb. 9 oz. 13 pwt. in grains.
13. Express 11℔ 7℥ 5ʒ 2℈ 13 gr. in grains.
14. Reduce 3 A. 59 sq. rd. 10 sq. yd. to square feet.
15. Express 365 d. 5 h. 48 min. 49.7 sec. in seconds.
16. Reduce 2 mi. 253 rd. 3 yd. 2 ft. 11 in. to inches.
17. Express $13\frac{11}{16}$ gal. in gills.
18. Express $\frac{4}{25}$ T. in drams.
19. Reduce $\frac{27}{64}°$ to seconds.
20. Reduce $\frac{7}{8}$ mi. to inches.
21. How many seconds are there in $\frac{5}{11}$ wk.?
22. How many square inches are there in $\frac{1}{16}$ A.?
23. Reduce £$\frac{105}{128}$ to farthings.
24. Reduce $\frac{73}{120}$ O. to minims.
25. Express $10\frac{79}{252}$ lb. troy in grains.
26. Express 9.53 bu. in pints.
27. Reduce .56 wk. to seconds.
28. How many drams are there in .004992 cwt.?
29. Reduce .03581 rd. to inches.
30. Express 8.604 sq. rd. in square feet.

162. Reduction Ascending.

Reduction Ascending is the process of expressing denominate numbers in terms of higher denominations.

1. Express 127 gi. in terms of higher denominations.

```
4)127 gi.
  2)31 pt. + 3 gi.
   4)15 qt. + 1 pt.
       3 gal. + 3 qt.
3 gal. 3 qt. 1 pt. 3 gi., Ans.
```

Since 4 gi. = 1 pt., there are as many pints in 127 gills as 4 is contained times in 127.

4 is contained in 127 31 times, with the remainder 3; whence, 127 gi. = 31 pt. 3 gi.

Since 2 pt. = 1 qt., there are as many quarts in 31 pints as 2 is contained times in 31; that is, 31 pt. = 15 qt. 1 pt.

Since 4 qt. = 1 gal., there are as many gallons in 15 quarts as 4 is contained times in 15; that is, 15 qt. = 3 gal. 3 qt.

Whence, 127 gi. = 3 gal. 3 qt. 1 pt. 3 gi.

2. Express 5870 ft. in terms of higher denominations.

$$
\begin{array}{l}
3)\underline{5870 \text{ ft.}} \\
\quad 1956 \text{ yd.} + 2 \text{ ft.} \\
\quad\quad \underline{2} \\
11)\underline{3912 \text{ half-yards.}} \\
320)\underline{355 \text{ rd.} + 7 \text{ half-yards}}, \text{ or } 3\tfrac{1}{2} \text{ yd., or } 3 \text{ yd. } 1\tfrac{1}{2} \text{ ft.} \\
\quad\quad 1 \text{ mi.} + 35 \text{ rd.}
\end{array}
$$

1 mi. 35 rd. 3 yd. $1\frac{1}{2}$ ft. + 2 ft. = 1 mi. 35 rd. 4 yd. $\frac{1}{2}$ ft., *Ans.*

Dividing 5870 ft. by 3, the result is 1956 yd. 2 ft.

To reduce 1956 yd. to rods, we divide it by $5\frac{1}{2}$, or $\frac{11}{2}$; that is, we multiply it by 2, and divide the result by 11 (Art. 104).

Now 1956 yd. × 2 = 3912 *half-yards;* dividing this by 11, the result is 355 rd. and 7 half-yards.

Also, 7 half-yards = $3\frac{1}{2}$ yd. = 3 yd. $1\frac{1}{2}$ ft.

Dividing 355 rd. by 320, we have 1 mi. 35 rd.

Therefore, 5870 ft. = 1 mi. 35 rd. 3 yd. $1\frac{1}{2}$ ft. + 2 ft.

But 1 ft. + 2 ft. = 3 ft. = 1 yd.

Whence, 5870 ft. = 1 mi. 35 rd. 4 yd. $\frac{1}{2}$ ft.

EXAMPLES.

Express in terms of higher denominations:

3. 975 pt. (dry meas.)
4. 2367 oz. (av.)
5. 6571 far.
6. 8027 gr. (apoth.)
7. 35798 ♏.
8. 1523 ft.
9. 2609 gi.
10. 29760 far.
11. 323515 sec.
12. 123549″.
13. 49326 gr. (troy.)
14. 45342 in.
15. 113292 sq. ft.
16. 800000 sec.
17. 525086 cu. in.
18. 951000 dr.
19. 334476 in.
20. 41253 sq. in.
21. 70000 in.
22. 89582 sq. ft.

ADDITION OF DENOMINATE NUMBERS.

163. 1. Find the sum of 17 lb. 10 oz. 15 pwt. 19 gr., 12 lb. 9 oz. 19 pwt. 20 gr., and 8 lb. 5 oz. 11 pwt. 15 gr.

17 lb.	10 oz.	15 pwt.	19 gr.
12	9	19	20
8	5	11	15
39 lb.	2 oz.	7 pwt.	6 gr., *Ans.*

The sum of 15 gr., 20 gr., and 19 gr., is 54 gr., or 2 pwt. 6 gr.; we then write 6 gr. under the column of grains.

The sum of 2 pwt., 11 pwt., 19 pwt., and 15 pwt., is 47 pwt., or 2 oz. 7 pwt.; we then write 7 pwt. under the column of pennyweights.

The sum of 2 oz., 5 oz., 9 oz., and 10 oz., is 26 oz., or 2 lb. 2 oz.; we then write 2 oz. under the column of ounces.

The sum of 2 lb., 8 lb., 12 lb., and 17 lb., is 39 lb.; we then write 39 lb. under the column of pounds.

The final result is 39 lb. 2 oz. 7 pwt. 6 gr.

2. Add 3 mi. 201 rd. 3 yd. 2 ft. 9 in., 4 mi. 76 rd. 5 yd. 0 ft. 11 in., 2 mi. 253 rd. 4 yd. 2 ft. 5 in., and 8 mi. 189 rd. 5 yd. 1 ft. 8 in.

3 mi.	201 rd.	3 yd.	2 ft.	9 in.
4	76	5	0	11
2	253	4	2	5
8	189	5	1	8
19 mi.	82 rd.	$2\frac{1}{2}$ yd.	1 ft.	9 in.
			1	6
19 mi.	82 rd.	3 yd.	0 ft.	3 in., *Ans.*

The sum of the inches is 33 in., or 2 ft. 9 in.

The sum of the feet is 7 ft., or 2 yd. 1 ft.

The sum of the yards is 19 yd., or 3 rd. $2\frac{1}{2}$ yd.

The sum of the rods is 722 rd., or 2 mi. 82 rd.

The sum of the miles is 19 mi.

But $\frac{1}{2}$ yd. = 1 ft. 6 in.; which, added to 19 mi. 82 rd. 2 yd. 1 ft. 9 in., gives 19 mi. 82 rd. 3 yd. 0 ft. 3 in.

EXAMPLES.

Add the following:

3. 12 gal. 3 qt. 1 pt. 2 gi., 7 gal. 1 qt. 0 pt. 1 gi., and 24 gal. 2 qt. 1 pt. 2 gi.

4. 4 cwt. 67 lb. 8 oz. 14 dr., 11 cwt. 49 lb. 15 oz. 9 dr., aud 2 cwt. 83 lb. 12 oz. 13 dr.

5. 34 bu. 3 pk. 7 qt. 1 pt., 46 bu. 1 pk. 5 qt. 1 pt., 55 bu. 2 pk. 6 qt. 0 pt., and 27 bu. 3 pk. 4 qt. 1 pt.

6. 71 d. 22 h. 18 min. 45 sec., 36 d. 16 h. 48 min. 30 sec., 60 d. 11 h. 32 min. 12 sec., 49 d. 9 h. 23 min. 54 sec., and 56 d. 17 h. 57 min. 29 sec.

7. 45° 38′ 40″, 123° 17′ 33″, 78° 44′ 55″, and 65° 46′ 18″.

8. 21 ℔ 8 ℥ 5 ʒ 1 ℈ 11 gr., 32 ℔ 10 ℥ 7 ʒ 2 ℈ 18 gr., 8 ℔ 9 ℥ 6 ʒ 2 ℈ 7 gr., and 16 ℔ 6 ℥ 4 ʒ 1 ℈ 14 gr.

9. 7 mi. 163 rd. 2 yd. 2 ft. 11 in., 2 mi. 313 rd. 5 yd. 1 ft. 7 in., 9 mi. 36 rd. 4 yd. 2 ft. 8 in., and 6 mi. 244 rd. 3 yd. 1 ft. 10 in.

10. £25 16*s.* 8*d.* 2 far., £86 6*s.* 11*d.* 3 far., £74 18*s.* 9*d.*, £100 15*s.* 10*d.* 1 far., £64 9*s.* 6*d.* 3 far., and £98 12*s.* 7*d.*

11. 13 T. 19 cwt. 81 lb. 9 oz. 14 dr., 8 T. 11 cwt. 64 lb. 12 oz. 7 dr., 21 T. 16 cwt. 92 lb. 11 oz. 10 dr., 15 T. 9 cwt. 57 lb. 8 oz. 13 dr., and 9 T. 15 cwt. 70 lb. 15 oz. 9 dr.

12. 45 cd. 123 cu. ft. 828 cu. in., 26 cd. 64 cu. ft. 1457 cu. in., and 73 cd. 98 cu. ft. 585 cu. in.

13. 6 lb. 11 oz. 14 pwt. 15 gr., 13 lb. 5 oz. 16 pwt. 8 gr., 28 lb. 8 oz. 13 pwt. 22 gr., and 19 lb. 6 oz. 15 pwt. 13 gr.

14. 3 sq. mi. 288 A. 59 sq. rd. 19 sq. yd. 8 sq. ft. 74 sq. in., 5 sq. mi. 446 A. 115 sq. rd. 23 sq. yd. 6 sq. ft. 109 sq. in., and 4 sq. mi. 610 A. 149 sq. rd. 28 sq. yd. 7 sq. ft. 141 sq. in.

15. 12 mi. 98 rd. 5 yd. 1 ft. 8 in., 9 mi. 212 rd. 4 yd. 2 ft. 10 in., 21 mi. 156 rd. 5 yd. 2 ft. 8 in., 18 mi. 303 rd. 3 yd. 1 ft. 9 in., 36 mi. 286 rd. 4 yd. 2 ft. 11 in., and 30 mi. 123 rd. 5 yd. 1 ft. 10 in.

SUBTRACTION OF DENOMINATE NUMBERS.

164. **1.** Subtract 5 T. 18 cwt. 43 lb. from 14 T. 12 cwt. 69 lb.

14 T.	12 cwt.	69 lb.
5	18	43
8 T.	14 cwt.	26 lb., *Ans.*

43 lb. from 69 lb. leaves 26 lb. Since 18 cwt. is greater than 12 cwt., we take 1 T., or 20 cwt., from the 14 T. of the minuend, leaving 13 T.; adding 20 cwt. to 12 cwt., we have 32 cwt.; then, 18 cwt. from 32 cwt. leaves 14 cwt.

Finally, 5 T. from 13 T. leaves 8 T.

2. Subtract 2 A. 103 sq. rd. 10 sq. yd. 8 sq. ft. 23 sq. in. from 3 A. 100 sq. rd. 10 sq. yd.

3 A.	100 sq. rd.	10 sq. yd.	0 sq. ft.	0 sq. in.
2	103	10	8	23
	156 sq. rd.	$29\frac{1}{4}$ sq. yd.	0 sq. ft.	121 sq. in.
			2	36
	156 sq. rd.	29 sq. yd.	3 sq. ft.	13 sq. in., *Ans.*

We take 1 sq. yd., or 9 sq. ft., or 8 sq. ft. 144 sq. in. from the 10 sq. yd. of the minuend; then, 23 sq. in. from 144 sq. in. leaves 121 sq. in., and 8 sq. ft. from 8 sq. ft. leaves 0 sq. ft.

10 sq. yd. from $39\frac{1}{4}$ sq. yd. leaves $29\frac{1}{4}$ sq. yd., and 103 sq. rd. from 259 sq. rd. leaves 156 sq. rd.

But $\frac{1}{4}$ sq. yd. = 2 sq. ft. 36 sq. in.; which, added to 156 sq. rd. 29 sq. yd. 0 sq. ft. 121 sq. in., gives 156 sq. rd. 29 sq. yd. 3 sq. ft. 13 sq. in.

EXAMPLES.

Subtract the following:

3. 2 bu. 3 pk. 7 qt. from 13 bu. 2 pk. 5 qt. 1 pt.

4. £87 13*s.* 9*d.* 1 far. from £105 13*s.* 8*d.* 3 far.

5. 49° 52′ 41″ from 87° 39′ 25″.

6. 38 gal. 3 qt. 1 pt. 3 gi. from 48 gal. 3 qt. 1 pt. 2 gi.

7. 42 lb. 7 oz. 18 pwt. 4 gr. from 52 lb. 7 oz.

8. 5 cu. yd. 26 cu. ft. 752 cu. in. from 13 cu. yd. 9 cu. ft.

9. 14 O. 13 f℥ 2 fʒ 48 ♏ from 24 O. 4 f℥ 7 fʒ 23 ♏.

10. 7 mi. 116 rd. 4 yd. 2 ft. from 15 mi. 0 rd. 3 yd. 1 ft.

11. 14 T. 16 cwt. 81 lb. 13 oz. 15 dr. from 15 T. 11 cwt.

12. 9 A. 147 sq. rd. 19 sq. yd. 5 sq. ft. from 15 A.

13. 1 d. 20 h. 31 min. 56 sec. from 6 d. 15 h. 48 min.

14. 11 mi. 250 rd. 2 yd. 2 ft. 3 in. from 28 mi. 45 rd. 2 yd. 1 ft. 9 in.

15. 4 A. 152 sq. rd. 28 sq. yd. 7 sq. ft. 130 sq. in. from 23 A. 153 sq. rd. 5 sq. yd. 8 sq. ft. 103 sq. in.

165. To Find the Difference in Time between Two Dates.

To find the *exact number of days* between two dates, we proceed as follows:

1. Find the exact number of days from July 12 to Oct. 5.

19	The number of days left in July is 19.
31	The number of days in August is 31, and in September, 30.
30	
5	From Oct. 1 to Oct. 5 is 5 days.
85 d., *Ans.*	Adding, the required number of days is 85.

Allowance must be made for *leap years* in finding the exact number of days between two dates.

2. Find the exact number of days from Feb. 17, 1880, to May 9, 1888.

Since 1880 and 1884 are leap years, the number of days from Feb. 17, 1880, to Feb. 17, 1888, is 8×365, plus 2, or 2922.

Since 1888 is a leap year, the number of days from Feb. 17, 1888, to May 9, 1888, is $12 + 31 + 30 + 9$, or 82.

Then, 2922 d. + 82 d. = 3004 d., *Ans.*

To find the time between two dates in years, months, and days, the following method is employed by business men:

3. Find the time from June 15, 1887, to April 8, 1892.

From June 15, 1887, to June 15, 1891, is 4 y.

From June 15, 1891, to March 15, 1892, is 9 mo.

From March 15, 1892, to April 8, 1892, the exact number of days is $16 + 8$, or 24.

Then the required result is 4 y. 9 mo. 24 d., *Ans.*

Note. In reckoning by the above method from the 31st day of a month to the 31st day of a month not having 31 days, we reckon to the *last day* of the latter month.

Thus, to find the time from March 31 to Oct. 17, we call from March 31 to Sept. 30 6 mo., and from Sept. 30 to Oct. 17 17 d.; the result is 6 mo. 17 d.

Again, to find the time from Jan. 28 to March 4, in an ordinary year, we call from Jan. 28 to Feb. 28 1 mo., and from Feb. 28 to March 4 4 d.; the result is 1 mo. 4 d.

But in a *leap year*, from Jan. 28 to Feb. 28 is 1 mo., and from Feb. 28 to March 4 is 5 d.; in this case, the time is 1 mo. 5 d.

EXAMPLES.

Find the exact number of days from:

4. March 27, 1886, to Dec. 10, 1886.

5. April 11, 1891, to Jan. 31, 1892.

6. Sept. 14, 1887, to June 5, 1888.

7. July 4, 1889, to May 30, 1891.

8. Nov. 20, 1887, to Aug. 8, 1889.

9. Feb. 18, 1883, to Oct. 1, 1892.

Find the time in years, months, and days from:

10. March 13, 1885, to Jan. 29, 1893.

11. Feb. 19, 1884, to June 8, 1889.

12. April 30, 1888, to March 22, 1890.

13. Aug. 31, 1881, to May 5, 1887.

14. Dec. 16, 1883, to Dec. 1, 1892.

15. Oct. 28, 1886, to March 18, 1888.

MULTIPLICATION OF DENOMINATE NUMBERS.

166. **1.** Multiply 2 mi. 135 rd. 5 yd. 2 ft. 10 in. by 7.

2 mi.	135 rd.	5 yd.	2 ft.	10 in.	
				7	
16 mi.	312 rd.	$2\frac{1}{2}$ yd.	1 ft.	10 in.	
			1	6	
16 mi.	312 rd.	3 yd.	0 ft.	4 in.,	*Ans.*

7×10 in. is 70 in., or 5 ft. 10 in.

7×2 ft., plus 5 ft., is 19 ft., or 6 yd. 1 ft.

7×5 yd., plus 6 yd., is 41 yd., or 7 rd. $2\frac{1}{2}$ yd.

7×135 rd., plus 7 rd., is 952 rd., or 2 mi. 312 rd.

7×2 mi., plus 2 mi., is 16 mi.

But $\frac{1}{2}$ yd. = 1 ft. 6 in.; which, added to 16 mi. 312 rd. 2 yd. 1 ft. 10 in., gives 16 mi. 312 rd. 3 yd. 0 ft. 4 in.

EXAMPLES.

Multiply the following:

2. 13 bu. 3 pk. 5 qt. 1 pt. by 3.
3. 26° 38′ 51.32″ by 8.
4. 25 gal. 2 qt. 1 pt. $3\frac{2}{3}$ gi. by 6.
5. 8 O. 13 f℥ 5 fʒ 31♏ by 7.
6. 18 mi. 275 rd. 5 yd. 2 ft. by 4.
7. 5 d. 19 h. 37 min. 42 sec. by 15.
8. £86 16*s.* 9*d.* $1\frac{1}{2}$ far. by 18.
9. 16 T. 9 cwt. 82 lb. 10 oz. 14.5 dr. by 5.
10. 3 cd. 87 cu. ft. 561 cu. in. by 11.
11. 12 lb. 7 oz. 18 pwt. 13 gr. by 20.
12. 213 rd. 1 yd. 2 ft. 6 in. by 9.
13. 5 A. 97 sq. rd. 22 sq. yd. 4 sq. ft. 75 sq. in. by 10.

DIVISION OF DENOMINATE NUMBERS.

167. 1. Divide 46 A. 52 sq. rd. 21 sq. yd. 6 sq. ft. 30 sq. in. by 6.

6)46 A. 52 sq. rd. 21 sq. yd. 6 sq. ft. 30 sq. in.
7 A. 115 sq. rd. 13 sq. yd. 6 sq. ft. 41 sq. in., *Ans.*

46 A. ÷ 6 = 7 A., with a remainder of 4 A., or 640 sq. rd.

640 sq. rd. + 52 sq. rd. = 692 sq. rd.; 692 sq. rd. ÷ 6 = 115 sq. rd., with a remainder of 2 sq. rd., or $60\frac{1}{2}$ sq. yd.

$60\frac{1}{2}$ sq. yd. + 21 sq. yd. = $81\frac{1}{2}$ sq. yd.; $81\frac{1}{2}$ sq. yd. ÷ 6 = 13 sq. yd., with a remainder of $3\frac{1}{2}$ sq. yd., or $31\frac{1}{2}$ sq. ft.

$31\frac{1}{2}$ sq. ft. + 6 sq. ft. = $37\frac{1}{2}$ sq. ft.; $37\frac{1}{2}$ sq. ft. ÷ 6 = 6 sq. ft., with a remainder of $1\frac{1}{2}$ sq. ft., or 216 sq. in.

216 sq. in. + 30 sq. in. = 246 sq. in.; 246 sq. in. ÷ 6 = 41 sq. in.

Then the result is 7 A. 115 sq. rd. 13 sq. yd. 6 sq. ft. 41 sq. in.

EXAMPLES.

Divide the following:

2. 35 bu. 3 pk. 6 qt. 1 pt. by 3.
3. 60 gal. 0 qt. 1 pt. 1 gi. by 11.
4. 22° 30′ by 8.
5. 234 T. 17 cwt. 89 lb. 6 oz. by 10.
6. 128 cu. yd. 1 cu. ft. 759 cu. in. by 5.
7. 40 lb. 8 oz. 8 pwt. $15\frac{1}{2}$ gr. by 7.
8. 273 rd. 4 yd. 0 ft. 10 in. by 2.
9. £147 9*s.* 9*d.* 2 far. by 15.
10. 451 gal. 1 qt. 0 pt. 3 gi. by 13.
11. 3 wk. 5 d. 6 h. 46 min. 12.8 sec. by 4.
12. 57 ℔ 10 ℥ 2 ʒ 1 ℈ 4 gr. by 12.
13. 19 mi. 91 rd. 2 yd. 1 ft. 6 in. by 9.
14. 22 A. 154 sq. rd. 4 sq. yd. 8 sq. ft. 114 sq. in. by 6.

168. To Multiply or Divide a Denominate Number by a Fraction.

1. Multiply 8 T. 10 cwt. 73 lb. 5 oz. 11 dr. by $\frac{3}{4}$.

We multiply the given denominate number by 3, and divide the result by 4.

	T.	cwt.	lb.	oz.	dr.
	8 T.	10 cwt.	73 lb.	5 oz.	11 dr.
					3
4)	25 T.	12 cwt.	20 lb.	1 oz.	1 dr.
	6 T.	8 cwt.	5 lb.	0 oz.	$4\frac{1}{4}$ dr., *Ans.*

To divide a denominate number by a fraction, multiply it by the denominator, and divide the result by the numerator.

To multiply a denominate number by a mixed number, multiply it by the integer and the fraction separately, and add the results.

To divide a denominate number by a mixed number, the divisor should be expressed in a fractional form.

To multiply or divide a denominate number by a decimal, the latter should be expressed as a common fraction.

EXAMPLES.

2. Multiply £14 16*s.* 9*d.* 3 far. by $\frac{11}{9}$.
3. Multiply 13 lb. 10 oz. 15 pwt. 9 gr. by $3\frac{1}{3}$.
4. Divide 2 T. 3 cwt. 46 lb. 13 oz. 6 dr. by $\frac{3}{7}$.
5. Divide 14 bu. 3 pk. 2 qt. 1 pt. by $2\frac{3}{5}$.
6. Divide 2 d. 9 h. 8 min. 42 sec. by $\frac{5}{6}$.
7. Multiply 7 mi. 210 rd. 4 yd. 2 ft. 9 in. by .4.
8. Divide 2 gal. 2 qt. 1 pt. 2.45 gi. by .35.
9. Multiply 12 cd. 25 cu. ft. 1365 cu. in. by $\frac{4}{7}$.
10. Divide 38° 58′ 16″ by 1.7.
11. Multiply 6 ℔ 5 ℥ 4 ʒ 0 ℈ 10 gr. by 2.5.

169. To Divide One Denominate Number by Another of the Same Kind.

To divide one denominate number by another of the same kind, express them in terms of the *same denomination*, and divide the results.

1. Divide 141 bu. 1 pk. 7 qt. by 7 bu. 3 pk. 3 qt. 1 pt.

The lowest denomination expressed in either of the given numbers is *pints;* reducing both dividend and divisor to pints, we have

141 bu. 1 pk. 7 qt. = 9054 pt.,

and 7 bu. 3 pk. 3 qt. 1 pt. = 503 pt.

$$
\begin{array}{r}
503)9054(18, \textit{Ans.} \\
\underline{503}\phantom{(18, \textit{Ans.}} \\
4024\phantom{(18, \textit{Ans.}} \\
\underline{4024}\phantom{(18, \textit{Ans.}}
\end{array}
$$

EXAMPLES.

Divide:

2. £35 5*s.* 8*d.* 3 far. by £1 8*s.* 2*d.* 3 far.
3. 266 gal. 1 qt. 1 pt. 1 gi. by 8 gal. 2 qt. 0 pt. 3 gi.
4. 52 T. 13 cwt. 87 lb. by 6 T. 11 cwt. 73 lb. 6 oz.
5. 69 d. 21 h. 54 min. 48 sec. by 5 d. 19 h. 49 min. 34 sec.
6. 10 mi. 183 rd. 4 yd. 0 ft. 3 in. by 260 rd. 1 yd. 1 ft. 9 in.

TO EXPRESS A FRACTION OR DECIMAL OF A SIMPLE NUMBER IN LOWER DENOMINATIONS.

170. **1.** Express $\frac{5}{7}$ mi. in lower denominations.

$\frac{5}{7}$ mi. $= \frac{5}{7} \times 320$ rd. $= \frac{1600}{7}$ rd. $= 228\frac{4}{7}$ rd.
$\frac{4}{7}$ rd. $= \frac{4}{7} \times \frac{11}{2}$ yd. $= \frac{22}{7}$ yd. $= 3\frac{1}{7}$ yd.
$\frac{1}{7}$ yd. $= \frac{1}{7} \times 3$ ft. $= \frac{3}{7}$ ft.
$\frac{3}{7}$ ft. $= \frac{3}{7} \times 12$ in. $= \frac{36}{7}$ in. $= 5\frac{1}{7}$ in.

Hence, $\frac{5}{7}$ mi. = 228 rd. 3 yd. 0 ft. $5\frac{1}{7}$ in., *Ans.*

2. Express .89 gal. in lower denominations.

$$\begin{array}{rl} .89 & \text{gal.} \\ 4 & \\ \hline 3.56 & \text{qt.} \\ 2 & \\ \hline 1.12 & \text{pt.} \\ 4 & \\ \hline 0.48 & \text{gi.} \end{array}$$

.89 gal. = .89 × 4 qt., or 3.56 qt.
.56 qt. = .56 × 2 pt., or 1.12 pt.
.12 pt. = .12 × 4 gi., or 0.48 gi.
Then, the required result is 3 qt. 1 pt. 0.48 gi.

3 qt. 1 pt. 0.48 gi., *Ans.*

EXAMPLES.

Express in lower denominations:

3. $\frac{7}{8}$ wk.
4. $\frac{5}{18}$ lb. troy.
5. $\frac{11}{15}$ gal.
6. .3 bu.
7. .871°.
8. .605 T.
9. $\frac{5}{12}$ O.
10. $\frac{8}{9}$ rd.
11. .7 sq. rd.
12. .293 lb.
13. £$\frac{6}{13}$.
14. $\frac{5}{11}$ cd.
15. .51 mi.
16. £.0955.
17. $\frac{4}{7}$ A.

TO EXPRESS A DENOMINATE NUMBER AS A FRACTION OR DECIMAL OF A SINGLE DENOMINATION.

171. 1. Express 71 rd. 0 yd. 1 ft. 10 in. as a fraction of a mile.

10 in. $= \frac{5}{6}$ ft.
$1\frac{5}{6}$ ft. $= (\frac{11}{6} \div 3)$ yd., or $\frac{11}{18}$ yd.
$\frac{11}{18}$ yd. $= (\frac{11}{18} \div \frac{11}{2})$ rd., or $\frac{1}{9}$ rd.
$71\frac{1}{9}$ rd. $= (\frac{640}{9} \div 320)$ mi., or $\frac{2}{9}$ mi., *Ans.*

2. Express 2 d. 4 h. 4 min. 48 sec. as a decimal of a week.

48 sec. $= \frac{4}{5}$ min. $= .8$ min.
4.8 min. $= (4.8 \div 60)$ h. $= .08$ h.
4.08 h. $= (4.08 \div 24)$ d. $= .17$ d.
2.17 d. $= (2.17 \div 7)$ wk. $= .31$ wk., *Ans.*

EXAMPLES.

Express:

3. 10 oz. 13 pwt. 8 gr. as a fraction of a pound.
4. 3*s.* 1*d.* 2 far. as a fraction of a pound.
5. 30′ 24″ as a fraction of a degree.
6. 2 pk. 2 qt. 1 pt. as a fraction of a bushel.
7. 16 h. 19 min. 12 sec. as a fraction of a day.
8. 2 qt. 1 pt. $1\frac{1}{3}$ gi. as a fraction of a gallon.
9. 69 cu. ft. 576 cu. in. as a fraction of a cord.
10. 4 yd. 1 ft. $2\frac{3}{5}$ in. as a fraction of a rod.
11. 71 lb. 6 oz. $13\frac{5}{7}$ dr. as a fraction of a hundred-weight.
12. 5 f℥ 0 fʒ $57\frac{3}{5}$ ♏ as a fraction of a pint.
13. 43 sq. rd. 19 sq. yd. 2 sq. ft. 36 sq. in. as a fraction of an acre.
14. 3 pk. 1 qt. $1\frac{1}{5}$ pt. as a decimal of a bushel.
15. 25′ 39″ as a decimal of a degree.
16. 3 qt. 1 pt. 3 gi. as a decimal of a gallon.
17. 4 d. 19 h. 55 min. 12 sec. as a decimal of a week.

18. 2*s.* 11*d.* 2.08 far. as the decimal of a pound.

19. 16 cu. ft. 1512 cu. in. as a decimal of a cubic yard.

20. 10 oz. 5 pwt. 4.8 gr. as a decimal of a pound.

21. 15 cwt. 6 lb. 6 oz. 6.4 dr. as a decimal of a ton.

22. 9 sq. yd. 0 sq. ft. 97.2 sq. in. as a decimal of a square rod.

23. 259 rd. 1 yd. 0 ft. 3.6 in. as a decimal of a mile.

TO EXPRESS ONE DENOMINATE NUMBER AS A FRACTION OR DECIMAL OF ANOTHER.

172. To express one denominate number as a fraction or decimal of another, reduce them to the same denomination, and divide the results.

1. Express 5 gal. 1 qt. 1 pt. 3 gi. as a fraction of 7 gal. 2 qt. 1 pt. 1 gi.

5 gal. 1 qt. 1 pt. 3 gi. = 175 gi.
7 gal. 2 qt. 1 pt. 1 gi. = 245 gi.
$\frac{175}{245} = \frac{5}{7}$, *Ans.*

2. Express 2 sq. rd. 21 sq. yd. $8\frac{1}{2}$ sq. ft. as a decimal of 3 sq. rd. 6 sq. yd. $4\frac{1}{4}$ sq. ft.

2 sq. rd. 21 sq. yd. $8\frac{1}{2}$ sq. ft. = 742 sq. ft.
3 sq. rd. 6 sq. yd. $4\frac{1}{4}$ sq. ft. = 875 sq. ft.
$\frac{742}{875} = \frac{106}{125} = .848$, *Ans.*

EXAMPLES.

Express:

3. 1 bu. 3 pk. 4 qt. 1 pt. as a fraction of 3 bu. 3 pk. 1 qt.

4. 1 d. 13 h. 30 min. as a fraction of 5 d.

5. £2 5*s.* 11*d.* 1 far. as a fraction of £4 18*s.* 5*d.* 1 far.

6. $2° 46' 30''$ as a fraction of $3° 4' 30''$.

7. 4 T. 13 cwt. $33\frac{1}{3}$ lb. as a fraction of 7 T.

8. 6 rd. 1 yd. 1 ft. 4 in. as a fraction of 7 rd. 4 yd. $1\frac{2}{3}$ ft.

9. 1 ℥ 4 ʒ 0 ℈ 15 gr. as a fraction of 2 ℥ 6 ʒ 0 ℈ 3 gr.

10. 1 lb. 5 oz. 1 pwt. 8 gr. as a fraction of 1 lb. 11 oz. 9 pwt. 8 gr.

11. 45 sq. rd. 27 sq. yd. $5\frac{3}{4}$ sq. ft. as a fraction of 80 sq. rd. 10 sq. yd. 5 sq. ft.

12. 2 gal. 3 qt. 0 pt. 3 gi. as a decimal of 5 gal. 1 qt. 1 pt. 3 gi.

13. 5 bu. 2 pk. 2 qt. 1 pt. as a decimal of 7 bu. 3 pk. 7 qt.

14. 2 lb. 5 oz. 9 dr. as a decimal of 2 lb. 7 oz. 1 dr.

15. 5 cu. ft. 1440 cu. in. as a decimal of 7 cu. ft. 1344 cu. in.

16. 1 h. 54 min. 20 sec. as a decimal of 2 h. 10 min. 40 sec.

17. 3 lb. 6 oz. 7 pwt. 22 gr. as a decimal of 5 lb. 7 oz. 16 pwt. 16 gr.

18. 2 mi. 310 rd. 2.2 yd. as a decimal of 11 mi.

19. 61 sq. rd. 21 sq. yd. $3\frac{3}{4}$ sq. ft. as a decimal of 154 sq. rd. 8 sq. yd. $1\frac{1}{2}$ sq. ft.

LONGITUDE AND TIME.

173. Since the earth revolves upon its axis once in 24 hours, if the sun crosses the meridian of any place at a certain time, it will cross a meridian 1° to the west of the first $\frac{1}{360}$ of 24 hours, or $\frac{1}{15}$ of an hour, later; that is, 4 minutes later.

Hence, the local time for places on the first meridian is 4 minutes faster than for places on the second.

In like manner, the local time on the first meridian is:

1 h. faster than on a meridian 15° to the west;
1 min. faster than on a meridian 15′ to the west;
1 sec. faster than on a meridian 15″ to the west;
4 sec. faster than on a meridian 1′ to the west;
$\frac{1}{15}$ sec. faster than on a meridian 1″ to the west.

1. The longitude of Boston is 71° 4′ W., and of Rome, 12° 27′ E.; what is the difference in time between the cities?

The difference in longitude is 71° 4′ + 12° 27′, or 83° 31′.

The difference in time for 1° being 4 min., for 83° it is 83 × 4 min.; that is, 332 min., or 5 h. 32 min.

The difference in time for 1′ being 4 sec., for 31′ it is 31 × 4 sec.; that is, 124 sec., or 2 min. 4 sec.

Therefore the difference in time is

5 h. 32 min. + 2 min. 4 sec., or 5 h. 34 min. 4 sec., *Ans.*

Note. If the sum of the longitudes of two places, in E. and W. longitude, respectively, is greater than 180°, the difference in longitude may be found by subtracting this sum from 360°.

2. If the local time at Paris is 5 h. 43 min. 56 sec. slower than at Calcutta, what is the difference in longitude between the two places?

The difference in longitude for 1 h. being 15°, for 5 h. it is 5 × 15°, or 75°.

The difference in longitude for 1 min. being 15′, for 43 min. it is 43 × 15′; that is, 645′, or 10° 45′.

The difference in longitude for 1 sec. being 15″, for 56 sec. it is 56 × 15″; that is, 840″, or 14′.

Then the difference in longitude is 75° + 10° 45′ + 14′, or 85° 59′, *Ans.*

EXAMPLES.

Find the difference in time between:

3. New York, lon. 74° 0′ W., and San Francisco, lon. 122° 27′ W.

4. St. Petersburg, lon. 30° 19′ E., and Valparaiso, lon. 71° 42′ W.

5. Calcutta, lon. 88° 19′ 2″ E., and St. Paul, lon. 93° 4′ 55″ W.

6. Berlin, lon. 13° 24′ 28″ E., and Sydney, lon. 152° 19′ 37″ E.

7. When it is 10.30 p.m. at Washington, lon. 77° 3′ W., what time is it at Constantinople, lon. 28° 49′ E.?

8. When it is 3.15 p.m. at Paris, lon. 2° 20′ 17″ E., what time is it at New Orleans, lon. 90° 2′ 28″ W.?

9. When it is 11.35 a.m. at Pekin, lon. 116° 24′ 17″ E., what time is it at Chicago, lon. 87° 34′ 53″ W.?

10. When it is 6.27 a.m. at Canton, lon. 113° 15′ 33″ E., what time is it at Greenwich, England?

Find the difference in longitude between two places whose difference in time is:

11. 5 h. 57 min.

12. 2 h. 34 min. 42 sec.

13. 11 h. 13 min. 5 sec.

14. 8 h. 45 min. 38 sec.

15. The local time at Sandy Hook is 4 h. 56 min. 4 sec. slower than that at Greenwich, England; what is the longitude of Sandy Hook?

16. When it is 4.25 p.m. at Rome, it is 9 h. 42 min. 24 sec. a.m. at Mobile; if the longitude of Rome is 12° 27′ E., what is the longitude of Mobile?

17. When it is 1.20 a.m. at St. Louis, lon. 90° 15′ 15″ W., it is 8 h. 35 min. $1\frac{2}{3}$ sec. a.m. at the Cape of Good Hope. What is the longitude of the Cape of Good Hope?

18. When it is 6.33 p.m. at Jerusalem, lon. 35° 30′ 48″ E., it is 11 h. 17 min. 13 sec. a.m. at Montreal. What is the longitude of Montreal?

19. When it is 3.55 a.m. at Constantinople, lon. 28° 58′ 40″ E., it is 6 h. 50 min. 38 sec. a.m. at Bombay. What is the longitude of Bombay?

PROBLEMS.

174. 1. How many lots, each containing 1 A. 36 sq. rd., can be made from a piece of land containing 20 A. 132 sq. rd.?

2. What is the value of 18 T. 11 cwt. 20 lb. of coal, at $ 5.25 a ton?

3. Which is the faster, a train which runs 225 rods a minute, or one which runs a mile in 85 seconds?

4. If a man can do a piece of work in 7 h. 34 min. 10 sec., how long will it take five men to do the work?

5. A note dated Oct. 15, 1889, was paid July 3, 1891. How long did it run?

6. A gentleman divided his estate of 35 acres equally between his six children. How much did each receive?

7. The sides of a field are, respectively, 8 rd. 1 yd. 2 ft. 7 in., 9 rd. 1 yd. 2 ft. 6 in., 12 rd. 1 yd. 0 ft. 11 in., and 10 rd. 1 yd. 2 ft. 3 in. What is the distance around the field?

8. The capacity of a tank is 13 cu. ft. 576 cu. in. How many barrels of water will it hold, if the capacity of the barrel is 2 cu. ft. 1152 cu. in.?

9. Find the value of 15 A. 68 sq. rd. of land at $225 an acre.

10. How many cannon balls, each weighing 86 lb. 10 oz. $10\frac{2}{3}$ dr., can be formed from a mass of iron weighing 13 cwt.?

11. A merchant buys 10 barrels of wine, each containing 32 gallons, at $3.00 a gallon, and sells it at $87\frac{1}{2}$ cents a quart. How much does he gain?

12. A pipe fills a tank at the rate of 3 qt. 1 pt. a second. If the capacity of the tank is 843 gal. 2 qt., how many minutes will it take to fill it?

13. How many house-lots, each containing 59 sq. rd. 26 sq. yd. $3\frac{1}{4}$ sq. ft., can be made from a piece of land whose area is 114100 sq. ft.?

14. If the pound avoirdupois contains 7000 grains troy, how many pennyweights are there in an ounce avoirdupois?

15. Find the sum of $\frac{1}{3}$ of £5 17*s.* 8*d.*, $\frac{1}{4}$ of £7 5*s.* 11*d.*, and $\frac{1}{5}$ of £3 10*s.* 9*d.*

16. What is the exact number of days from Sept. 21, 1886, to April 5, 1892?

17. A man agreed to build 105 rods of fence. On the first day he built 29 rd. 2 yd., on the second day 33 rd. 4 yd. 2 ft., and on the third day 27 rd. 1 yd. How much remained to be built at the end of the third day?

18. What is the value of 26 square chains of land, at 12 cents a square foot?

19. Express 2 lb. 11 oz. 17 pwt. 14 gr. in apothecaries' weight.

20. If the pound avoirdupois contains 7000 grains troy, how many drams are there in an ounce troy?

21. What is the value of 17160 sq. ft. of land, at $800 an acre?

22. If coal is bought by the long ton at $5.60 a ton, and sold by the short ton at $6.25 a ton, what is the profit on 25 tons?

23. A mass of granite weighs 62 lb. 3.8 oz.; an equal mass of water weighs 23 lb. 15 oz. How many times is granite as heavy as water?

24. If the English sovereign be worth $4.87½, what is the value in cents of 3*s.* 8*d.*?

25. If a train travels at the rate of 36 miles an hour, what is its rate in feet per second?

26. Express 3 lb. 13 oz. 8 dr. in apothecaries' weight.

27. If the quart dry measure contains 67⅕ cubic inches, how many cubic feet are there in a bushel?

28. The cost of a certain article in English money is 3*s.* 11*d.* If the sovereign be worth $4.86, what is the value of the article in cents?

29. What is the value of a dozen silver spoons, each weighing 3 oz. 13 pwt. 12 gr., if silver be worth $0.90 an ounce?

30. The capacity of a tank is 18 cu. ft. 516 cu. in. If the tank can be emptied by a pipe in 3 min. 24 sec., how many cubic inches does it empty in one second?

31. Express 8 ℥ 1 ʒ 1 ℈ 5 gr. in avoirdupois weight.

32. If a man can do a piece of work in 13 h. 39 min. 12 sec., what part of the work can he do in 9 h. 6 min. 8 sec.?

33. If the liquid quart contains 57.75 cubic inches, how many cubic feet are there in a hogshead?

34. Two places on the equator are in longitude 53° 20′ E., and longitude 71° 55′ W., respectively. Find the distance between them in miles, if the length of a degree of the earth's equator be taken as $69\frac{1}{6}$ miles.

35. A man who travelled from A to B, found on arriving there that his watch was 17 min. 23 sec. faster than the local time. If the longitude of A is 83° 28′ W., what is the longitude of B?

36. If a bushel of wheat weighs 60 pounds, what is the value of a carload of wheat weighing 9 T. 13 cwt., at 93 cents a bushel?

37. Which is the greater, $\frac{3}{4}$ of 2 gal. 3 qt. 1 pt., or $\frac{5}{8}$ of 3 gal. 2 qt.; and how much?

38. If the pound avoirdupois contains 7000 grains troy, express the pound troy as a decimal of a pound avoirdupois.

39. If a furnace burns 6 T. 6 cwt. of coal from Nov. 19 to Feb. 11, how many pounds does it burn per day?

40. If a train travels 704 inches a second, how many miles does it travel in 18 min. 54 sec.?

41. A vessel, filled to the brim with water, weighs, with its contents, 45 pounds. A mass of copper is thrown into it, displacing 13 lb. 5 oz. of water. If copper is 8.8 times as heavy as water, how much does the vessel now weigh?

42. A certain township contains 4 sq. mi. 173 A. 120 sq. rd. of improved, and 2 sq. mi. 223 A. 90 sq. rd. of unimproved land. What part of the township is unimproved?

43. The local time at two places on the equator differs by 3 h. 50 min. 24 sec. What is the distance in miles between the places, if the length of a degree of the equator is $69\frac{1}{6}$ miles?

44. If gold be worth $20.25 an ounce troy, what is the value of a mass of gold weighing 28 lb. 11 oz. avoirdupois?

45. Express 9 oz. 8 dr. in troy weight.

46. If a cubic foot of water weighs $62\frac{1}{2}$ pounds, and lead is 11.44 times as heavy as water, how many cubic inches will there be in a piece of lead weighing 35 lb. 12 oz.?

47. The cost of a certain article is $2.70. Find its value in English money, if the sovereign be worth $4.86.

48. A train leaves A for B, 26 miles distant, travelling at the rate of 192 rods a minute. Five minutes later, another train leaves B for A, travelling at the rate of 14 yd. 2 ft. a second. How many miles from A will they meet?

49. If a gallon contains 231 cu. in., and a bushel 2150.42 cu. in., express a liquid quart as a decimal of a dry quart.

50. If a cubic foot of water weighs 1000 ounces, and iron is 7.6 times as heavy as water, what is the weight in pounds of 405 cubic inches of iron?

51. The sides of a field are, respectively, 3 ch. 17 li., 2 ch. 83 li., 3 ch. 44 li., and 2 ch. 68 li. Find the distance around the field in feet.

52. A and B buy a piece of land, containing 8 A. 155 sq. rd., for $7168. If A pays $2048, and B $5120, how much land should each receive?

53. If the moon revolves around the earth in 27 d. 7 h. 42 min., over how many degrees of its orbit does it pass in 1 d. 15 h. 30 min.?

54. If a rail weighs 63 pounds to the yard, how many tons of rail will be required to lay a mile of single track railway?

XIII. THE METRIC SYSTEM.

Note. The teacher may, at his option, reserve the chapter on the Metric System until the remaining chapters of the book have been taken, or omit it entirely.

175. In the **Metric System** of measures, two consecutive denominations of the same kind (Art. 158) are so related that the greater is *ten times* the less.

Note. The Metric System is used exclusively in several foreign countries. In the United States, it is employed in the sciences, and the Coast Survey; it is also used to a certain extent in the Mint and General Post Office.

176. *Subdivisions* of the principal units are expressed by writing before the name of the unit the prefixes:

Milli-, to denote *one-thousandth* of the unit;
Centi-, to denote *one-hundredth* of the unit;
Deci-, to denote *one-tenth* of the unit.

Multiples of the principal units are expressed by writing before the name of the unit the prefixes:

Deka-, to denote *ten* times the unit;
Hekto-, to denote *one hundred* times the unit;
Kilo-, to denote *one thousand* times the unit;
Myria-, to denote *ten thousand* times the unit.

Only a few of the denominations are much used; these will be indicated in the following tables by printing them in larger type.

177. Measures of Length.

The principal unit of length is the **Meter**; it is approximately equal to one ten-millionth part of the distance, measured on a meridian, from the equator to the pole.

TABLE.

10 millimeters (^{mm})	= 1 centimeter.	(^{cm})
10 centimeters	= 1 decimeter.	(^{dm})
10 decimeters	= 1 meter.	(^{m})
10 meters	= 1 dekameter.	(^{Dm})
10 dekameters	= 1 hektometer.	(^{Hm})
10 hektometers	= 1 kilometer.	(^{Km})
10 kilometers	= 1 myriameter.	(^{Mm})

EQUIVALENTS.

1 centimeter	= .3937 in.	1 inch	= 2.54^{cm}.
1 meter	= 39.37 in.	1 foot	= 30.48^{cm}.
1 meter	= 1.0936 yd.	1 rod	= 5.029^{m}.
1 kilometer	= .6214 mi.	1 mile	= 1.6093^{Km}.

Note 1. The *millimeter* and *centimeter* are used for very small measurements in the arts and sciences; the *meter*, in measuring cloth, short distances, etc.; the *kilometer*, in measuring long distances.

Note 2. A kilometer is approximately $\frac{5}{8}$ of a mile.

178. Measures of Area.

The principal unit of area is the **Square Meter**; that is, the area of a square whose side is one meter.

TABLE.

100 square millimeters ($^{sq\,mm}$)	= 1 square centimeter.	($^{sq\,cm}$)
100 square centimeters	= 1 square decimeter.	($^{sq\,dm}$)
100 square decimeters	= 1 square meter.	($^{sq\,m}$)
100 square meters	= 1 square dekameter.	($^{sq\,Dm}$)
100 square dekameters	= 1 square hektometer.	($^{sq\,Hm}$)
100 square hektometers	= 1 square kilometer.	($^{sq\,Km}$)

The following terms are also used:

A centar (^{ca}) = a square meter.
An ar (^{a}) = a square dekameter, or 100 centars.
A hektar (^{Ha}) = a square hektometer, or 100 ars.

EQUIVALENTS.

$1^{sq\ cm}$ = .155 sq. in.	1 sq. in. = $6.452^{sq\ cm}$.
$1^{sq\ m}$ = 1.196 sq. yd.	1 sq. yd. = $.8361^{sq\ m}$.
$1^{sq\ Km}$ = .3861 sq. mi.	1 sq. rd. = $.2529^{a}$.
1^{a} = 3.954 sq. rd.	1 sq. mi. = $2.59^{sq\ Km}$.
1^{Ha} = 2.471 A.	1 A. = $.4047^{Ha}$.

Note. The *square meter* is used in measuring ordinary surfaces; the *square kilometer*, in measuring the areas of countries; the *ar* and *hektar*, in measuring land.

179. Measures of Volume.

The principal unit of volume is the **Cubic Meter**; that is, the volume of a cube whose edge is one meter.

TABLE.

1000 cubic millimeters ($^{cu\ mm}$) = 1 cubic centimeter. ($^{cu\ cm}$)
1000 cubic centimeters = 1 cubic decimeter. ($^{cu\ dm}$)
1000 cubic decimeters = 1 cubic meter. ($^{cu\ m}$)

The following terms are also used:

A decister (^{dst}) = .1 cubic meter.
A ster (st) = 1 cubic meter, or 10 decisters.

EQUIVALENTS.

$1^{cu\ cm}$ = .06103 cu. in.	1 cu. in. = $16.387^{cu\ cm}$.
$1^{cu\ m}$ = 1.308 cu. yd.	1 cu. yd. = $.7645^{cu\ m}$.
1^{st} = .2759 cord.	1 cord = 3.624^{st}.

Note. The *cubic meter* is used in measuring ordinary solids; the *ster*, in measuring wood.

180. Measures of Capacity.

The principal unit of capacity is the **Liter**, which is equal to a cubic decimeter.

TABLE.

10 milliliters (^{ml})	= 1 **centiliter**.	(^{cl})
10 centiliters	= 1 deciliter.	(^{dl})
10 deciliters	= 1 **liter**.	(^{l})
10 liters	= 1 dekaliter.	(^{Dl})
10 dekaliters	= 1 **hektoliter**.	(^{Hl})
10 hektoliters	= 1 kiloliter.	(^{Kl})

EQUIVALENTS.

1 liter	= 61.022 cu. in.	1 hektoliter	= 2.837 bu.
1 liter	= 1.0567 liq. qt.	1 liquid qt.	= .9463^{l}.
1 liter	= .9081 dry qt.	1 dry qt.	= 1.101^{l}.
1 hektoliter	= 3.531 cu. ft.	1 gallon	= 3.785^{l}.
1 hektoliter	= 26.417 gal.	1 bushel	= .3524^{Hl}.

Note. The *liter* is used in measuring liquids and small fruit; the *hektoliter*, in measuring grain, vegetables, and liquids in casks.

181. Measures of Weight.

The principal unit of weight is the **Gram**; that is, the weight of a cubic centimeter (milliliter) of distilled water at its greatest density (39.2° Fahrenheit).

TABLE.

10 milligrams (^{mg})	= 1 **centigram**.	(^{cg})
10 centigrams	= 1 decigram.	(^{dg})
10 decigrams	= 1 **gram**.	(^{g})
10 grams	= 1 dekagram.	(^{Dg})
10 dekagrams	= 1 hektogram.	(^{Hg})
10 hektograms	= 1 **kilogram**.	(^{Kg})

10 kilograms	= 1 myriagram.	(^{Mg})
10 myriagrams	= 1 quintal.	(^{Q})
10 quintals	= 1 metric ton.	(^{T})

EQUIVALENTS.

1 gram	= 15.432 gr.	1 oz. troy	= 31.1035^{g}.
1 gram	$\doteq$ .03527 oz. av.	1 oz. av.	= 28.35^{g}.
1 kilogram	= 2.2046 lb. av.	1 lb. av.	= .4536^{Kg}.
1 metric ton	= 1.1023 T.	1 grain	= .0648^{g}.

Note 1. The *quintal* is very seldom used.

Note 2. The *gram* is used in weighing letters, precious metals, and jewels, and in mixing medical prescriptions; the *kilogram*, in weighing ordinary articles; the *metric ton*, in weighing heavy articles.

Note 3. A *kilogram* is the weight of a cubic decimeter (liter), and a metric ton of a cubic meter (kiloliter), of distilled water at its greatest density.

Note 4. Of the United States coins, the nickel five-cent piece weighs 5 grams, and two silver half-dollars weigh 25 grams.

Note 5. A kilogram is approximately 2$\frac{1}{5}$ pounds avoirdupois.

EXAMPLES.

182. 1. How many centimeters are there in a dekameter?

2. How many milligrams are there in a hektogram?

3. How many square centimeters are there in a square meter?

4. How many square meters are there in a hektar?

5. How many cubic millimeters are there in a cubic decimeter?

6. How many deciliters are there in a cubic meter?

7. How many hektograms are there in a metric ton?

8. How many millimeters are there in a kilometer?

9. How many square decimeters are there in a square hektometer?

10. How many cubic millimeters are there in a ster?

11. How many milliliters are there in a dekaliter?

METRIC NUMBERS.

183. A **Metric Number** is a denominate number (Art. 160) expressed in denominations of the metric system.

A metric number is usually expressed in terms of a single denomination of the same kind.

Thus, $7^{Km}\,3^{Hm}\,9^{Dm}\,1^{m}\,2^{dm}\,8^{cm}$ may be expressed in the forms 73.9128^{Hm}, 73912.8^{dm}, etc.

To *read* a metric number when expressed in terms of a single denomination, the name of the denomination may be given to the number to the left of the decimal point, and the name of the smallest denomination to the portion to the right of the decimal point.

Thus, 73.9128^{Hm} may be read either "73, and 9128 ten-thousandths hektometers," or "73 hektometers, and 9128 centimeters."

184. Reduction of Metric Numbers.

A metric number may be expressed as a decimal of the next lower denomination by moving the decimal point one place to the right; as a decimal of the next lower denomination but one by moving the decimal point two places to the right; and so on. (Compare Art. 144.)

Thus, $73.9128^{Hm} = 739.128^{Dm} = 7391.28^{m}$, etc.

A metric number may be expressed as a decimal of the next higher denomination by moving the decimal point one place to the left; as a decimal of the next higher denomination but one by moving the decimal point two places to the left; and so on. (Compare Art. 145.)

Thus, $5.301^{mg} = .5301^{cg} = .05301^{dg}$, etc.

The above rules do not apply to the table of measures of area and volume given in Arts. 178 and 179; in these cases, the decimal point must be moved *two* and *three* places, respectively.

Thus, $85.64^{sq\,Dm} = 8564^{sq\,m} = 856400^{sq\,dm}$, etc.;

$780^{cu\,mm} = .78^{cu\,cm} = .00078^{cu\,dm}$, etc.

EXAMPLES.

1. Express 4.721^{g} as a decimal of a centigram.

2. Express $.07319^{Km}$ as a decimal of a decimeter.

3. Express 8943^{cl} as a decimal of a kiloliter.

4. Express 32.75^{m} as a decimal of a hektometer, of a dekameter, and of a millimeter.

5. Express $.05487^{sq\ Hm}$ as a decimal of a square meter, of a square kilometer, and of a square centimeter.

6. Express 132^{dl} as a decimal of a hektoliter, of a liter, and of a centiliter.

7. Express $.00651^{Ha}$ in ars, and in centars.

8. Express $.09078^{Mg}$ in metric tons, in hektograms, and in decigrams.

9. Express $.732^{Dm}$ in kilometers, in meters, and in centimeters.

10. Express $4615^{cu\ mm}$ as a decimal of a cubic decimeter, and of a cubic centimeter.

11. Express $.002505^{Kl}$ in dekaliters, in deciliters, and in milliliters.

12. How many decisters are there in $1834^{cu\ dm}$?

13. Express 64.94^{cg} as a decimal of a kilogram, of a dekagram, and of a milligram.

14. What is the weight in hektograms of a cubic centimeter of distilled water at its greatest density?

15. What is the weight in decigrams of a centiliter of distilled water at its greatest density?

16. Express $21.2^{sq\ cm}$ in centars.

17. Express $.908^{cu\ cm}$ in centiliters.

18. Find the weight in myriagrams of $14.04^{cu\ m}$ of water.

19. Express 10.7^{Ha} in square kilometers.

20. How many deciliters of water will weigh $.000686^{cg}$?

21. Express 7.13^{Kl} in cubic dekameters.

22. Express $863^{\text{cu cm}}$ in decisters.

23. Express $.0359^{\text{cu Dm}}$ in hektoliters.

24. How many dekaliters of water will weigh 5.257^{T}?

25. Express $.48352^{\text{dl}}$ of water in cubic millimeters, and find its weight in centigrams.

185. **Addition, Subtraction, Multiplication, and Division of Metric Numbers.**

If two metric numbers be expressed as decimals of the same denomination, they may be added or subtracted by the methods of Arts. 119 or 120; the result being a decimal of the same denomination.

They may also be divided by the method of Art. 127.

Again, a metric number may be multiplied or divided by the methods of Arts. 121 or 127; the result being a decimal of the same denomination.

1. Add 3.561^{Hl}, $.0903^{\text{Dl}}$, 47.07^{l}, and 862.8^{cl}.

Expressing each metric number as the decimal of a liter, we have

$$\begin{array}{r} 356.1^{\text{l}} \\ .903 \\ 47.07 \\ 8.628 \\ \hline 412.701^{\text{l}}, \textit{ Ans.} \end{array}$$

2. Divide 3.71622^{Dg} by 72.3^{cg}.

Expressing 3.71622^{Dg} as the decimal of a centigram, we have

$$\begin{array}{l} 72.3)3716.22(51.4, \textit{ Ans.} \\ \quad\quad\;\; 3615 \\ \quad\quad\;\; \overline{\;1012} \\ \quad\quad\quad\; 723 \\ \quad\quad\quad\; \overline{2892} \\ \quad\quad\quad\; 2892 \\ \quad\quad\quad\; \overline{} \end{array}$$

EXAMPLES.

3. Add 5621^{cl}, $.01906^{Hl}$, $.3027^{Kl}$, and 4.249^{dl}.
4. Add $.153^{Dg}$, 78.26^{mg}, 8.53^{dg}, and $.006097^{Mg}$.
5. Add 47.6^{m}, 83.057^{Dm}, $.04829^{Hm}$, and 9720.4^{cm}.
6. Add $3.114^{sq\,Dm}$, $30135^{sq\,cm}$, and $.00598^{sq\,m}$.
7. Subtract 89.7^{Dg} from 2.8164^{Kg}.
8. Subtract 2.482^{mm} from $.50168^{m}$.
9. Subtract $8.5819^{cu\,dm}$ from $9516.27^{cu\,cm}$.
10. Subtract 60.433^{dl} from 4.307^{Hl}.
11. Multiply 5.03^{Dm} by 2.7, and express the result as a decimal of a hektometer.
12. Multiply $.8259^{sq\,dm}$ by 30.8, and express the result as a decimal of a square meter.
13. Multiply 43.7^{mg} by .519, and express the result as a decimal of a centigram.
14. Multiply $.002846^{Hl}$ by .0733, and express the result as a decimal of a deciliter.
15. Divide 93.98^{ml} by 25.4^{dl}.
16. Divide $.0737505^{Kg}$ by 40.5, and express the result as a decimal of a dekagram.
17. Divide 4.686^{Hm} by 825^{m}.
18. Divide $.0602784^{cu\,cm}$ by .644, and express the result as a decimal of a cubic millimeter.

186. To express a Metric Number in ordinary Units, or an ordinary Denominate Number in Metric Units.

Problems of this kind may be solved by means of the tables of equivalents given in Arts. 177 to 181.

1. Express 3^{Dm} in feet.

We have, $3^{Dm} = 30^{m}$.

But by Art. 177, a meter is equivalent to 1.0936 yd.

Then, $3^{Dm} = 30 \times 1.0936$ yd.

$= 32.808$ yd. $= 98.424$ ft., *Ans.*

2. Express 135 lb. 10 oz. 8 dr. in hektograms.

We have, 135 lb. 10 oz. 8 dr. = 2170.5 oz.

But by Art. 181, an ounce avoirdupois = 28.35^{g}.

Then, 2170.5 oz. av. = 2170.5×28.35^{g}

= 61533.675^{g}.

Whence, 135 lb. 10 oz. 8 dr. = 615.33675^{Hg}, *Ans.*

EXAMPLES.

3. How many decimeters are there in a yard?
4. How many dekaliters are there in a peck?
5. How many hundredweights are there in a myriagram?
6. How many cubic yards are there in a decister?
7. How many square decimeters are there in a square foot?
8. How many cubic feet are there in a cubic decimeter?
9. How many gills are there in a centiliter?
10. How many square feet are there in a centar?
11. How many dekagrams are there in a dram?
12. How many feet are there in a dekameter?
13. How many deciliters are there in a liquid pint?
14. How many hektograms are there in a pound troy?
15. How many cubic yards are there in a kiloliter?
16. Express 31^{Hm} in rods.
17. Express 4.7 pwt. in centigrams.
18. Express 75.1^{cl} in cubic inches.
19. Express .642 sq. mi. in hektars.
20. Express 29 cu. ft. in hektoliters.
21. Express $.0083^{T}$ in hundredweights.
22. Express $.93^{Dg}$ in drams.
23. Express 5 cu. ft. in cubic decimeters.
24. Express 288 cu. ft. in decisters.

25. Express .0153 pt. dry measure in centiliters.

26. Express $.584^{\text{sq Dm}}$ in square rods.

27. Express 10 bu. 2 pk. 5 qt. in dekaliters.

28. Express 5 yd. 1 ft. 11 in. in dekameters.

29. Express 8 sq. yd. 5 sq. ft. 112 sq. in. in square meters.

30. Express 20 cu. ft. 1440 cu. in. in sters.

31. Express 2 lb. 13 oz. 4 dr. in centigrams.

32. Express 3 pk. 7 qt. 1 pt. in cubic decimeters.

33. Express 4 gal. 1 qt. 0 pt. 2 gi. in deciliters.

34. Express 12 cd. 96 cu. ft. in cubic dekameters.

35. Express 8 lb. 3 oz. 10 pwt. in kilograms.

36. Express 5 A. 68 sq. rd. 13 sq. yd. in hektars.

37. Express 1 mi. 181 rd. 3 yd. 2 ft. 6 in. in meters.

MISCELLANEOUS PROBLEMS.

187. **1.** How many dekameters are there in a chain?

2. How many square rods are there in a square hektometer?

3. If a ream of paper is $.7872^{\text{dm}}$ in thickness, what is the thickness in millimeters of a single sheet?

4. How much is gained by buying 3.64^{Ha} of land at $ 1025 a hektar, and selling it at 13¼ cents a square meter?

5. How many dry pints are there in a deciliter?

6. How many dekagrams are there in a scruple?

7. If a horse eats 2^{Dl} of oats in a week, how many weeks will 3 bushels last him?

8. A merchant buys 150^{m} of silk for $ 247.70, and sells it at $ 1.75 a yard. How much does he gain?

9. How much do I lose by buying 49.763^{Kl} of grain, at $ 2.88 a hektoliter, and selling it at 2.39 cents a cubic decimeter?

10. How many pounds, apothecaries' weight, are there in a kilogram?

11. If sulphuric acid is 1.84 times as heavy as water, what is the weight in dekagrams of 26^{l} of the acid?

12. If alcohol is .791 times as heavy as water, how many dekaliters of alcohol will it take to weigh 3061.17^{Hg}?

13. How many deciliters are there in a fluid dram?

14. How many ars are there in a square chain?

15. How many links are there in a decimeter?

16. A merchant buys 300^{l} of wine at \$0.83 a liter. At what price per gallon must he sell it to gain \$50?

17. A tank contains 4.2966^{T} of water. How long will it take to empty it by a pipe through which pass 1.54^{Dl} a second?

18. A block of stone weighs 15 T. 4 cwt. 20 lb. How many cubic meters does it contain, if it is 2.6 times as heavy as an equal bulk of water?

19. How many centiliters are there in a gill?

20. A merchant buys silk at the rate of \$2.35 a yard. At what price per meter must he sell it, so as to neither gain nor lose by the transaction?

21. The scale of a map is 36^{mm} to a kilometer. If the measured distance between two places on the map is 48.276^{cm}, what is the actual distance between them in hektometers?

22. How many decigrams are there in a pennyweight?

23. How many drams are there in a dekagram?

24. What is the weight in kilograms of a cubic foot of water?

25. A tank, containing 738.4^{Mg} of water, has two taps. One fills it at the rate of $.017^{cum}$ a minute, and the other empties the contents at the rate of 5^{dl} a second. How long will it take to empty the tank if both pipes are opened?

26. Find the weight of a pint of water in dekagrams.

27. If a train runs at the rate of 51^{Km} an hour, what is its rate in rods a minute?

28. How many minims are there in a centiliter?

29. If it costs \$ 5 to travel 384^{Km} by rail, what is the rate of fare in cents per mile?

30. If a man walks at the rate of 3.6 miles an hour, how many minutes will it take him to walk a kilometer?

31. How many cubic inches of water weigh a hektogram?

32. How many gallons of water weigh a metric ton?

33. If a horse travels at the rate of 11.2^{Km} an hour, how many miles can he travel in 4 h. 39 min.?

34. A tank can be filled with water in 2 h. 47 min. by a tap through which pass 75^{Hg} a minute. If the tank is filled with kerosene, how much are the contents worth at 28 cents a dekaliter?

35. If a cubic yard of masonry weighs 3500 lb., what is the weight of a cubic decimeter in kilograms?

36. If a cubic inch of bronze weighs .3032 lb., how many cubic centimeters will it take to weigh a hektogram?

37. If a cubic inch of mercury weighs 7.88 oz. avoirdupois, how much does a liter weigh in kilograms?

38. If sea-water is 1.026 times as heavy as fresh water, how many cubic yards of sea-water will it take to weigh a metric ton?

39. If a rail weighs 69 pounds to the yard, how many kilograms does it weigh to the meter?

40. Two places on the equator are in lon. 42° 37′ 57″ E., and lon. 78° 34′ 3″ W., respectively. Find the distance between them in kilometers, if a degree of the equator be taken as $69\frac{1}{6}$ miles.

XIV. INVOLUTION AND EVOLUTION.

188. **Involution** is the process of raising a number to any required power (Art. 55).

This may be effected by taking the given number as a factor as many times as there are units in the exponent of the required power (Art. 55).

1. Find the value of $(\frac{3}{5})^4$.

We have, $(\frac{3}{5})^4 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{81}{625}$, *Ans.*

The following rule is evident from the above:

To raise a fraction to any power, raise both numerator and denominator to the required power, and divide the first result by the second.

EXAMPLES.

Find the values of the following:

2. 87^2.	**4.** 103^3.	**6.** 12^5.	**8.** $(\frac{10}{7})^3$.	**10.** $(\frac{4}{11})^4$.
3. 61^3.	**5.** 25^4.	**7.** $(\frac{1}{3})^4$.	**9.** $(3\frac{3}{4})^2$.	**11.** $(\frac{5}{6})^5$.

189. If one number is the square of another, the second number is said to be the **Square Root** of the first.

Thus, since $9 = 3^2$, 3 is the square root of 9.

190. If one number is the cube of another, the second number is said to be the **Cube Root** of the first.

Thus, since $8 = 2^3$, 2 is the cube root of 8.

191. In like manner, if one number is the fourth power of another, the second number is said to be the **Fourth Root** of the first; and so on.

192. **Evolution** is the process of finding any required root of a number.

193. The **Radical Sign**, $\sqrt{}$, when written over a number, indicates some root of the number.

Thus, $\sqrt{25}$ indicates the square root of 25;
$\sqrt[3]{27}$ indicates the cube root of 27;
$\sqrt[4]{81}$ indicates the fourth root of 81; etc.

The **Index** of a root is the number written over the radical sign to indicate what root of the number is taken.

Thus, in $\sqrt[3]{27}$, the index is 3.

194. If one number is a power of another, the first number is said to be *a perfect power of the degree denoted by the exponent of the power.*

A perfect power of the second degree is called a *perfect square*, and a perfect power of the third degree is called a *perfect cube.*

Thus, since $7^2 = 49$, 49 is a perfect square.

195. It is evident from Ex. 1, Art. 188, that to find any root of a fraction, each of whose terms is a perfect power of the degree denoted by the index of the required root, we *extract the required root of both numerator and denominator, and divide the first result by the second.*

196. 1. Extract the cube root of 343.

We find a number whose cube is equal to 343.

The number is 7; hence, $\sqrt[3]{343} = 7$, *Ans.*

2. Extract the fourth root of $\frac{256}{625}$.

By Art. 195, $\sqrt[4]{\dfrac{256}{625}} = \dfrac{\sqrt[4]{256}}{\sqrt[4]{625}} = \dfrac{4}{5}$, *Ans.*

EXAMPLES.

Find the values of the following:

3. $\sqrt{121}$.
4. $\sqrt[3]{125}$.
5. $\sqrt[4]{1296}$.
6. $\sqrt[5]{1024}$.
7. $\sqrt{\frac{169}{25}}$.
8. $\sqrt[3]{\frac{27}{64}}$.
9. $\sqrt[4]{\frac{16}{81}}$.
10. $\sqrt[5]{\frac{243}{32}}$.

SQUARE ROOT.

197. Let it be required to find the square of 74.

We have, $74 = 70 + 4.$

To multiply $70 + 4$ by itself, we multiply it by 70, and then by 4, and add the second result to the first.

$$\begin{aligned} 70 \times (70 + 4) &= 70^2 + 70 \times 4 \\ 4 \times (70 + 4) &= 70 \times 4 + 4^2 \end{aligned}$$

Whence, $74^2 = 70^2 + 2 \times 70 \times 4 + 4^2.$

That is, *the square of any number of two figures is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

198. It follows from Art. 197 that

$$\begin{aligned} 74^2 - 70^2 &= 2 \times 70 \times 4 + 4^2 \\ &= (2 \times 70 + 4) \times 4. \end{aligned}$$

That is, *if from the square of any number of two figures the square of the tens be subtracted, the remainder is equal to twice the tens, plus the units, multiplied by the units.*

199. We have $1^2 = 1$, $9^2 = 81$, $10^2 = 100$, $99^2 = 9801$, etc.

That is, the square of any number of one figure contains either one or two figures; the square of any number of two figures contains either three or four figures; etc.

Hence, *if a point be placed over every second figure of any number, beginning at the units' place, the number of points shows the number of figures in its square root.*

200. 1. Find the square root of 6889.

$$\begin{array}{rr|l} & \dot{6}8\dot{8}9 & \underline{80 + 3 = 83}, \textit{ Ans.} \\ 80^2 = & 6400 & \\ \hline \text{Trial divisor, } 2 \times 80 = 160 & 489 & \\ 3 & & \\ \text{Complete divisor, } 163 & \underline{489} & \end{array}$$

Pointing the number by the rule of Art. 199, we find that there are two figures in its square root.

The greatest perfect square in 68 is 64, which is the square of 8; then 8 is the tens' figure of the root.

Subtracting the square of 80, or 6400, from 6889, the remainder is 489.

By Art. 198, this is equal to 2×80, plus the units' figure of the root, multiplied by the units' figure of the root.

That is, 489 is equal to 160, plus the units' figure, multiplied by the units' figure.

Then, to obtain the units' figure, we must divide 489 by 160 plus the units' figure.

Now we can find an *approximate* value of the units' figure by dividing 489 by 160.

The quotient is 3+ ; we then infer that the units' figure is 3.

Adding 3 to the *trial-divisor*, 160, we obtain the *complete divisor*, 163; multiplying this by 3, the product is 489, which, subtracted from 489, leaves no remainder.

Then the required square root is 83.

Omitting the ciphers for the sake of brevity, and condensing the process, it will stand as follows:

$$
\begin{array}{r|l}
 & 6\dot{8}8\dot{9} \;\underline{|\;83} \\
 & 64 \\
\hline
163 & 489 \\
 & \underline{489}
\end{array}
$$

From the above example, we derive the following rule:

Separate the number into periods by pointing every second figure beginning with the units' place.

Find the greatest square in the left hand period, and write its square root as the first figure of the root; subtract the square of the first root-figure from the first period, and to the result annex the next period.

Divide this remainder, omitting the last figure, by twice the part of the root already obtained, and annex the quotient to the root, and also to the trial-divisor.

Multiply the complete divisor by the root-figure last obtained, and subtract the product from the remainder.

If other periods remain, we may regard the part of the root already found as tens with respect to the next root-figure, and proceed as before; doubling the part of the root already found for the next trial-divisor.

If any root-figure is 0, annex 0 to the trial-divisor, and annex to the remainder the next period. (Compare Ex. 2.)

2. Find the square root of 6270016.

$$
\begin{array}{r|l}
 & \dot{6}2\dot{7}0\dot{0}1\dot{6} \;\underline{|\;2504,}\ \textit{Ans.} \\
 & 4 \\
\hline
45 & 227 \\
 & 225 \\
\hline
5004 & 20016 \\
 & 20016 \\
\hline
\end{array}
$$

In this case, the second trial-divisor is 50, and the second remainder is 200.

Since 50 is not contained in 20, the third root-figure is 0.

We then annex 0 to the trial-divisor 50, giving 500, and annex to the remainder the next period, 16.

If, on multiplying any complete divisor by the root-figure last obtained, the product is greater than the remainder, the figure of the root last obtained is too great, and one less must be substituted for it.

201. Square Root of Decimals.

We have $1.5^2 = 2.25$, $.61^2 = .3721$, etc.

That is, the square of a decimal of one place is a decimal of two places; the square of a decimal of two places is a decimal of four places; etc.

Hence, *if a point be placed over every second figure of any decimal, beginning with the units' place, and extending in either direction, the number of points to the left of the decimal point shows the number of places in the integral, and the number of points to the right, the number of places in the decimal, portion of the square root.*

202. The rule of Art. 200 may be used to find the square root of a decimal, provided that the decimal point is inserted in its proper position in the root.

Example. Find the square root of 14.3641.

```
      14.3641 | 3.79, Ans.
       9
 67  | 536
     | 469
 749 | 6741
     | 6741
```

EXAMPLES.

203. Find the square roots of the following:

1. 6084.	**7.** .247009.	**13.** 811440.64.
2. .9409.	**8.** .00521284.	**14.** 33443089.
3. 457.96.	**9.** 33.1776.	**15.** 58.384881.
4. 273529.	**10.** .01170724.	**16.** .0022448644.
5. .081796.	**11.** 446.0544.	**17.** 738482.4225.
6. 6544.81.	**12.** .68112009.	**18.** .9733203649.

204. Square Root of an Imperfect Square.

If there is a final remainder, the number has no exact square root; but we may continue the process by annexing periods of ciphers, and obtain an approximate value of the root, correct to any desired number of decimal places.

1. Find the square root of 12 to four decimal places.

```
        12.00000000 | 3.4641+, Ans.
         9
 64    | 300
       | 256
 686   | 4400
       | 4116
 6924  | 28400
       | 27696
 69281 | 70400
       | 69281
```

EXAMPLES.

Extract the square roots of the following to four places of decimals:

2. 7.	**4.** 17.3.	**6.** .1.	**8.** 375.8329.
3. 31.	**5.** .08.	**7.** 2.08627.	**9.** 684.625.

To find the approximate square root of a fraction whose denominator is, and whose numerator is not, a perfect square, we may divide the approximate square root of the numerator by the square root of the denominator.

If the denominator is not a perfect square, the fraction should be reduced to an equivalent fraction whose denominator is a perfect square.

10. Find the square root of $\frac{3}{8}$ to four places of decimals.

$$\sqrt{\frac{3}{8}}=\sqrt{\frac{6}{16}}=\frac{\sqrt{6}}{4}=\frac{2.4494+}{4}=.6123+, \textit{ Ans.}$$

Extract the square roots of the following to four places of decimals:

11. $\frac{11}{4}$.	**13.** $\frac{13}{9}$.	**15.** $\frac{5}{12}$.	**17.** $\frac{7}{18}$.	**19.** $\frac{41}{32}$.
12. $\frac{3}{16}$.	**14.** $\frac{1}{5}$.	**16.** $\frac{9}{11}$.	**18.** $\frac{13}{72}$.	**20.** $\frac{79}{108}$.

CUBE ROOT.

205. Let it be required to find the cube of 74.

By Art. 197, $74^2=70^2+2\times 70\times 4+4^2$.

To multiply the above result by 74, we multiply it by 70, and then by 4, and add the second result to the first.

$$\begin{array}{lr} 70\times(70^2+2\times70\times4+4^2)= & 70^3+2\times70^2\times4+\quad 70\times4^2 \\ 4\times(70^2+2\times70\times4+4^2)= & \underline{70^2\times4+2\times70\times4^2+4^3} \\ \text{Whence,}\qquad\qquad & 74^3=70^3+3\times70^2\times4+3\times70\times4^2+4^3. \end{array}$$

That is, *the cube of any number of two figures is equal to the cube of the tens, plus three times the square of the tens times the units, plus three times the tens times the square of the units, plus the cube of the units.*

206. It follows from Art. 205 that

$$74^3 - 70^3 = 3 \times 70^2 \times 4 + 3 \times 70 \times 4^2 + 4^3$$
$$= (3 \times 70^2 + 3 \times 70 \times 4 + 4^2) \times 4.$$

That is, *if from the cube of any number of two figures the cube of the tens be subtracted, the remainder is equal to three times the square of the tens, plus three times the tens times the units, plus the square of the units, multiplied by the units.*

207. We have, $1^3 = 1$, $9^3 = 729$, $10^3 = 1000$, $99^3 = 970299$, etc.

That is, the cube of any number of one figure contains either one, two, or three figures; the cube of any number of two figures contains either four, five, or six figures; etc.

Hence, *if a point be placed over every third figure of any number, beginning at the units' place, the number of points shows the number of figures in its cube root.*

208. 1. Find the cube root of 74088.

$$\begin{array}{lrr|l}
 & & 7\dot{4}08\dot{8} & \underline{40 + 2} = 42, \textit{ Ans.} \\
 & 40^3 = & 64000 & \\
\hline
\text{Trial divisor,} & 3 \times 40^2 = 4800 & 10088 & \\
 & 3 \times 40 \times 2 = \ \ 240 & & \\
 & 2^2 = \ \ \ \ \underline{4} & & \\
 & \text{Complete divisor, } 5044 & \underline{10088} &
\end{array}$$

Pointing the number by the rule of Art. 207, we find that there are two figures in its cube root.

The greatest perfect cube in 74 is 64, which is the cube of 4; then 4 is the tens' figure of the root.

Subtracting the cube of 40, or 64000, from 74088, the remainder is 10088.

By Art. 206, this is equal to 3×40^2, plus $3 \times 40 \times$ the units' figure of the root, plus the square of the units' figure, multiplied by the units' figure.

That is, 10088 is equal to 4800, plus $3 \times 40 \times$ the units' figure, plus the square of the units' figure, multiplied by the units' figure.

Then to obtain the units' figure, we must divide 10088 by 4800, plus $3 \times 40 \times$ the units' figure, plus the square of the units' figure.

Now we can find an *approximate* value of the units' figure by dividing 10088 by 4800.

The quotient is 2+ ; we then infer that the units' figure is 2.

Adding to the *trial-divisor*, 4800, $3 \times 40 \times 2$, or 240, and 2^2, or 4, we obtain the *complete divisor*, 5044 ; multiplying this by 2, the product is 10088, which, subtracted from 10088, leaves no remainder.

Then the required cube root is 42.

Omitting the ciphers for the sake of brevity, and condensing the process, it will stand as follows:

$$\begin{array}{r|l}
 & 7\dot{4}08\dot{8} \;|\; \underline{42} \\
 & 64 \\ \hline
4800 & 10088 \\
240 & \\
\underline{4} & \\
5044 & \underline{10088}
\end{array}$$

From the above example, we derive the following rule:

Separate the number into periods by pointing every third figure, beginning with the units' place.

Find the greatest cube in the left-hand period, and write its cube root as the first figure of the root; subtract the cube of the first root-figure from the first period, and to the result annex the next period.

Divide this remainder by three times the square of the part of the root already found, with two ciphers annexed, and write the quotient as the next figure of the root.

Add to the trial-divisor three times the product of the last root-figure by the part of the root previously found, with one cipher annexed, and the square of the last root-figure.

Multiply the complete divisor by the figure of the root last obtained, and subtract the product from the remainder.

If other periods remain, we may regard the part of the root already found as tens with respect to the next root-figure, and proceed as before; taking three times the square of the part of the root already found, with two ciphers annexed, for the next trial-divisor.

If any root-figure is 0, annex two ciphers to the trial-divisor, and annex to the remainder the next period. (Compare Ex., Art. 211.)

If, on multiplying any complete divisor by the root-figure last obtained, the product is greater than the remainder, the figure of the root last obtained is too great, and one less must be substituted for it.

2. Find the cube root of 480048687.

	$4\dot{8}00\dot{4}8\dot{6}87$	783, *Ans.*
	343	
14700	137048	
1680		
64		
16444	131552	
1825200	5496687	
7020		
9		
1832229	5496687	

209. In the above example, the first complete divisor is

$$3\times 70^2+3\times 70\times 8+8^2.$$

The second trial-divisor is 3×78^2, with two ciphers annexed.

Now by Art. 197, $3\times 78^2=3\times(70^2+2\times 70\times 8+8^2)$

$$\begin{aligned}&=3\times 70^2+6\times 70\times 8+3\times 8^2\\&=3\times 70^2+3\times 70\times 8+8^2\\&\quad+(3\times 70\times 8+2\times 8^2).\end{aligned}$$

That is, *the second trial-divisor may be obtained by adding to the preceding complete divisor the second number and twice the third number required to form it, and annexing two ciphers to the result.*

The above rule holds for any trial-divisor, and will be found to save much labor in extracting cube roots.

210. Cube Root of Decimals.

We have $1.1^3 = 1.331$, $.25^3 = .015625$, etc.

That is, the cube of a decimal of one place is a decimal of three places; the cube of a decimal of two places is a decimal of six places; etc.

Hence, *if a point be placed over every third figure of any decimal beginning with the units' place, and extending in either direction, the number of points to the left of the decimal point shows the number of places in the integral, and the number of points to the right, the number of places in the decimal, portion of the cube root.*

211. The rule of Art. 208 may be used to find the cube root of a decimal, provided that the decimal point is inserted in its proper position in the root.

Example. Find the cube root of 1073.741824.

$$\dot{1}07\dot{3}.74\dot{1}82\dot{4} \;\underline{|\; 10.24}, \; \textit{Ans.}$$

	1
30000	73741
600	
4	
30604	61208
600	12533824
8	
3121200	
12240	
16	
3133456	12533824

Since 300 is not contained in 73, the second root-figure is 0.

We then annex two ciphers to the trial-divisor 300, giving 30000, and annex to the remainder the next period, 741.

The second trial-divisor is obtained by the rule of Art. 209; adding to the preceding complete divisor, 30604, the first number, 600, and twice the second number, 8, required to form it, we have 31212; annexing two ciphers to this, the result is 3121200.

EXAMPLES.

212. Find the cube roots of the following:

1. 29791.
2. 97.336.
3. .681472.
4. 1860867.
5. 1.481544.
6. .000941192.
7. 8.242408.
8. 51478848.
9. 10077.696.
10. .517781627.
11. 116.930169.
12. .031855013.
13. .724150792.
14. 1039509.197.
15. .000152273304.
16. 395446.904.
17. 196.629718375.
18. .277550577667.

213. Cube Root of an Imperfect Cube.

If there is a final remainder, the number has no exact cube root; but we may obtain an approximate value of the root, correct to any desired number of decimal places.

1. Find the cube root of 3 to three decimal places.

```
                3.000000000 | 1.442+, Ans.
                1
       ------------------
       300     | 2000
       120     |
        16     |
       ---     |
       436     | 1744
       ----------------------
       120     |  256000
        32     |
       ---     |
       58800   |
        1680   |
          16   |
       -----   |
       60496   |  241984
       --------------------------
        1680   |  14016000
          32   |
       -------
       6220800 |
```

EXAMPLES.

Find, to three places of decimals, the cube roots of:

2. 2.
3. 6.
4. 7.2.
5. 41.
6. 169.

To find the approximate cube root of a fraction whose denominator is, and whose numerator is not, a perfect cube, we may divide the approximate cube root of the numerator by the cube root of the denominator.

If the denominator is not a perfect cube, the fraction should be reduced to an equivalent fraction whose denominator is a perfect cube.

7. Find the cube root of $\frac{1}{9}$ to three places of decimals.

$$\sqrt[3]{\frac{1}{9}} = \sqrt[3]{\frac{3}{27}} = \frac{\sqrt[3]{3}}{3} = \frac{1.442+}{3} = .480+, \textit{ Ans.}$$

Find the cube roots of the following to three places of decimals:

8. $\frac{5}{8}$. **9.** $\frac{5}{4}$. **10.** $\frac{7}{27}$. **11.** $\frac{2}{3}$. **12.** $\frac{19}{25}$.

13. Find the cube root of $\frac{13}{9}$ to four places of decimals.

14. Find the cube root of $\frac{35}{72}$ to four places of decimals.

214. If the index of a root is the product of two or more numbers, we may obtain the result by *successive extractions of the simpler roots.*

Thus, since $4 = 2 \times 2$, we may find the fourth root of a number by taking the square root of its square root.

Again, since $6 = 3 \times 2$, we may find the sixth root of a number by taking the cube root of its square root.

EXAMPLES.

1. Find the fourth root of 1185921.
2. Find the fourth root of 33362176.
3. Find the fourth root of 59969536.
4. Find the fourth root of 69799526416.
5. Find the sixth root of 13841287201.
6. Find the sixth root of 75418890625.

XV. MENSURATION.

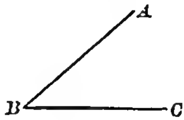
215. **Mensuration** is the process of measuring the lengths of lines, the areas of surfaces, and the volumes of solids.

PLANE FIGURES.

216. The **Area** of a surface is the number of times that it contains another surface, taken as the unit of measurement.

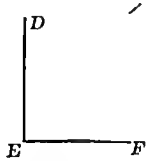
Thus, the statement that the area of a surface is 8 square inches, signifies that a square inch is contained in the surface 8 times.

Two surfaces are said to be *equivalent* when they have equal areas.

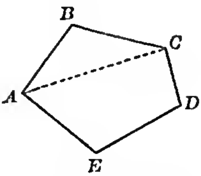


217. An **Angle** is the figure formed by two straight lines which meet at a point; as ABC.

The point of meeting B is called the *vertex* of the angle, and the lines AB and BC are called its *sides*.



A *right angle* is the angle formed by two straight lines which are perpendicular to each other; as DEF.



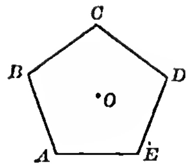
218. A **Polygon** is a plane figure bounded by straight lines; as $ABCDE$.

The bounding lines, AB, BC, etc., are called the *sides* of the polygon, and their sum is called the *perimeter*.

The *angles* of the polygon are the angles ABC, BCD, etc., formed by the consecutive sides; and their vertices B, C, etc., are called the *vertices* of the polygon.

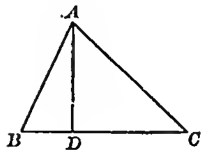
A *diagonal* is a line joining any two vertices which are not consecutive; as AC.

219. A **Regular Polygon** is one whose sides are all equal, and whose angles are all equal; as *ABCDE.*



The point *O*, equally distant from the vertices, is called the *centre* of the polygon.

220. A **Triangle** is a polygon of three sides; as *ABC.*

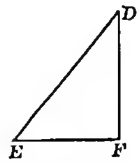


The *base* of the triangle is the side *BC* on which it is supposed to stand; and the vertex *A*, opposite to the base, is called the *vertex* of the triangle.

Note. Either side of a triangle may be regarded as the base.

The *altitude* of the triangle is the perpendicular *AD* drawn from the vertex to the base.

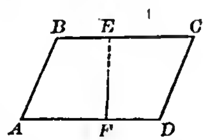
A *right triangle* is one which has a right angle; as *DEF*, which has a right angle at *F*.



The side *DE*, opposite to the right angle, is called the *hypotenuse.*

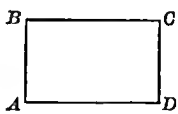
221. A **Quadrilateral** is a polygon of four sides.

222. A **Parallelogram** is a quadrilateral whose opposite sides are parallel; as *ABCD.*



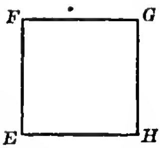
The *base* of the parallelogram is the side *AD* on which it is supposed to stand; the *altitude* is the perpendicular distance *EF* between the base and the side parallel to the base.

223. A **Rectangle** is a parallelogram whose angles are all right angles; as *ABCD.*



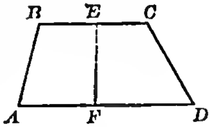
The sides *AB* and *AD* are called the *dimensions* of the rectangle.

A **Square** is a rectangle whose sides are equal; as $EFGH$.



224. A **Trapezoid** is a quadrilateral two of whose sides are parallel; as $ABCD$.

The parallel sides AD and BC are called the *bases* of the trapezoid; and the perpendicular distance between them EF is called the *altitude*.



225. Areas of Polygons.

The proofs of the following principles may be found in any text-book on Geometry:

1. *The area of a triangle is equal to one-half the product of its base and altitude.*

2. *The area of a parallelogram (or rectangle) is equal to the product of its base and altitude.*

3. *The area of a square is equal to the square of one of its sides.*

4. *The area of a trapezoid is equal to one-half the sum of its bases, multiplied by its altitude.*

It is important to observe that, in finding the product of two *lines,* such as the base and altitude of a triangle, their lengths must be expressed in terms of the *same unit,* and the area is obtained in terms of the *square* of this unit.

Thus, to multiply 3 feet by 7 inches, we must first express 3 feet in inches.

Now 3 feet = 36 inches; and multiplying 36 inches by 7 inches, the product is 252 square inches.

EXAMPLES.

226. 1. Find the area of a trapezoid whose bases are 43 in. and 35 in., respectively, and altitude 21 in.

By Art. 225, 4, the area of the trapezoid is $\frac{1}{2} \times (43 + 35) \times 21$, or 819 sq. in., *Ans.*

2. The area of a triangle is $1\frac{1}{4}$ sq. yd.; if its altitude is 40 in., what is its base in feet?

By Art. 225, 1, the base of a triangle is equal to twice its area, divided by its altitude.

We have, $1\frac{1}{4}$ sq. yd. $= \frac{45}{4}$ sq. ft., and 40 in. $= \frac{10}{3}$ ft.

Now $$\frac{45}{2} \div \frac{10}{3} = \frac{\overset{9}{\cancel{45}}}{2} \times \frac{3}{\underset{2}{\cancel{10}}} = \frac{27}{4} = 6\tfrac{3}{4}.$$

Whence, the required base is $6\frac{3}{4}$ ft., *Ans.*

3. What is the side of a square whose area is $7\frac{1}{9}$ sq. rd.?

By Art. 225, 3, the side of a square is equal to the square root of its area.

Now $$\sqrt{7\tfrac{1}{9}} = \sqrt{\tfrac{64}{9}} = \tfrac{8}{3} = 2\tfrac{2}{3}.$$

Hence, the required side is $2\frac{2}{3}$ rd., *Ans.*

4. Find the area in square inches of a triangle whose base is $3\frac{1}{2}$ ft., and altitude $2\frac{1}{3}$ ft.

5. Find the area in square feet and square inches of a square, each side of which is 3 ft. 10 in.

6. Find the area in square yards of a parallelogram whose base is $7\frac{1}{2}$ ft., and altitude 64 in.

7. Find the base in feet of a triangle whose area is $14\frac{1}{2}$ sq. ft., and altitude 87 in.

8. Find the altitude in yards of a rectangle whose area is 1782 sq. ft., and base 3 rd.

9. The area of a square is 20 sq. rd. 20 sq. yd.; what is its side in feet?

10. Find the area in square inches of a trapezoid whose bases are $8\frac{3}{4}$ ft. and $5\frac{5}{6}$ ft., respectively, and altitude $3\frac{3}{5}$ ft.

11. Find the altitude in yards of a triangle whose area is 3 sq. ft., and base 27 in.

12. Find the base in inches of a rectangle whose area is $3\frac{1}{8}$ sq. yd., and altitude $4\frac{1}{2}$ ft.

13. Find the area in acres of a square field whoso side is 396 ft.

14. Find the area of a floor whose length is 15 ft. 4 in., and width 12 ft. 10 in.

15. A triangular house-lot contains 3 acres. If its base is 500 feet, what is its altitude in rods?

16. Find the width of a rectangular field whose area is 15 acres, and length 75 rods.

17. If the side of a square field is 14 rd. 3 yd., how much is the field worth at $242 an acre?

18. Find the altitude in feet of a trapezoid whose area is $139\frac{1}{2}$ sq. ft., and bases 19 ft. and 12 ft., respectively.

19. How many bricks, each 8 inches long and $4\frac{1}{2}$ inches wide, will bo required to lay a sidewalk $7\frac{1}{2}$ feet wide and 220 feet long?

20. A rectangular garden, 64 feet long and 35 feet wide, is surrounded by a walk 3 feet wide. How many square feet are there in the walk?

21. A map is $1\frac{1}{3}$ ft. long, and 1 ft. wide. If the scale of the map is $2\frac{1}{2}$ miles to an inch, how many square miles of country does it represent?

22. Find the length in yards of a rectangular field whose area is 7 acres, and width 363 feet.

23. Find the lower base in rods of a trapezoid whose area is 1 A. 50 sq. rd., upper base 99 yd., and altitude 165 ft.

24. The dimensions of a rectangular floor are 22 ft. 6 in., and 16 ft. 9 in. What will it cost to cover it with oil-cloth, at $1.52 a square yard?

25. How many paving-stones, each 7 inches long and 4 inches wide, will be required to pave two miles of street, 63 ft. in width?

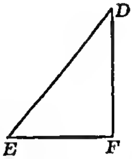
26. A field is 10 rd. 5 yd. long, and 13 rd. $3\frac{1}{2}$ yd. wide. How much is it worth at $605 an acre?

27. A man sold a rectangular field for $3\frac{1}{3}$ cents a square foot, receiving the sum of \$ 5288.40; if the field was 28 rd. 2 yd. long, what was its width?

28. If the area of a field is 2 A. 32 sq. rd. 26 sq. yd. 2 sq. ft., and its width 15 rd. 2 yd. 2 ft., what is its length?

29. A field is 18 rd. 3 yd. 2 ft. wide, and 33 rd. 1 yd. $1\frac{1}{2}$ ft. long. How much is it worth at \$ 484 an acre?

227. It is proved in Geometry that



In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Note. This means that, *if the sides are all expressed in terms of the same unit*, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

It follows from the above that

In a right triangle, the square of either side about the right angle is equal to the square of the hypotenuse, minus the square of the other side.

EXAMPLES.

228. **1.** The sides about the right angle of a right triangle are 5 in. and 1 ft., respectively; find the hypotenuse in inches.

We have $\quad 1 \text{ ft.} = 12 \text{ in.}$

But $\quad \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13.$

Whence, the hypotenuse is 13 in., *Ans.*

2. The hypotenuse of a right triangle is $2\frac{5}{6}$ in., and one of the sides about the right angle is $1\frac{1}{3}$ in. Find the other side.

We have
$$\sqrt{(2\tfrac{5}{6})^2 - (1\tfrac{1}{3})^2} = \sqrt{(\tfrac{17}{6})^2 - (\tfrac{4}{3})^2}$$
$$= \sqrt{\tfrac{289}{36} - \tfrac{16}{9}} = \sqrt{\tfrac{225}{36}} = \tfrac{15}{6} = 2\tfrac{1}{2}.$$

Whence, the required side is $2\frac{1}{2}$ in., *Ans.*

3. The diagonal of a square is 2 feet; find the approximate value of its side to three places of decimals.

By Art. 227, the square of the diagonal of a square is equal to twice the square of its side.

Then 4 sq. ft. is equal to twice the square of the side.

Hence, the side is equal to the square root of 2 sq. ft.

The approximate value of $\sqrt{2}$ to three decimal places is 1.404.

Whence, the required side is 1.404+ ft., *Ans.*

4. The sides about the right angle of a right triangle are 7 in. and 2 ft., respectively. Find the hypotenuse in inches.

5. The sides about the right angle of a right triangle are 1 rd. $3\frac{1}{2}$ yd., and 7 rd. $1\frac{1}{2}$ yd., respectively. Find the hypotenuse.

6. The hypotenuse of a right triangle is $3\frac{1}{3}$ ft., and one of the sides about the right angle is $\frac{2}{3}$ yd. Find the other side in inches.

7. The hypotenuse of a right triangle is 1 yd. 2 ft. 1 in., and one of the sides about the right angle is 11 in. Find the other side.

8. The diagonal of a square is 15 in. Find the approximate value of its side to four places of decimals.

9. The sides about the right angle of a right triangle are 10 in. and 7 in., respectively. Find the approximate value of the hypotenuse to four places of decimals.

10. What is the length of the longest straight line that can be drawn on a floor whose length is 17 ft. 8 in., and width 13 ft. 3 in.?

11. How far from a tower 35 ft. high must the foot of a ladder 37 ft. long be placed, so as to exactly reach the top of the tower?

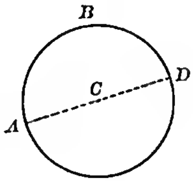
12. A tree was broken off 12 ft. above the ground, and fell so that its top lay 47 ft. 3 in. from the foot of the tree, the end where it was broken resting on the stump. What was the height of the tree?

13. A vessel sails due east at the rate of $6\frac{2}{5}$ miles an hour, and another sails due south at the rate of 12 miles an hour. How far apart are they at the end of 7 h. 45 min.?

14. A ladder $37\frac{1}{2}$ feet long is placed so that it just reaches a window $22\frac{1}{2}$ ft. above the street; and when turned about its foot, just reaches a window 36 ft. above the street on the other side. Find the width of the street.

15. If the area of a square is 33 sq. ft. 50 sq. in., what is the length of its diagonal?

229. A **Circle** is a portion of a plane bounded by a curved line, all points of which are equally distant from a point within called the *centre;* as *ABD.*



The bounding curve is called the *circumference* of the circle.

A *radius* is a straight line drawn from the centre to the circumference, as *CD*; a *diameter* is a straight line drawn through the centre, having its extremities in the circumference; as *AD.*

230. Measurement of the Circle.

It is proved in Geometry that, approximately,

1. *The circumference of a circle is equal to its diameter multiplied by* 3.1416.

2. *The area of a circle is equal to the square of its radius multiplied by* 3.1416.

Note. The above rules give the circumference in terms of the unit in which the diameter is expressed, and the area in terms of the square of the unit in which the radius is expressed.

The following rules are also useful:

3. *The circumference of a circle is approximately equal to twice its radius multiplied by* 3.1416.

4. *The area of a circle is approximately equal to one-fourth the square of its diameter multiplied by* 3.1416.

EXAMPLES.

231. 1. What is the circumference of a circle whose diameter is 7 inches?

By Art. 230, 1, the required circumference is

$$7 \times 3.1416 = 21.9912 \text{ in., } \textit{Ans.}$$

2. Find the diameter of a circle whose area is 35 sq. ft.

By Art. 230, 4, the square of the diameter of a circle is equal to four times the area divided by 3.1416.

We have $\frac{140}{3.1416} = 44.5632+.$

The square root of 44.5632+ is 6.67+.

Whence, the required diameter is 6.67+ ft., *Ans.*

3. The radius of a circle is 7 inches. Find its circumference and area.

4. The diameter of a circle is 50 ft. Find its circumference in yards.

5. The circumference of a circle is 33 rods. Find its radius.

6. Find the diameter in inches of a circle whose area is one square foot.

7. If the diameter of the earth is 7912 miles, what is the distance around it?

8. A wheel is 2 ft. 3 in. in diameter. How many miles does it travel in revolving 2000 times?

9. How many acres are there in a circular field whose diameter is 500 feet?

10. A horse is tied by a rope 31 ft. 6 in. long; over how many square yards of ground can he graze?

11. A wheel turns 29 times in travelling 154 yd. 2 ft. Find its diameter in inches.

12. The floor of a room 12 ft. 3 in. long, and 10 ft. 8 in. wide, has two circular openings whose radii are 2 ft. 1 in., and 1 ft. 5 in., respectively. Find the area of floor remaining.

13. A circular pond, 100 ft. in diameter, is surrounded by a walk 4 feet wide. Find the area of the walk.

14. If a wheel is 5 feet in diameter, how many times does it revolve in running 27 miles?

15. The diameter of a circle is 10 inches. What is the side of an equivalent square (Art. 216)?

16. The side of a square is 8 feet. What is the circumference of an equivalent circle?

17. Two plots of ground, one a square, the other a circle, contain each 70,686 sq. ft. How much longer is the perimeter of the square than the circumference of the circle?

SOLIDS.

232. The *volume* of a solid is the number of times that it contains another solid, adopted as the unit of measurement.

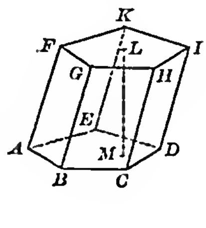
Thus, the statement that the volume of a solid is 6 cu. ft., means that a cubic foot is contained in the solid 6 times.

Two solids are said to be *equivalent* when they have equal volumes.

233. A **Polyedron** is a solid bounded by planes.

The bounding planes are called the *faces* of the polyedron; their intersections are called the *edges*, and the intersections of the edges are called the *vertices*.

234. A **Prism** is a polyedron two of whose faces are equal and parallel, the other faces being parallelograms; as *A–I*.



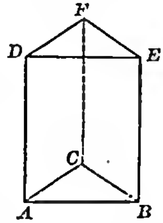
The equal and parallel faces, *ABCDE* and *FGHIK*, are called the *bases* of the prism, and the remaining faces the *lateral faces*.

The lateral faces taken together form the *lateral surface* of the prism; and their intersections, *AF*, *BG*, etc., are called the *lateral edges*.

The *lateral area* is the area of the lateral surface.

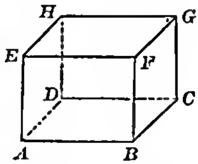
The *altitude* is the perpendicular distance LM between the planes of the bases.

235. A **Right Prism** is one whose lateral edges are perpendicular to its bases; as ABC–DEF.



The lateral faces are rectangles, and the lateral edges are equal to the altitude.

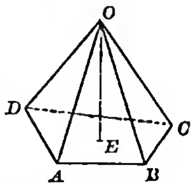
236. A **Rectangular Parallelopiped** is a right prism whose six faces are all rectangles; as A–G.



The *dimensions* are the three edges which meet at any vertex.

A **Cube** is a rectangular parallelopiped whose six faces are all squares.

237. A **Pyramid** is a polyedron bounded by a polygon and a series of triangles having a common vertex; as O–$ABCD$.



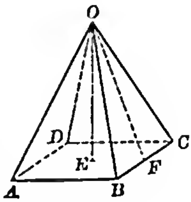
The polygon $ABCD$ is called the *base* of the pyramid; and the common vertex O of the triangular faces is called the *vertex*.

The triangular faces are called the *lateral faces*, and taken together form the *lateral surface*.

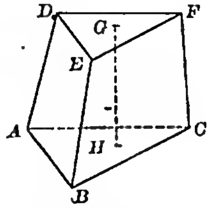
The intersections OA, OB, etc., of the lateral faces are called the *lateral edges*; and the area of the lateral surface is called the *lateral area*.

The *altitude* is the perpendicular OE drawn from the vertex to the base.

238. A **Regular Pyramid** is one whose base is a regular polygon, and whose vertex lies in the perpendicular erected at the centre of the base; as O–$ABCD$.

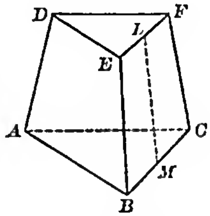


The *slant height* of a regular pyramid is the altitude of any one of its lateral faces; that is, it is the straight line drawn from the vertex to the middle point of any side of the base; as *OF*.



239. A **Frustum** of a pyramid is that portion of a pyramid included between the base and a plane parallel to the base; as *ABC–DEF*.

The *altitude* of the frustum is the perpendicular distance between the planes of its bases; as *GH*.



240. The *slant height* of a frustum of a regular pyramid is the straight line joining the middle points of the parallel sides of any lateral face; as *LM*.

241. Lateral Areas and Volumes of Polyedrons.

The proofs of the following principles may be found in any text-book on Solid Geometry:

1. *The lateral area of a right prism (or rectangular parallelopiped) is equal to the perimeter of its base multiplied by its altitude.*
2. *The volume of a prism (or rectangular parallelopiped) is equal to the area of its base multiplied by its altitude.*
3. *The volume of a cube is equal to the cube of one of its edges.*
4. *The lateral area of a regular pyramid is equal to the perimeter of its base multiplied by one-half its slant height.*
5. *The volume of a pyramid is equal to the area of its base multiplied by one-third its altitude.*
6. *The lateral area of a frustum of a regular pyramid is equal to one-half the sum of the perimeters of its bases, multiplied by its slant height.*

7. *The volume of a frustum of a pyramid is equal to the sum of the areas of its bases, plus the square root of the product of the areas of its bases, multiplied by one-third its altitude.*

To multiply an *area* by a *length*, the area must be expressed in terms of the square of the unit in which the length is expressed, and the product is obtained in terms of the cube of this unit.

Thus, to multiply 2 sq. ft. by 10 in., we must first express the 2 sq. ft. in square inches.

Now 2 sq. ft. = 288 sq. in.; and multiplying 288 sq. in. by 10 in., the product is 2880 cu. in.

EXAMPLES.

242. **1.** Find the lateral area of a regular pyramid, the perimeter of whose base is 17 in., and slant height 8 in.

By Art. 241, 4, the required lateral area is

$$\tfrac{1}{2} \times 17 \times 8, \text{ or } 68 \text{ sq. in., } \textit{Ans.}$$

2. The volume of a rectangular parallelopiped is 693 cu. ft., and the dimensions of its base are 11 ft. and 7 ft. Find its altitude, and the area of its entire surface.

The area of the base is 7×11, or 77 sq. ft.

By Art. 241, 2, the altitude of the parallelopiped is equal to its volume, divided by the area of its base.

Hence, the required altitude $= \frac{693}{77}$, or 9 ft.

By Art. 241, 1, the lateral area is equal to

$$2 \times (7 + 11) \times 9, \text{ or } 324 \text{ sq. ft.}$$

The area of the two bases is 2×77, or 154 sq. ft.

Hence, the area of the entire surface is

$$324 \text{ sq. ft.} + 154 \text{ sq. ft., or } 478 \text{ sq. ft., } \textit{Ans.}$$

3. Find the volume of a frustum of a pyramid whose lower base is a square 7 in. on a side, upper base 4 in. on a side, and altitude 6 in.

The areas of the bases are 49 sq. in. and 16 sq. in., respectively.

Now, $\sqrt{49 \times 16} = \sqrt{784} = 28.$

Hence, the square root of the product of the areas of the bases is 28 sq. in.

Then by Art. 241, 7, the required volume is

$$\tfrac{1}{3} \times (49 + 16 + 28) \times 6, \text{ or } 186 \text{ cu. in., } Ans.$$

4. Find the lateral area and volume of a prism whose altitude is 11 in., having for its base a right triangle whose sides are 5 in., 12 in., and 13 in.

5. Find the volume and area of the entire surface of a cube whose edge is $3\frac{1}{4}$ inches.

6. Find the lateral area of a regular pyramid whose base is a square 6 ft. on a side, and slant height 12 feet.

7. The volume of a prism, whose base is a square, is 637 cu. ft., and its altitude is 13 ft. Find the length of each side of the base, and the lateral area.

8. What is the volume of a pyramid whose altitude is 21 in., having for its base a right triangle whose sides are 8 in., 15 in., and 17 in.?

9. Find the lateral area of a frustum of a regular pyramid whose lower base is a square 9 ft. on a side, upper base 5 ft. on a side, and slant height 14 ft.

10. The volume of a pyramid, whose base is a square, is 847 cu. in., and its altitude is 21 in. Find the length of each side of the base.

11. Find the volume of a frustum of a pyramid whose lower base is a rectangle 15 in. by 6 in., upper base 5 in. by 2 in., and altitude 15 in.

12. The lateral area of a regular pyramid is 1680 sq. in. The base is a triangle whose sides are all equal, and the slant height is 35 in. Find the length of each side of the base.

13. A box is 13 in. long, 12 in. wide, and 7 in. deep. Find its volume, and the area of its entire surface.

14. The volume of a box is 84 cu. ft., and the dimensions of its bottom are 7 ft. and 4 ft. Find its depth, and the area of its entire surface.

15. A wagon 7 ft. long and 4 ft. wide is piled with wood to a depth of $5\frac{1}{2}$ ft. What is the value of the wood at \$7.04 a cord?

16. The lateral area of a frustum of a regular pyramid is 936 sq. in. The lower base is a square 18 in. on a side, and the upper base is 6 in. on a side. Find the slant height of the frustum.

17. The volume of a cube is $4\frac{17}{27}$ cu. ft. What is the area of its entire surface in square inches?

18. What will be the cost of a pile of wood $35\frac{1}{2}$ ft. long, $6\frac{1}{2}$ ft. high, and 4 ft. wide, at \$7.68 a cord?

19. A monument whose height is 12 ft., is in the form of a pyramid with a square base, 2 ft. $10\frac{1}{2}$ in. on a side. Find its weight, at 180 lb. to the cubic foot.

20. How many bricks, each 8 in. long, $2\frac{3}{4}$ in. wide, and 2 in. thick, will be required to build a wall 18 ft. long, 3 ft. high, and 11 in. thick?

21. What must be the length of a pile of wood that is 3 ft. 9 in. wide, and 5 ft. 4 in. high, to contain 5 cords?

22. A monument is in the form of a frustum of a square pyramid 8 ft. in height, surmounted by a square pyramid 2 ft. in height. If each side of the lower base of the frustum is 3 ft., and each side of the upper base 2 ft., find the volume of the monument.

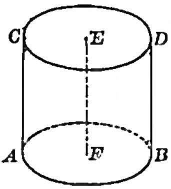
23. The volume of a frustum of a pyramid is 210 cu. in. The lower base is a right triangle whose sides are 6 in., 8 in., and 10 in., and the upper base a right triangle whose sides are 3 in., 4 in., and 5 in. Find the altitude of the frustum.

24. A box made of 2 in. plank, without a cover, measures on the outside 3 ft. 2 in. long, 2 ft. 3 in. wide, and 1 ft. 6 in. deep. How many cubic feet of material were used in its construction?

25. The base of a square pyramid is 14 in. on a side, and the altitude is 24 in. Find its lateral area and volume.

26. The water in a certain ditch flows at the rate of $2\frac{3}{4}$ miles an hour. If the ditch is 3 ft. wide, and 2 ft. deep, how many cubic feet of water pass through it in one day?

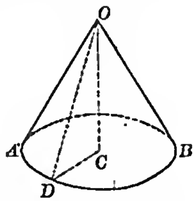
243. A **Cylinder** is a solid bounded by two parallel circles, and a curved surface all points of which are equally distant from a straight line within called the *axis*; as *A–D*.



The parallel circles *AB* and *CD* are called the *bases* of the cylinder, and the curved surface is called the *lateral surface*.

The *lateral area* is the area of the lateral surface, and the *altitude* is the perpendicular distance *EF* between the bases.

244. A **Cone** is a solid bounded by a circle, and a curved surface which tapers uniformly to a point called the *vertex*; as *OAB*.

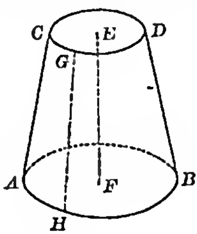


The circle *AB* is called the *base* of the cone, and the curved surface is called the *lateral surface*.

The *lateral area* is the area of the lateral surface, and the *altitude* is the perpendicular *OC* from the vertex to the base.

The *slant height* is the straight line drawn from the vertex to any point in the circumference of the base; as *OD*.

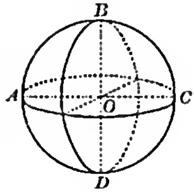
245. A **Frustum** of a cone is that portion of a cone included between the base and a plane parallel to the base; as *A–D*.



The *altitude* of the frustum is the perpendicular distance between the bases; as *EF*.

The *slant height* is that portion of the slant height of the cone included between the bases of the frustum; as *GH*.

246. A Sphere is a solid bounded by a curved surface, all points of which are equally distant from a point within called the *centre*; as *ABD*.



A *radius* is a straight line drawn from the centre to the surface, as *OA*; a *diameter* is a straight line drawn through the centre, having its extremities in the surface, as *AC*.

247. Measurement of the Cylinder, Cone, and Sphere.

It is proved in Geometry that:

1. *The lateral area of a cylinder is equal to the circumference of its base multiplied by its altitude; or, approximately, to twice the radius of its base, times its altitude, times* 3.1416.

2. *The volume of a cylinder is equal to the area of its base multiplied by its altitude; or, approximately, to the square of the radius of its base, times its altitude, times* 3.1416.

3. *The lateral area of a cone is equal to the circumference of its base multiplied by one-half its slant height; or, approximately, to the radius of its base, times its slant height, times* 3.1416.

4. *The volume of a cone is equal to the area of its base multiplied by one-third its altitude; or, approximately, to the square of the radius of its base, times one-third its altitude, times* 3.1416.

5. *The lateral area of a frustum of a cone is equal to one-half the sum of the circumferences of its bases, multiplied by its slant height; or, approximately, to the sum of the radii of its bases, times its slant height, times* 3.1416.

6. *The volume of a frustum of a cone is equal to the sum of the areas of its bases, plus the square root of the product of the areas of its bases, multiplied by one-third its altitude; or, approximately, to the sum of the squares of the radii of its bases, plus the product of the radii of its bases, times one-third its altitude, times* 3.1416.

Also, approximately,

7. *The area of a sphere is equal to the square of its diameter, or four times the square of its radius, multiplied by* 3.1416.

8. *The volume of a sphere is equal to one-sixth the cube of its diameter, or four-thirds the cube of its radius, multiplied by* 3.1416.

Note. The second paragraph on page 163 applies with equal force to the above rules.

EXAMPLES.

248. 1. Find the lateral area and volume of a cylinder whose altitude is 9 in., and radius of base 4 in.

By Art. 247, 1, the required lateral area is

$$8 \times 9 \times 3.1416, \text{ or } 226.1952 \text{ sq. in.}$$

By Art. 247, 2, the required volume is

$$16 \times 9 \times 3.1416, \text{ or } 452.3904 \text{ cu. in.}$$

2. Find the radius of a sphere whose area is 452.3904 sq. in.

By Art. 247, 7, the square of the radius of a sphere is equal to its area divided by 4 times 3.1416, or 12.5664.

Hence, the square of the required radius is

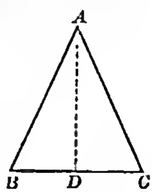
$$\frac{452.3904}{12.5664}, \text{ or } 36 \text{ sq. in.}$$

The square root of 36 is 6; whence the required radius is 6 in., *Ans.*

3. The volume of a cone is 1005.312 cu. ft., and the radius of its base is 8 ft. Find its altitude and slant height.

By Art. 247, 4, the altitude of a cone is equal to three times its volume, divided by 3.1416 times the square of the radius of its base.

Hence, the required altitude is $\frac{3015.936}{3.1416 \times 64}$, or 15 ft.



In the right triangle *ABD*, by Art. 227, the square of *AB* is equal to the sum of the squares of *AD* and *BD*.

But $8^2 + 15^2 = 64 + 225 = 289$, which is the square of 17.

Hence, the required slant height is 17 ft.

4. What is the volume of a frustum of a cone whose altitude is 12 in., and radii of bases 6 in. and 2 in., respectively?

By Art. 247, 6, the required volume is

$$(36 + 4 + 12) \times 4 \times 3.1416, \text{ or } 653.4528 \text{ cu. in., } Ans.$$

5. Find the lateral area and volume of a cylinder whose altitude is 7 in., and radius of base 3 in.

6. Find the area and volume of a sphere whose radius is 4 in.

7. Find the lateral area of a frustum of a cone whose slant height is 14 ft., and radii of bases 9 ft. and 2 ft., respectively.

8. Find the lateral area and volume of a cone whose altitude is 12 in., and radius of base 5 in.

9. Find the volume of a frustum of a cone whose altitude is 9 ft., and radii of bases 10 ft. and 6 ft., respectively.

10. The lateral area of a cone is 188.496 sq. in., and the radius of its base is 6 in. Find its slant height and altitude.

11. The area of a sphere is 314.16 sq. in. Find its diameter and volume.

12. The volume of a cylinder is 2412.7488 cu. in., and its altitude is 12 in. Find the radius of its base.

13. The lateral area of a frustum of a cone is 603.1872 sq. in., and the radii of its bases are 5 in. and 11 in., respectively. Find its slant height.

14. Assuming the earth to be a sphere 7900 miles in diameter, find its area and volume.

15. A tent, in the shape of a cone, has a slant height of 16 feet, and a diameter at the base of 24 feet. How many square yards of material were used in its construction?

16. The volume of a frustum of a cone is 779.1168 cu. ft., and the radii of its bases are 10 ft. and 2 ft., respectively. Find its altitude.

17. What will it cost to gild a ball 25 inches in diameter, at $13.50 a square foot?

18. The altitude of a cone is 9 in., and the radius of its base is 7 in. Find the altitude of an equivalent cylinder (Art. 232), the diameter of whose base is 10 in.

19. How many cubic feet are there in a log of wood 20 feet long, whose girth is 3 feet?

20. A basin is in the shape of a hemisphere whose diameter is $2\frac{2}{3}$ yards. How many cubic feet of water will it hold?

21. How many cubic feet of metal are there in a hollow iron tube 18 ft. long, whose outer diameter is 7 in., and thickness 1 in.?

22. Find the radius of a sphere, whose surface is equivalent to the lateral surface of a cylinder, whose altitude is 8 ft., and radius of base 4 ft.

23. The volume of a sphere is 7238.2464 cu. in. Find its radius.

24. A cylindrical vessel, 8 in. in diameter, is filled to the brim with water. A ball is immersed in it, displacing water to the depth of $2\frac{1}{4}$ in. Find the diameter of the ball.

25. The outer diameter of a spherical shell is 9 in., and its thickness is 1 in. What is its weight, if a cubic inch of the metal weighs $\frac{1}{8}$ lb.?

26. How many cubic feet are there in a column whose length is 22 ft., diameter of larger end 10 in., and diameter of smaller end 7 in.?

27. The altitude of a frustum of a cone is 6 ft., and the radii of its bases are 3 ft. and 2 ft., respectively. What is the diameter of an equivalent sphere?

28. If a gallon contains 231 cu. in., what must be the depth of a cylindrical measure, 3 in. in diameter, to hold a quart?

APPLICATIONS OF MENSURATION.

249. Capacity of Bins, Tanks, and Cisterns.

The following equivalents are to be used in the examples of the present article:

1 bushel = 2150.42 cu. in.	1 gallon = 231 cu. in.
1 bushel = $1\frac{1}{4}$ cu. ft.	$7\frac{1}{2}$ gallons = 1 cu. ft.

1. How many bushels of grain can be put into a bin 5 ft. 3 in. long, 3 ft. 6 in. wide, and 4 ft. 1 in. deep?

We have

5 ft. 3 in. = 63 in., 3 ft. 6 in. = 42 in., and 4 ft. 1 in. = 49 in.

Then the volume of the bin = $63 \times 42 \times 49$, or 129654 cu. in.

Since one bushel contains 2150.42 cu. in., as many bushels can be put into the bin as 2150.42 is contained times in 129654, which is 60.29+, *Ans.*

EXAMPLES.

2. How many bushels of grain can be put into a bin 7 ft. long, 3 ft. wide, and 4 ft. deep?

3. If a ton of coal occupies 38 cu. ft., how many tons can be put into a bin 8 ft. long, $5\frac{1}{4}$ ft. wide, and $6\frac{1}{3}$ ft. deep?

4. How many gallons will a tank hold which is 2 ft. 6 in. long, 2 ft. wide, and 1 ft. 9 in. deep?

5. How many gallons of water can be put into a cylindrical tank whose diameter is 50 in., and depth 65 in.?

6. How deep must a bin be that is 6 ft. long and 4 ft. wide, to hold 84 bushels of grain?

7. What must be the depth of a cubical bin to hold 100 bushels of wheat?

8. If a tank is 66 inches wide and 42 inches deep, how long must it be to hold 1000 gallons?

9. How deep must a cistern be, whose diameter is 60 in., to hold 800 gallons?

10. How many bushels of oats can be put into a bin 4 ft. 7 in. long, 3 ft. 5 in. wide, and 3 ft. 10 in. deep?

11. If a tank 8 ft. long, 5 ft. wide, and 3 ft. deep, is filled with oil, how much is the oil worth at 13 cents a gallon?

12. What must be the diameter of a cylindrical tank, whose depth is 55 inches, to hold 640 gallons?

13. How deep must a bin be that is 7 ft. 2 in. long, and 5 ft. 7 in. wide, to hold 150 bushels of rye?

14. A cubical bin, 5 ft. 3 in. deep, is filled with wheat. What is its value at $0.96 per bushel?

15. To what depth must a cistern 38 inches in diameter be filled, to hold 304 gallons?

16. A well is 3 ft. in diameter, and 32 ft. deep. How many barrels of $31\frac{1}{2}$ gallons each will it contain?

17. How many bushels can be put into a cylindrical receptacle whose diameter is 3 ft. 6 in., and depth 5 ft. 4 in.?

18. What must be the diameter of a cistern, of depth 4 feet, to hold 400 gallons?

19. If a tank holds 500 gallons of water, how many bushels of grain can be put into it?

250. Carpeting Rooms.

1. A floor is 15 ft. 4 in. long, and 11 ft. 9 in. wide. How much will it cost to cover it with carpeting, each length 2 ft. 8 in. wide, at 63 cents a yard, no allowance being made for waste in matching the pattern?

If the strips are laid *lengthwise* of the room, as many strips will be required as 2 ft. 8 in. is contained times in 11 ft. 9 in.

32 in. is contained in 141 in. 4 times, with a remainder of 13 in.

Then *five* strips will be required.

The total length of the five strips is 15 ft. 4 in. × 5, or 76 ft. 8 in.; that is, $15\frac{5}{9}$ yd.

Then the cost will be $15\frac{5}{9}$ × $0.63, or $9.80, *Ans.*

Note. It will be understood, in the following examples, that the strips are laid *lengthwise* of the room, unless the contrary is specified.

EXAMPLES.

2. How many yards of carpeting $\frac{5}{8}$ of a yard wide will be required for a floor 17 ft. long and 14 ft. wide?

3. How much will it cost to cover a floor 15 ft. long and 11 ft. 3 in. wide with oil-cloth, at 40 cents a square yard?

4. How many yards of carpeting 2 ft. 7 in. wide will be required for a floor 20 ft. 11 in. long and 16 ft. 4 in. wide, if the strips run lengthwise of the room? How many if the strips run across the room?

5. How much will it cost to cover a floor 14 ft. 3 in. square with straw matting, in strips one yard wide, at 44 cents a yard?

6. A floor is 16 ft. 11 in. long and 12 ft. 1 in. wide. How much will it cost to cover it with carpeting, each length 2 ft. 5 in. wide, at 75 cents a yard?

7. How many yards of carpeting 2 ft. 10 in. wide, will be required for a floor 16 ft. 9 in. long and 13 ft. 8 in. wide, if there is a waste of $4\frac{1}{2}$ inches in each strip in matching the pattern?

8. Which way should the strips run to carpet most economically a floor 18 ft. 3 in. long and 15 ft. 4 in. wide, the strips being 2 ft. 9 in. wide?

9. How much will it cost to cover a floor 21 ft. 6 in. long and 18 ft. 4 in. wide with carpeting 2 ft. 6 in. wide, at 87 cents a yard, if there is a waste of $\frac{1}{8}$ of a yard in each strip in matching the pattern?

10. Which will be the cheaper, to cover a floor 19 ft. 7 in. long and 14 ft. 9 in. wide with matting, in strips 2 ft. 8 in. wide, laid lengthwise, at 42 cents a yard, or to cover it with oil-cloth at 48 cents a square yard?

251. Plastering and Papering.

1. How much will it cost to plaster a room 16 ft. long, 13 ft. wide, and 9 ft. high, at 40 cents a square yard, allowing 64 sq. ft. for doors and windows?

The area of the four walls is $2 \times (16 + 13) \times 9$, or 522 sq. ft.

The area of the ceiling is 16×13, or 208 sq. ft.

Then the total area to be plastered is $522 + 208 - 64$, or 666 sq. ft.; that is, 74 sq. yd.

Hence, at 40 cents a square yard, the total cost will be

$$74 \times \$0.40 = \$29.60, \textit{ Ans.}$$

2. How many rolls of paper, $1\frac{2}{3}$ ft. wide, 9 yards to a roll, will be required to paper a room 19 ft. long, 14 ft. wide, and $9\frac{1}{6}$ ft. high, allowing for two doors, each 3 ft. wide and $7\frac{1}{3}$ ft. high, four windows, each 3 ft. wide and $5\frac{1}{2}$ ft. high, and a base-board 9 in. wide?

The area of the four walls is $2 \times (19 + 14) \times 9\frac{1}{6}$, or 605 sq. ft.

The area of the two doors is $2 \times 3 \times 7\frac{1}{3}$, or 44 sq. ft.

The area of the four windows is $4 \times 3 \times 5\frac{1}{2}$, or 66 sq. ft.

The length of the base-board is the distance around the room, less the width of the two doors; that is, $66 - 6$, or 60 ft.

Then the area of the base-board is $60 \times \frac{3}{4}$, or 45 sq. ft.

Thus, the total area to be deducted from the area of the four walls is $44 + 66 + 45$, or 155 sq. ft.

Hence, the area to be papered is $605 - 155$, or 450 sq. ft.

The area of each roll is $1\frac{2}{3} \times 27$, or 45 sq. ft.

Then, as many rolls will be required as 45 is contained times in 450; that is, 10 rolls, *Ans.*

EXAMPLES.

3. How much will it cost to plaster a room 18 ft. long, 15 ft. wide, and 10 ft. high, at 39 cents a square yard, allowing 102 sq. ft. for doors and windows?

4. How much will it cost to plaster a room 20 ft. 8 in. long, 16 ft. 3 in. wide, and 9 ft. 6 in. high, at 48 cents a square yard, allowing 128 sq. ft. 36 sq. in. for doors and windows?

5. How many rolls of paper, $1\frac{3}{8}$ ft. wide, 12 yards to a roll, will be required to paper a room 17 ft. long, 12 ft. wide, and 9 ft. high, no allowance being made for doors or windows?

6. How much will it cost to paper a room 14 ft. square, and $8\frac{1}{2}$ ft. high, with paper 1 ft. 10 in. wide, 10 yards to a roll, at 75 cents a roll; allowing 80 sq. ft. for doors and windows?

7. A room 16 ft. long, 12 ft. wide, and 9 ft. high, has three doors, each 3 ft. wide and 7 ft. high, two windows, each 3 ft. wide and 5 ft. 9 in. high, and a base-board 9 inches wide. How much will it cost to plaster it at 44 cents a square yard?

8. How much will it cost to paper a room 15 ft. long, 11 ft. wide, and 10 ft. high, with paper 1 ft. 11 in. wide, 11 yards to a roll, at $1.10 a roll, allowing 175 sq. ft. for doors, windows, and base-board?

9. What will it cost to plaster a hemispherical dome, whose diameter is 60 ft., at 50 cents a square yard?

10. How many rolls of paper, 1 ft. $11\frac{1}{2}$ in. wide, 12 yards to a roll, will be required to paper a room 21 ft. long, $15\frac{1}{2}$ ft. wide, and $8\frac{3}{4}$ ft. high, allowing for three doors, each 3 ft. wide and $6\frac{3}{4}$ ft. high, three windows, each $2\frac{3}{8}$ ft. wide and $5\frac{1}{2}$ ft. high, and a base-board 1 ft. wide?

11. A room 23 ft. 6 in. long, 15 ft. 4 in. wide, and 9 ft. 9 in. high, has three doors, each 3 ft. wide and 6 ft. 8 in. high, four windows, each 2 ft. 9 in. wide and 5 ft. 9 in. high, and is surrounded by a base-board 9 in. wide. How much will it cost to plaster it at 36 cents a square yard?

12. Find the cost of papering a room 18 ft. 4 in. long, 14 ft. 6 in. wide, and 9 ft. 6 in. high, with paper 1 ft. 9 in. wide, $11\frac{1}{3}$ yards to a roll, at $1.19 a roll; allowing for two doors, each 3 ft. wide and 7 ft. high, and two windows, each 2 ft. 9 in. wide and 6 ft. high?

252. **Board Measure.**

A board one inch or less in thickness is said to have as many **Board Feet** as there are square feet in its surface.

If it is more than an inch in thickness, the number of board feet is found by multiplying the number of square feet in its surface by the number of inches in its thickness.

In measuring a board that tapers, the width is taken as one-half the sum of the widths of the two ends.

Boards are usually sold at a certain price per hundred (C.) or per thousand (M.) board feet.

1. Find the cost of 24 planks, each 22 ft. 8 in. long, 21 in. wide, and $2\frac{1}{4}$ in. thick, at \$25 per M.

22 ft. 8 in. $= \frac{68}{3}$ ft., 21 in. $= \frac{7}{4}$ ft., and $2\frac{1}{4}$ in. $= \frac{9}{4}$ in.

Then the total number of board feet is

$$\frac{\overset{17}{\cancel{68}}}{\cancel{3}} \times \frac{7}{\cancel{4}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{4}} \times \overset{6}{\cancel{24}}, \text{ or } 2142.$$

At \$25 per M., the total cost will be

$$2.142 \times \$25, \text{ or } \$53.55, \textit{Ans.}$$

EXAMPLES.

2. Find the number of board feet in a board 16 ft. 6 in. long, 14 in. wide, and 1 in. thick.

3. Find the number of board feet in a board 10 ft. long, 11 in. wide, and $\frac{3}{4}$ in. thick.

4. Find the number of board feet in a piece of timber 25 ft. 9 in. long, 9 in. wide, and 8 in. thick.

5. Find the number of board feet in a plank 18 ft. 8 in. long, 1 ft. 5 in. wide, and $3\frac{1}{2}$ in. thick.

6. Find the number of board feet in a tapering plank 15 ft. 4 in. long, 2 ft. 3 in. wide at one end, and 1 ft. 11 in. wide at the other, and $3\frac{3}{8}$ in. thick.

7. Find the cost of 45 spruce joists, each 14 ft. long, 6 in. wide, and 4 in. thick, at \$14 per M.

8. Find the cost of 150 boards, each 11 ft. 8 in. long, 5 in. wide, and $\frac{5}{8}$ in. thick, at \$18.30 per M.

9. Find the cost of 30 planks, each 17 ft. 4 in. long, 1 ft. 10 in. wide, and $2\frac{3}{4}$ in. thick, at \$2.55 per C.

10. Find the cost of a plank-walk 75 ft. long, 2 ft. 6 in. wide, and $\frac{7}{8}$ in. thick, at \$31.50 per M.

11. Find the cost of 75 planks, each 12 ft. 10 in. long, 1 ft. 7 in. wide at one end, and 1 ft. 1 in. wide at the other, and $1\frac{3}{4}$ in. thick, at \$15 per M.

253. Measurement of Round Timber.

To find the side of the squared timber that can be sawed from a log, *multiply the diameter of the smaller end by* .707.

To find the number of board feet in the squared timber that can be sawed from a log, *multiply together one-half the length in feet, the diameter of the smaller end in feet, and the diameter of the smaller end in inches.*

1. Find the side, and the number of board feet, in the squared timber that can be sawed from a log whose length is $15\frac{1}{6}$ ft., and diameter at the smaller end 18 in.

By the first of the above rules, the side is

$$18 \text{ in.} \times .707, \text{ or } 12.726 \text{ in.}$$

By the second rule, the number of board feet is

$$\frac{1}{2} \times 15\tfrac{1}{6} \times \frac{3}{2} \times 18 = \frac{1}{2} \times \frac{91}{\cancel{6}} \times \frac{3}{2} \times \overset{3}{\cancel{18}} = \frac{819}{4} = 204\tfrac{3}{4}.$$

EXAMPLES.

Find the side, and the number of board feet, in the squared timber that can be sawed from a log whose length is:

2. 18 ft., and diameter 1 ft.
3. 21 ft., and diameter of smaller end $1\frac{1}{8}$ ft.
4. $17\frac{1}{3}$ ft., and diameter 15 in.
5. 15 ft. 9 in., and diameter 1 ft. 2 in.
6. 23 ft. 10 in., and diameter of smaller end 1 ft. 7 in.

254. Specific Gravity.

The **Specific Gravity** of a substance is the number of times that the weight of a certain portion of the substance contains the weight of an equal bulk of water.

For example, a cubic foot of copper weighs 8.8 times as much as a cubic foot of water; hence, the specific gravity of copper is 8.8.

In the following examples, the weight of a cubic foot of water is taken as 1000 oz., or 62.5 lb.

1. What is the weight of a cubic foot of iron, if its specific gravity is 7.53?

Since a cubic foot of water weighs 62.5 lb., and iron is 7.53 times as heavy as water, a cubic foot of iron will weigh

62.5 lb. × 7.53, or 470.625 lb., *Ans.*

2. A mass of granite (specific gravity 2.6) weighs 7800 lb.; how many cubic feet does it contain?

Since one cubic foot of granite weighs 62.5 lb. × 2.6, or 162.5 lb., to weigh 7800 lb. will take as many cubic feet of granite as 162.5 is contained times is 7800.

Dividing 7800 by 162.5, the result is 48 cu. ft., *Ans.*

EXAMPLES.

3. Find the weight in pounds of a cubic foot of copper (specific gravity 8.81).

4. Find the weight in pounds of a cubic yard of brickwork (specific gravity 1.8).

5. Find the weight in pounds of 5 cu. ft. 288 cu. in. of yellow pine (specific gravity .46).

6. Find the weight in ounces of a cubic inch of mercury (specific gravity 13.596).

7. If a mass of iron (specific gravity 7.68) weighs 6 T., how many cubic feet does it contain?

8. If a mass of tin (specific gravity 7.5) weighs 18750 lb., how many cubic feet does it contain?

9. If a certain bulk of alcohol (specific gravity .791) weighs $197\frac{3}{4}$ oz., how many cubic inches does it contain?

10. If a piece of gold (specific gravity 19.4) weighs 37 lb. 14 oz. 4 dr., how many cubic inches does it contain?

11. If a cubic foot of glass weighs 170 lb., find its specific gravity.

12. If a cubic yard of oak weighs 1485 lb., find its specific gravity.

13. If 3 cu. ft. 432 cu. in. of silver weighs 2132 lb. 13 oz. avoirdupois, find its specific gravity.

14. If a cubic inch of brass weighs 77.2864 dr., find its specific gravity.

255. Geometrical Explanation of Square and Cube Root.

Square Root.

Let it be required to find the square root of 1296.

Let *ACEG* be a square containing 1296 sq. in.

To find its side in inches.

Fig. 1

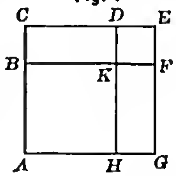


Fig. 2

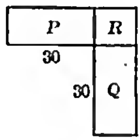
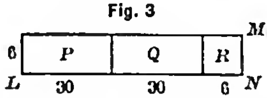


Fig. 3



Since a square whose side is 30 in. contains 900 sq. in., and a square whose side is 40 in. contains 1600 sq. in., the side of the given square must be between 30 and 40 in.

Thus the tens' figure of the root is 3.

Removing from the given square the square *ABKH*, whose side is 30 in., there remains an irregular figure, shown in Fig. 2, composed of two rectangles *P* and *Q*, and a square *R*, whose united area is 1296 − 900, or 396 sq. in.

The rectangles and the square may be arranged as shown in Fig. 3, forming a rectangle *LM*, whose altitude is the units' figure of the root.

Now the altitude of a rectangle is equal to its area divided by its base.

Since the base of each of the rectangles P and Q is 30 in., the base of the rectangle LM is something more than 60 in.

If we divide the area of LM, 396 sq. in., by its approximate base, 60 in., we obtain something more than 6 in. as the approximate altitude.

If, now, we make trial of 6 in. as the altitude of the rectangle, the base LN is 60 in. + 6 in., or 66 in.; and multiplying this by the altitude, 6 in., the result is 396 sq. in.

But this is just the area of the irregular figure of Fig. 2.

We then conclude that the units' figure of the root is 6; whence, the required root is 30 + 6, or 36.

The above process is exactly in accordance with the Rule of Art. 200.

Cube Root.

Let it be required to find the cube root of 13824.

Fig. 1

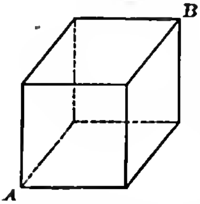


Fig. 2

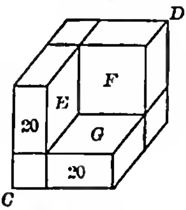
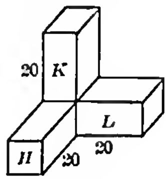


Fig. 3



Let AB be a cube containing 13824 cu. in.

To find its edge in inches.

Since a cube whose edge is 20 in. contains 8000 cu. in., and a cube whose edge is 30 in. contains 27000 cu. in., the edge of the given cube must be between 20 and 30 in.

Thus the tens' figure of the root is 2.

Removing from the given cube a cube whose edge is 20 in., there remains an irregular solid CD, whose volume is 13824 − 8000, or 5824 cu. in.

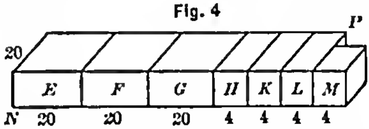
Removing from CD the three solids E, F, and G, there remains an irregular solid, shown in Fig. 3, composed of three rectangular parallelopipeds, H, K, and L, and a cube.

The solids E, F, G, H, K, and L, and the cube M, may be arranged as shown in Fig. 4, forming an irregular solid NP, whose altitude is the units' figure of the root.

Now the altitude of this solid is equal to its volume divided by the area of its base.

Since the area of the base of each of the solids E, F, and G is 20^2, or 400 sq. in., the sum of the areas of their bases is 3×400, or 1200 sq. in.

Fig. 4



Then the area of the base of the solid NP is something more than 1200 sq. in.

If we divide the volume of NP, 5824 cu. in., by its approximate area of base, 1200 sq. in., we obtain something more than 4 in. as the approximate altitude.

If, now, we make trial of 4 in. as the altitude of the solid, the area of the base of each of the solids H, K, and L is 4×20, or 80 sq. in., and the sum of the areas of their bases is 3×80, or 240 sq. in.

Also, the area of the base of the cube M is 4^2, or 16 sq. in.

Then the area of the base of the solid NP is $1200 + 240 + 16$, or 1456 sq. in.; and multiplying this by the altitude, 4 in., the result is 5824 cu. in.

But this is just the volume of the irregular solid of Fig. 2.

We then conclude that the units' figure of the root is 4; whence, the required root is $20 + 4$, or 24.

The above process is exactly in accordance with the rule of Art. 208.

PROBLEMS IN MENSURATION INVOLVING THE METRIC SYSTEM.

Note. The remainder of the present chapter may be omitted by those who have not previously taken the chapter on the Metric System.

256. Mensuration of Plane Figures.

1. Find the area in square decimeters of a rectangle whose base is 8.9^{m}, and altitude 735^{cm}.

2. Find the altitude in meters of a triangle whose area is $16.8^{sq\,Dm}$, and base $.096^{Km}$.

3. The hypotenuse of a right triangle is 41^{m}, and one of the sides about the right angle is $.9^{Dm}$. Find the other side in hektometers.

4. Find the base in dekameters of a parallelogram whose area is $89.1^{\text{sq cm}}$, and altitude 55^{mm}.

5. Find the circumference in meters of a circle whose radius is 23^{Km}.

6. Find the area in square hektometers of a triangle whose base is 528^{dm}, and altitude 3.59^{Dm}.

7. The sides about the right angle of a right triangle are 18^{m} and 33^{dm}, respectively. Find the hypotenuse in dekameters.

8. The diameter of a circle is 13^{dm}. Find its area in square centimeters.

9. Find the side in decimeters of a square whose area is $143641^{\text{sq mm}}$.

10. Find the area in square decimeters of a trapezoid whose bases are 3.51^{m} and 4852^{mm}, respectively, and altitude $.0295^{\text{Hm}}$.

11. Find the radius in dekameters of a circle whose area is $.19635^{\text{sq Hm}}$.

12. Find the diameter in centimeters of a circle whose circumference is 12^{mm}.

13. Find the lower base in centimeters of a trapezoid whose area is $3.6^{\text{sq m}}$, upper base $.13^{\text{Dm}}$, and altitude 18^{dm}.

14. Find the area in ars of a rectangular field 253.8^{dm} long and 13.9^{m} wide.

15. Find the area in centars of a floor $.5498^{\text{Dm}}$ long and 467^{cm} wide.

16. A circular grass-plot, 17^{m} in diameter, is surrounded by a walk 18^{dm} wide. Find the area of the walk in centars.

17. A triangular house-lot contains $.241853^{\text{Ha}}$. If its base is 7.34^{Dm}, what is its altitude in meters?

18. The area of a square field is 77.2641^{Ha}. Find its side in dekameters.

19. What is the length of the longest straight line that can be drawn in a rectangular field whose length is 204^m and width 85^m?

20. If the diameter of a wheel is 75^{cm}, how many times will it revolve in travelling 12.9591^{Km}?

21. The side of a square field is 87.2^m. How much is the field worth at $ 8750 a hektar?

22. A vessel sails due north at the rate of 9.3^{Km} an hour, and another sails due west at the rate of 12.4^{Km} an hour. How far apart are they at the end of 4 h. 12 min.?

23. Two circles whose radii are 25^{Dm} and 173^m have the same centre. How many square hektometers of area are included between their circumferences?

24. How many acres are there in a rectangular field whose length is 8.9^{Dm}, and width $.75^{Hm}$?

25. A cow, tied by a rope to a stake, can graze over $415.4766^{sq\,m}$ of ground. What is the length of the rope?

26. Find the diagonal in hektometers of a square whose area is $.1764^{sq\,Km}$.

27. A man sold a rectangular field for 28 cents a centar, receiving the sum of $ 915.04. If the field was $.43^{Hm}$ wide, what was its length in dekameters?

28. The floor of a room is 6^m long and 43^{dm} wide, and has a circular opening $.17^{Dm}$ in diameter. Find the number of square centimeters in the floor.

29. A rectangular garden is surrounded by a walk 1.2^m wide, containing $248.16^{sq\,m}$. If the garden is 64^m long, what is its width?

30. A circular field contains a hektar. What is its diameter in dekameters?

31. The diagonal of a square is 76^m. Find the approximate length of its side.

Mensuration of Solids.

1. Find the lateral area and volume of a prism whose altitude is 7^{dm}, having for its base a right triangle whose sides are 3^{dm}, 4^{dm}, and 5^{dm}.

2. Find the area and volume of a sphere whose radius is 3^{cm}.

3. Find the lateral area and volume of a cone whose altitude is 24^{m}, and radius of base 7^{m}.

4. The volume of a rectangular parallelopiped is $1920^{cu\ cm}$, and the dimensions of its base are 15^{cm} and 8^{cm}. Find its altitude and the area of its entire surface.

5. The volume of a pyramid is $1320^{cu\ dm}$. The base is a right triangle whose sides are 10^{dm}, 24^{dm}, and 26^{dm}. Find the altitude of the pyramid.

6. Find the lateral area of a frustum of a cone, whose slant height is 8^{m}, radius of lower base 9^{m}, and radius of upper base 3^{m}.

7. The area of a sphere is $201.0624^{sq\ cm}$. Find its radius and volume.

8. The base of a regular pyramid is a square whose area is $67.24^{sq\ dm}$, and its slant height is 579^{cm}. Find its lateral area in square meters.

9. Find the volume of a frustum of a pyramid whose lower base is a rectangle 12^{Dm} by 3^{Dm}, upper base 8^{Dm} by 2^{Dm}, and altitude 16.8^{Dm}.

10. The volume of a frustum of a cone is $1894.3848^{cu\ m}$, and the radii of its bases are 11^{m} and 5^{m}. Find its altitude.

11. Find the lateral area in square millimeters, and the volume in cubic centimeters, of a cylinder whose altitude is 81^{mm}, and radius of base 9^{cm}.

12. The lateral area of a cone is $565.488^{sq\ m}$, and the radius of its base is 12^{m}. Find its slant height, altitude, and volume.

13. Find the lateral area of a frustum of a regular pyramid whose lower base is a square 15^m on a side, upper base 9^m on a side, and slant height 8.49^m.

14. A basin is in the shape of a hemisphere whose diameter is 4.3^{dm}. How many deciliters of water will it contain? What is the weight of this water in decigrams?

15. A wood pile is 2.5^m long, 13^{dm} wide, and 125^{cm} high. How much is it worth at $3.36 a ster?

16. What will be the cost of gilding a ball 65^{cm} in diameter, at $1.50 a square decimeter?

17. A room is 5^m long, 48^{dm} wide, and 326^{cm} high. How many dekaliters of air does it contain?

18. How much will it cost to dig a ditch 7^{Dm} long, 18^{dm} wide, and 12^{dm} deep, at 75 cents a ster?

19. A wood pile, 19^{dm} long and 142^{cm} wide, contains 3.72324^{st}. Find its height in meters.

20. Find the number of cubic meters in a tapering piece of timber 9^m long, one of which is 5^{dm} square, and the other 32^{cm} square.

21. A bar of iron is 6.2^m long, 8^{cm} wide, and 13^{mm} thick. Find its weight in kilograms, if iron is 7.53 times as heavy as water.

22. A cylindrical boiler is 4^m long, and 13^{dm} in diameter. How many liters of water will it contain?

23. A trench is 42^m long, 8^{dm} deep, 2.1^m wide at the top, and 1.7^m wide at the bottom. How many hektoliters of water will it contain?

24. How many bricks, each 2^{dm} long, 82^{mm} wide, and 4.8^{cm} thick, will be required to build a wall 37^m long, 41^{cm} wide, and 19.2^{dm} high?

25. A spherical shell, 5^{cm} thick, has an outside diameter of 4^{dm}. How many cubic centimeters of metal does it contain?

26. A brass rod is 3.5^{m} long and 4^{cm} in diameter. Find its weight in dekagrams, if brass is 8.3 times as heavy as water.

27. Find the weight in kilograms of the water that can be put into a pail in the shape of a frustum of a cone, whose depth is 24^{cm}, diameter at the top 32^{cm}, and diameter at the bottom 22^{cm}.

28. A cannon-ball, 23^{cm} in diameter, is dropped into a cubical box filled with water, whose depth is 23^{cm}. How many centiliters of water will be left in the box?

29. A hopper, in the form of an inverted frustum of a pyramid, holds 1.364^{Hl} of grain. It is 1^{m} square at the top, and 2^{dm} square at the bottom. Find its depth in meters.

30. A projectile consists of two hemispheres, connected by a cylinder. If the altitude and diameter of the cylinder are 2^{dm} and 13^{cm}, respectively, find the number of cubic decimeters in the projectile.

Capacity of Bins, Tanks, and Cisterns, Carpeting, Plastering, and Papering.

1. A tank is 3.7^{m} long, $.98^{m}$ wide, and 1.6^{m} deep. How many liters of water will it contain?

2. How many hektoliters of wheat can be put into a bin 2.3^{m} long, 12^{dm} wide, and 18^{dm} deep?

3. How many meters of carpeting 8^{dm} wide will be required for a floor 6.59^{m} long and 5^{m} wide?

4. How much will it cost to plaster a room 5^{m} long, 4^{m} wide, and 2.9^{m} high, at 45 cents a square meter, allowing $10.8^{sq\,m}$ for doors and windows?

5. What must be the depth in meters of a tank 24^{dm} long and 78^{cm} wide, to hold 35.9124^{Hl} of water?

6. What must be the length in decimeters of a bin 1.88^{m} deep and $.13^{Dm}$ wide, to hold 657.436^{Dl} of grain?

7. A cylindrical cistern whose diameter is 3^{m} is filled with water to a depth of 19^{dm}. How many hektoliters of water does it contain, and what is the weight of the water in hektograms?

8. How many rolls of paper 7^{dm} wide, 11^{m} to a roll, will be required to paper a room 5.8^{m} long, 4.7^{m} wide, and 2.8^{m} high, allowing $8.75^{sq\,m}$ for doors and windows?

9. A cubical tank holds 491.3^{Mg} of water. What is its depth in meters?

10. A hemispherical dome is 25^{m} in diameter. How much will it cost to plaster it at 48 cents a square meter?

11. What will it cost to cover a floor 7.9^{m} long and 5.8^{m} wide with carpeting 78^{cm} wide, at 95 cents a meter, if there is a waste of 5^{cm} in each strip in matching the pattern?

12. A tank 5.1^{m} long, 2.3^{m} wide, and 1.7^{m} deep, is filled by a pipe through which pass 289^{dl} of water a minute. How long will it take to fill it?

13. How many rolls of paper 68^{cm} wide, 10^{m} to a roll, will be required to paper a room 7^{m} long, 5.5^{m} wide, and 3.2^{m} high, with two doors, each 88^{cm} wide and 22^{dm} high, and four windows, each 86^{cm} wide and 17.5^{dm} high?

14. A well is 13^{dm} in diameter, and 12^{m} deep. How many kiloliters of water will it hold?

15. A cylindrical tank, 16^{dm} deep, holds 15393.84^{Kg} of water. What is its diameter in meters?

16. How much will it cost to plaster a room 7.2^{m} long, 4.9^{m} wide, and 3^{m} high, at 42 cents a square meter, allowance being made for two doors, each 9^{dm} wide and 21^{dm} high, three windows, each 8^{dm} wide and 18^{dm} high, and a baseboard 2^{dm} wide?

17. If a metric ton of coal occupies $1.184^{cu\,m}$, how many metric tons can be put into a bin 37^{dm} long, 12^{dm} wide, and 36^{dm} deep?

18. Which way should the strips run to carpet most economically a floor 7.8^{m} long and 6.2^{m} wide, the strips being 84^{cm} wide?

19. A bin 2.3^{m} long, 1.1^{m} wide, and 1.48^{m} deep is filled with grain. How much is it worth at $ 2.75 a hektoliter? If the grain weighs .83 times as much as an equal bulk of water, what is the weight of the contents in kilograms?

20. A cylindrical cistern, 25^{dm} deep, holds 636.174^{Dl} of water. Find the diameter of the cistern in meters.

21. How much will it cost to cover a floor 5.6^{m} long, and 4.9^{m} wide, with carpeting 75^{cm} wide, at 87 cents a meter, if the strips run lengthwise of the room? How much if the strips run across the room?

22. A tank 84^{cm} deep, with a square bottom, contains 869.4^{Kg} of sulphuric acid. If the acid is 1.84 times as heavy as water, what is the length of each side of the bottom in centimeters?

23. To what depth must a cylindrical cistern 113^{cm} in diameter be filled, to hold a metric ton of water?

24. How much will it cost to paper a room 6.4^{m} long, 5.4^{m} wide, and 3.1^{m} high, with paper 58^{cm} wide, 10.7^{m} to a roll, at 84 cents a roll, allowing for three doors, each 85^{cm} wide and 2^{m} high, two windows, each 82^{cm} wide and 17^{dm} high, and a base-board 3^{dm} wide?

25. A cylindrical tank, 18^{dm} deep, contains 2862.783^{Kg} of oil. If the oil is .9 as heavy as water, find the diameter of the tank in meters.

257. Specific Gravity.

If the specific gravity of any substance is 8.7, a cubic centimeter of the substance will weigh 8.7 times as much as a cubic centimeter of water; that is, it will weigh 8.7^{g}.

A cubic decimeter (or liter) of the substance will weigh 1000×8.7^{g}, or 8.7^{Kg}.

A cubic meter of the substance will weigh 1000×8.7^{Kg}, or 8.7^{T}.

It follows from the above that the specific gravity of any substance is:

1. *The number of grams in the weight of a cubic centimeter of the substance.*

2. *The number of kilograms in the weight of a cubic decimeter (or liter) of the substance.*

3. *The number of metric tons in the weight of a cubic meter of the substance.*

EXAMPLES.

1. Find the weight in kilograms of a bar of aluminum (specific gravity 2.57) 8^{dm} long, 2^{cm} wide, and 7^{mm} thick.

The volume of the bar is $8^{dm} \times .2^{dm} \times .07^{dm}$, or $.112^{cu\ dm}$.

But since the specific gravity of the substance is 2.57, a cubic decimeter of it weighs 2.57^{Kg}.

Hence, the required weight is $.112 \times 2.57^{Kg}$, or $.28784^{Kg}$, *Ans.*

2. Find the number of cubic centimeters in a piece of silver (specific gravity 10.5) weighing $.2625^{Hg}$.

$.2625^{Hg}$ is the same as 26.25^{g}.

Since one cubic centimeter of silver weighs 10.5^{g}, there will be as many cubic centimeters in 26.25^{g} as 10.5 is contained times in 26.25.

Hence, $26.25 \div 10.5 = 2.5^{cu\ cm}$, *Ans.*

3. If 3^{Hl} of sulphuric acid weigh 552^{Kg}, what is its specific gravity?

If 3^{Hl} weigh 552^{Kg}, one liter will weigh $\frac{1}{300}$ of 552^{Kg}, or 1.84^{Kg}.

Hence, the specific gravity of the acid is 1.84, *Ans.*

4. Find the weight in grams of $5^{cu\ dm}$ of cork (specific gravity .24).

5. Find the weight in hektograms of 15^{Dl} of petroleum (specific gravity .878).

6. How many cubic centimeters are there in a piece of lead (specific gravity 11.4) weighing 2131.8^{dg}?

7. How many hektoliters of alcohol (specific gravity .791) does it take to weigh 10203.9^{Hg}?

8. If $8^{cu\ Dm}$ of ice weigh 7360^{T}, what is its specific gravity?

9. If 4.8^{cl} of mercury weigh 65260.8^{cg}, what is its specific gravity?

10. How many metric tons of sand (specific gravity 1.42) can be put into a box 1.5^{m} long, 19^{dm} wide, and 57^{cm} deep?

11. Find the weight in decigrams of a bar of platinum (specific gravity 21.5) 5.9^{dm} long, 2.4^{cm} wide, and 3.8^{mm} thick.

12. A block of stone (specific gravity 2.9), 1.2^{m} long and 8.37^{dm} wide, weighs 1.893294^{T}. Find its thickness in centimeters.

13. A rod of zinc 5^{m} long, and 2^{cm} in diameter, weighs 11.15268^{Kg}. Find its specific gravity.

14. A brick 2^{dm} long, 62^{mm} wide, and 2.4^{cm} thick, weighs 6.19008^{Hg}. Find its specific gravity.

15. A cylindrical tank, 12^{dm} deep, is filled with linseed oil (specific gravity .94). If the weight of the oil is 717.604272^{Kg}, what is the diameter of the tank in decimeters?

16. Find the weight in kilograms of a tapering bar of steel (specific gravity 7.8), 5.3^{m} long, 3^{cm} in diameter at one end, and 2^{cm} in diameter at the other.

XVI. RATIO AND PROPORTION.

258. The **Ratio** of one number to another is the quotient obtained by dividing the first number by the second.

Thus, the ratio of 3 to 5 is $\frac{3}{5}$; it is also expressed $3:5$.

259. The first term of a ratio is called the *antecedent*, and the second term the *consequent*.

Thus, in the ratio $3:5$, 3 is the antecedent, and 5 the consequent.

260. The ratio of *two quantities of the same kind, when expressed in terms of the same unit*, is the ratio of the numbers by which they are expressed.

Thus, the ratio of \$10 to \$7 is $10:7$.

The ratio of 3 bushels to 19 pecks is the ratio of 12 pecks to 19 pecks, or $12:19$.

Note. No ratio can exist between quantities which are not of the same kind, such as feet and pounds.

261. A ratio being an expression of division, it follows from Arts. 87 and 93 that

Both terms of a ratio may be multiplied or divided by the same number, without altering the value of the ratio.

Thus, the ratio $21:35$, by dividing each term by 7, is equal to the ratio $3:5$.

262. A ratio is said to be *compounded* of two or more others when its antecedent is the product of their antecedents, and its consequent the product of their consequents.

Thus, the ratio compounded of $\left.\begin{matrix}3:5\\7:4\end{matrix}\right\}$ is $3 \times 7 : 5 \times 4$, or $21:20$.

EXAMPLES.

263. Simplify the following ratios:

1. $98:245$.	**3.** \$7.13 : \$11.47.	**5.** $1089:1573$.
2. $\frac{10}{27}:\frac{25}{36}$.	**4.** $4\frac{1}{8}:4\frac{7}{12}$.	**6.** $3\frac{8}{35}:2\frac{8}{77}$.

7. What is the ratio of 25 lb. 7 oz. av. to 32 lb. 6 oz. av.?

8. What is the ratio of £5 6s. 3d. to £3 3s. 9d.?

9. What is the ratio of 3 rd. 5 yd. 1 ft. 5 in. to 5 rd. 0 yd. 2 ft. 3 in.?

10. Which is the greater, 12 : 13 or 25 : 27?

Find the ratios compounded of:

11. $\left.\begin{array}{r}2:3\\6:7\\14:5\end{array}\right\}$. **12.** $\left.\begin{array}{r}8:9\\11:16\\12:11\end{array}\right\}$. **13.** $\left.\begin{array}{r}27:25\\28:45\\20:21\end{array}\right\}$. **14.** $\left.\begin{array}{r}68:57\\76:69\\92:85\end{array}\right\}$.

PROPORTION.

264. A **Proportion** is an equality of ratios.

Thus, the statement that the ratio of 3 to 5 is equal to the ratio of 6 to 10 forms a proportion, which may be written in the forms

$$3:5=6:10,\ \tfrac{3}{5}=\tfrac{6}{10},\ \text{or}\ 3:5::6:10.$$

265. The first and fourth terms of a proportion are called the *extremes*, and the second and third terms the *means*.

Thus, in the proportion $3:5=6:10$, 3 and 10 are the extremes, and 5 and 6 the means.

266. In a proportion in which the means are equal, either mean is called a **Mean Proportional** between the first and fourth terms, and the fourth term is called a **Third Proportional** to the first and second terms.

Thus, in the proportion $4:6=6:9$, 6 is a mean proportional between 4 and 9, and 9 is a third proportional to 4 and 6.

A **Fourth Proportional** to three numbers is the fourth term of a proportion, whose first three terms are the three numbers in their order.

Thus, in the proportion $3:5=6:10$, 10 is a fourth proportional to 3, 5, and 6.

PROPERTIES OF PROPORTIONS.

267. Let us consider the proportion

$$3:5=6:10, \text{ or } \tfrac{3}{5}=\tfrac{6}{10}.$$

If each of these equal fractions be multiplied by the product of the two denominators, 5×10, the products will evidently be equal; that is,

$$5\times 10\times \tfrac{3}{5}=5\times 10\times \tfrac{6}{10}.$$

Cancelling 5 in the first product, and 10 in the second, we have

$$10\times 3=5\times 6. \tag{1}$$

Hence, *in any proportion, the product of the extremes is equal to the product of the means.*

268. If each of the equal products in (1), Art. 267, be divided by 3, the quotients will evidently be equal.

That is, $$\frac{10\times 3}{3}=\frac{5\times 6}{3}; \text{ or, } 10=\frac{5\times 6}{3}.$$

Hence, *in any proportion, either extreme is equal to the product of the means divided by the other extreme.*

In like manner, *either mean is equal to the product of the extremes divided by the other mean.*

269. It follows from Art. 267 that the square of the mean proportional between two numbers is equal to the product of the numbers.

Hence, *the mean proportional between two numbers is equal to the square root of their product.*

EXAMPLES.

270. 1. Find a fourth proportional to 7, 10, and 21.

By Art. 268, the required fourth proportional is

$$\frac{10\times 21}{7}, \text{ or } 30, \textit{ Ans.}$$

2. Find a mean proportional between $\frac{3}{4}$ and $\frac{12}{25}$.

By Art. 269, the required mean proportional is

$$\sqrt{\tfrac{3}{4} \times \tfrac{12}{25}} = \sqrt{\tfrac{9}{25}} = \tfrac{3}{5}, \text{ } Ans.$$

3. Find a fourth proportional to 65, 80, and 91.

4. Find a third proportional to 25 and 30.

5. Find a mean proportional between 12 and 48.

6. Find a third proportional to $\frac{3}{4}$ and $\frac{5}{6}$.

7. Find a fourth proportional to $\frac{20}{21}$, $\frac{3}{5}$, and $\frac{8}{7}$.

8. Find a mean proportional between $5\frac{5}{8}$ and $18\frac{9}{10}$.

9. What is the second term of a proportion whose first, third, and fourth terms are $5\frac{1}{7}$, $4\frac{4}{5}$, and $1\frac{5}{9}$?

10. Find a fourth proportional to $\frac{55}{84}$, $\frac{88}{75}$, and $\frac{125}{51}$.

PROBLEMS IN PROPORTION.

271. 1. If 35 yards of cloth cost $78.75, how much will 47 yards cost?

It is convenient to arrange the proportion so that the required answer shall be the *fourth term*, and the quantity which is of the same kind as the answer the third term.

In the present case, since *cost* is required, the third term will be $78.75.

Let the fourth term be represented by x.

The other two quantities, 35 yards and 47 yards, are taken to form the first ratio; and since 47 yards will evidently cost more than 35 yards, the first term of the proportion should be 35 yards, and the second term 47 yards.

The proportion will then stand as follows:

$$35 \text{ yards} : 47 \text{ yards} = \$78.75 : x.$$

The value of x may be obtained by means of the principle of Art. 268; that is, by dividing the product of the means by the other extreme.

In order to avoid the multiplication of *yards* by *dollars*, we may consider the terms of each ratio replaced by the *numbers* which express how many times they contain the unit.

Thus, $35 : 47 = 78.75 : x.$

Whence, $x = \frac{47 \times 78.75}{35} = 105.75.$

Then the required cost is \$105.75, *Ans.*

2. If 27 men can do a piece of work in 15 days, how many days will it take 36 men to do it?

Since the answer is to be *days*, we make 15 days the third term.

Now 36 men will evidently require *less* time than 27 men to perform the work; we therefore make 36 men the first term, and 27 men the second term.

Then, omitting reference to the units,

$$36 : 27 = 15 : x.$$

Or, $x = \frac{27 \times 15}{36} = \frac{45}{4} = 11\frac{1}{4}.$

Then the required time is $11\frac{1}{4}$ days, *Ans.*

From the above examples, we derive the following

RULE.

Make the quantity which is of the same kind as the answer the third term.

If, from the nature of the problem, the answer is to be greater than the third term, make the smaller of the remaining quantities the first, and the greater the second term; but if the answer is to be less than the third term, make the greater of the quantities the first term, and the smaller the second term.

Finally, divide the product of the means by the given extreme.

EXAMPLES.

3. If 23 gallons of molasses cost $\$4.37$, how much will 29 gallons cost?

4. If 51 bushels of grain cost $\$21.42$, how much will 38 bushels cost?

5. If 41 men can do a piece of work in 35 days, how many days will it take 28 men to do it?

6. If 37 men can do a piece of work in 63 days, how many days will it take 42 men to do it?

7. If a man travels 705 miles in 25 days, how many miles will he travel in 23 days?

8. If a certain amount of provisions will last 50 men 121 days, how many days will it last 77 men?

9. If $4\frac{7}{8}$ bushels of oats cost $\$2.87\frac{1}{2}$, how much will $7\frac{4}{5}$ bushels cost?

10. If a man can perform a certain journey in 84 hours, travelling $9\frac{5}{8}$ miles an hour, how many hours will it take him travelling $16\frac{1}{8}$ miles an hour?

11. A bankrupt pays 76 cents on a dollar. If a certain creditor receives $\$251.75$, what was his original claim?

12. If a field 26 rods long and 10 rods wide be worth $\$650$, how much will a field 25 rods long and 18 rods wide be worth?

13. If 234 men can do a piece of work in $11\frac{2}{3}$ hours, how many men will it take to do it in $9\frac{3}{4}$ hours?

14. If $5\frac{1}{2}$ yards of cloth cost $\$8.25$, how much will $13\frac{3}{5}$ yards cost?

15. If a man 5 ft. $9\frac{3}{4}$ in. in height casts a shadow 3 ft. $2\frac{3}{4}$ in. in length, what is the height of a steeple that casts a shadow 88 ft. $6\frac{1}{2}$ in. in length?

16. If $8\frac{1}{3}$ tons of coal cost $\$36\frac{3}{4}$, how many tons can be bought for $\$39\frac{1}{5}$?

17. If a train travels 6468 yards in $5\frac{1}{4}$ minutes, what is its rate in miles an hour?

18. If £2 8*s.* 8*d.* be worth \$11.92, how much is £6 1*s.* 8*d.* worth?

19. If a horse travels 65 rods in 104 seconds, how many minutes will it take him to travel a mile?

20. A pipe which discharges 3 qt. 1 pt. 2 gi. in one second, empties a tank in 5 min. 57 sec. How long will it take a pipe which discharges 2 qt. 0 pt. 1 gi. in one second to empty the tank?

21. If a train performs a certain journey in $2\frac{2}{3}$ hours, travelling at the rate of 2750 feet a minute, how long will it take it travelling at the rate of $18\frac{1}{3}$ yards a second?

22. A garrison of 357 men had food for 112 days; but some reinforcements having been received, the food lasted only 98 days. How many men were received as reinforcements?

23. If 3 cu. ft. 1008 cu. in. of water weigh 223 lb. $15\frac{1}{3}$ oz., how much will 4 cu. ft. 720 cu. in. of water weigh?

24. A piece of work was to have been done by 45 men in 83 days; but after 27 days, 21 men were sent away. How long did it take the remaining 24 men to complete the work?

25. If a field 10 rd. 3 yd. long and 7 rd. 4 yd. wide be worth \$$147\frac{9}{10}$, how much will a field 9 rd. 3 yd. long and 8 rd. 4 yd. wide be worth?

COMPOUND PROPORTION.

272. A **Compound Proportion** is a proportion one of whose ratios is a *compound ratio* (Art. 262); as,

$$\left.\begin{matrix} 3:5 \\ 7:4 \end{matrix}\right\} = 42:40.$$

PROBLEMS.

273. 1. If 4 men can build 64 feet of wall in 8 days of 9 hours each, how many men will it take to build 80 feet of wall in 3 days of 10 hours each?

Since the answer is to be *men*, we make 4 men the third term.

If the answer depended only on the number of feet built, it would be *greater* than the third term, for it will take longer to build 80 feet than 64 feet; hence the first ratio is 64 : 80.

Again, if the answer depended only on the number of days worked, it would be *greater* than the third term, for it will take more men to build the wall in 3 days than in 8 days; hence the second ratio is 3 : 8.

Also, if the answer depended only on the number of hours worked each day, it would be *less* than the third term, for it will take fewer men to build the wall working 10 hours than working 9 hours a day; hence the third ratio is 10 : 9.

The work now stands as follows:

$$\left.\begin{array}{r} 64 : 80 \\ 3 : 8 \\ 10 : 9 \end{array}\right\} = 4 : x.$$

Whence, $$x = \frac{4 \times \overset{8}{\cancel{80}} \times \cancel{8} \times \overset{3}{\cancel{9}}}{\underset{8}{\cancel{64}} \times \cancel{3} \times \cancel{10}} = 12 \text{ men, } \textit{Ans.}$$

2. If 10 men can build 108 feet of wall in 6 days, how many feet can 15 men build in 5 days?

3. If 22 men can do a piece of work in 3 days of 9 hours each, how many days of 11 hours each will it take 15 men to do the same work?

4. If 18 men can build 57 rods of ditch in 26 days, how many men will it take to build 38 rods in 39 days?

5. If 16 men can do a piece of work in 9 days of $8\frac{1}{3}$ hours each, how many men will it take to do the same work in 6 days of 10 hours each?

6. If 13 horses consume 65 bushels of oats in 24 days, how many bushels will 7 horses consume in 32 days?

7. If 5 men can build 200 feet of fence in 4 days, in how many days can 7 men build 350 feet?

8. If a pasture of 13 acres will feed 9 cows for $4\frac{1}{3}$ months, how many cows will 21 acres feed for $5\frac{1}{4}$ months?

9. If a man can travel 156 miles in 6 days of $8\frac{2}{3}$ hours each, how many days of $12\frac{3}{4}$ hours each will it take him to travel 85 miles?

10. If 6 men can do a piece of work in 9 days of 10 hours each, in how many hours a day can 12 men do the same work in 5 days?

11. If a man can travel 125 miles in 5 days of $6\frac{2}{3}$ hours each, how far can he travel in 8 days of $7\frac{1}{5}$ hours each?

12. If 8 horses consume $4\frac{1}{2}$ tons of hay in 32 days, how many days will $6\frac{3}{4}$ tons last 9 horses?

13. If a 10-cent loaf of bread weighs $7\frac{1}{2}$ ounces, when wheat is \$ $1\frac{1}{10}$ a bushel, what ought a 5-cent loaf to weigh when wheat is \$ $1\frac{1}{4}$ a bushel?

14. If a man can travel 63 miles in 3 days of 7 hours each, how many hours a day must he travel to cover 105 miles in 4 days?

15. If 6 men can build 102 yards of wall in 17 days of 7 hours each, how many yards can 7 men build in 8 days of 9 hours each?

16. If a piece of metal 7 feet long, $4\frac{1}{2}$ inches wide, and 5 inches thick, weighs 550 pounds, how much will a piece of the same metal weigh that is 12 feet long, $5\frac{1}{4}$ inches wide, and 3 inches thick?

17. If 7 men can build 140 feet of fence in 3 days of $8\frac{1}{3}$ hours each, in how many hours a day can 6 men build 240 feet of fence in 5 days?

18. If a bin 8 feet long, $4\frac{1}{3}$ feet wide, and $2\frac{1}{2}$ feet deep, holds $67\frac{1}{2}$ bushels, how wide must a bin be made that is 20 feet long and $4\frac{1}{6}$ feet deep, to hold 450 bushels?

19. If 13 men can build 39 rods of ditch in 4 days of $10\frac{1}{2}$ hours each, how many days of $9\frac{1}{3}$ hours each will it take 11 men to build 55 rods?

20. If a piece of iron 18 inches long, 8 inches wide, and $1\frac{1}{2}$ inches thick, weighs 54 pounds, how much will a piece weigh that is 15 inches long, 12 inches wide, and $1\frac{3}{4}$ inches thick?

21. If a tank 7 feet long, 4 feet wide, and $2\frac{1}{3}$ feet deep, holds $8\frac{1}{3}$ hogsheads of water, how many hogsheads will a tank hold that is 6 feet long, 5 feet wide, and $3\frac{1}{2}$ feet deep?

22. If a piece of stone 6 feet long, $1\frac{1}{2}$ feet wide, and 2 inches thick, weighs 240 pounds, how long must a piece of the same stone be that is $2\frac{1}{3}$ feet wide, and 3 inches thick, to weigh 840 pounds?

23. If 3 men can do $\frac{5}{12}$ of a certain piece of work in 5 days of 12 hours each, how many men will it take to do $\frac{4}{9}$ of the work in 6 days of 8 hours each?

24. If a tank 6 feet long, 3 feet wide, and 2 feet deep, holds $4\frac{1}{4}$ hogsheads of water, how deep must a tank be that is 8 feet long, and $5\frac{1}{3}$ feet wide, to contain $22\frac{2}{3}$ hogsheads of water?

25. If 16 men can do $\frac{2}{3}$ of a certain piece of work in 20 days of 7 hours each, in how many days of 8 hours each can 12 men do $\frac{3}{4}$ of the work?

26. If 12 blocks of stone, each 3 feet long, $1\frac{1}{2}$ feet wide, and 6 inches thick, weigh together 1035 pounds, how much will 18 blocks of the same stone weigh, each $2\frac{1}{2}$ feet long, 1 foot wide, and 8 inches thick?

27. If 14 men can build a wall 80 feet long, 4 feet wide, and 6 feet high, in 24 days, how many days will it take 18 men to build a wall 72 feet long, 5 feet wide, and 8 feet high?

28. If 45 men can dig a ditch 100 feet long, 20 feet wide, and 8 feet deep, in 5 days, how many men will it take to dig a ditch 120 feet long, 32 feet wide, and 9 feet deep, in 6 days?

29. If 6 men can build a wall 72 feet long, 5 feet wide, and 8 feet high, in 8 days of 10 hours each, how many men will it take to build a wall 60 feet long, 4 feet wide, and 9 feet high, in 5 days of 9 hours each?

30. If 24 men can dig a trench 96 feet long, 10 feet wide, and 6 feet deep, in 6 days of 8 hours each, how many days of 11 hours each will it take 21 men to dig a trench 80 feet long, 11 feet wide, and 7 feet deep?

PARTITIVE PROPORTION.

274. Partitive Proportion is the process of dividing a number into parts proportional to certain given numbers.

PROBLEMS.

275. 1. Divide 98 into parts proportional to 2, 3, 4, and 5.

The first part is to be $\frac{2}{5}$, the second part $\frac{3}{5}$, and the third part $\frac{4}{5}$, of the fourth part.

Whence, the sum of all the parts must be $\frac{2}{5} + \frac{3}{5} + \frac{4}{5} + \frac{5}{5}$, or $\frac{14}{5}$ of the fourth part.

But the sum of all the parts is 98.

Whence, the fourth part must be $\frac{5}{14}$ of 98, or 35; and the first, second, and third parts are, respectively, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ of 35, or 14, 21, and 28, *Ans.*

In the above example, the required parts are, respectively, $\frac{2}{14}$, $\frac{3}{14}$, $\frac{4}{14}$, and $\frac{5}{14}$ of 98.

In any similar case, we form fractions having the given numbers for their numerators, and the sum of these numbers for their common denominator.

2. Divide 54 into parts proportional to $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.

Reducing the fractions to their least common denominator, this is the same as dividing 54 into parts proportional to $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{10}{12}$; or, into parts proportional to 8, 9, and 10.

$8 + 9 + 10 = 27.$

Whence, the required parts are, respectively, $\frac{8}{27}$ of 54, $\frac{9}{27}$ of 54, and $\frac{10}{27}$ of 54; or, 16, 18, and 20, *Ans.*

3. Divide 105 into parts proportional to 2, 5, and 8.

4. Divide 324 into parts proportional to $3\frac{1}{2}$ and $4\frac{3}{5}$.

5. Divide 455 into parts proportional to 3, 5, 7, 9, and 11.

6. Divide 2466 into parts proportional to $1\frac{1}{3}$, $1\frac{1}{4}$, $1\frac{1}{6}$, and $1\frac{1}{7}$.

7. Divide 406 into two parts, the second of which shall be seven times the first.

8. An alloy contains 30 parts copper, 18 parts zinc, and 7 parts tin. How many pounds of each metal will be required to make 935 pounds of the alloy?

9. Three men, A, B, and C, invested money in a certain enterprise; A putting in \$1250, B \$1600, and C \$2040. If they gain \$1467, how shall it be divided?

10. A certain explosive mixture contains 16 parts saltpetre, 3 parts charcoal, and 5 parts sulphur. How many pounds of each substance will be required to make 274 pounds of the mixture?

11. Divide 1386 into four parts, such that the second part shall be twice the first, the third part three times the second, and the fourth part four times the third.

12. A man divided the sum of \$163.52 between his four children, in proportion to the numbers 8, 12, 15, and 21. How much did each receive?

13. Air contains 21 parts oxygen to 79 parts nitrogen. How many cubic feet of each gas are there in a receptacle 5 ft. 4 in. long, 5 ft. 3 in. wide, and 4 ft. 6 in. deep?

14. Divide 66 into three parts, such that the second part shall be $\frac{4}{5}$ of the first, and the third part $\frac{4}{5}$ of the second.

15. A certain sum of money was divided between three men, in proportion to the numbers 41, 32, and 19. If the first man received $ 51.25, what was the amount divided?

16. A certain number is divided into parts proportional to the numbers 3, 6, 11, and 18, such that the fourth part exceeds the third by 35. What are the parts?

SIMILAR SURFACES AND SOLIDS.

276. Similar surfaces or solids are those which have the same *form*.

Thus, two circles of unequal diameters are similar surfaces.

It is proved in Geometry that:

1. *Corresponding lines in similar surfaces or similar solids are proportional.*

Thus, in two circles, the circumference of the first is to the circumference of the second as the diameter of the first is to the diameter of the second.

2. *The areas of similar surfaces are to each other as the squares of their corresponding lines.*

Thus, the area of one circle is to the area of another as the square of the radius of the first is to the square of the radius of the second.

3. *The volumes of similar solids are to each other as the cubes of their corresponding lines.*

Thus, the volume of one sphere is to the volume of another as the cube of the diameter of the first is to the cube of the diameter of the second.

EXAMPLES.

1. If the volume of a pyramid whose altitude is 7 in. is 686 cu. in., what is the volume of a similar pyramid whose altitude is 12 inches?

Let x represent the volume of the second pyramid.

Then, since the volumes of similar solids are to each other as the cubes of their corresponding lines, we have

$$686 : x = 7^3 : 12^3.$$

Whence, $x = \frac{686 \times 12^3}{7^3}$, or 3456 cu. in., *Ans.*

2. If the area of a rectangle whose base is 12 ft. is 96 sq. ft., what is the base of a similar rectangle whose area is 216 sq. ft.?

Let x represent the base of the second rectangle.

Then, since the areas of similar surfaces are to each other as the squares of their corresponding lines, we have

$$96 : 216 = 12^2 : x^2.$$

Whence, $x^2 = \frac{216 \times 12^2}{96} = 324.$

Therefore, $x = \sqrt{324}$, or 18 ft., *Ans.*

3. If the area of a parallelogram whose base is 15 in. is 375 sq. in., what is tho area of a similar parallelogram whose base is 18 in.?

4. If the area of a triangle whose altitude is 12 in. is 72 sq. in., what is the altitude of a similar triangle whose area is 32 sq. in.?

5. If the volume of a frustum of a cone whose altitude is 16 in. is 360 cu. in., what is the volume of a similar frustum of a cone whose altitude is 20 in.?

6. If the volume of a prism whose altitude is 9 ft. is 171 cu. ft., what is the altitude of a similar prism whose volume is $50\frac{2}{3}$ cu. ft.?

7. If the area of a trapezoid whose altitude is 1 ft. 10 in. is 4 sq. ft. 104 sq. in., what is the altitude of a similar trapezoid whose area is 2 sq. ft. $94\frac{1}{2}$ sq. in.?

8. If the volume of a cylinder the radius of whose base is 5 in. is 625 cu. in., what is the radius of the base of a similar cylinder, whose volume is 1715 cu. in.?

9. Two bins of the same form contain, respectively, 375 and 648 bushels of wheat. If the first bin is 3 ft. 9 in. deep, what is the depth of the second?

10. If a circular plate of silver whose diameter is 8 inches is worth $76.80, what is the diameter of a similar plate whose value is $97.20?

11. If it costs $26.25 to concrete the floor of a cellar whose length is 35 ft., how much will it cost to concrete the floor of a similar cellar whose length is 42 ft.?

12. If a sphere 6 inches in diameter weighs 47 lb. 4 oz., what is the weight of a sphere of the same material whose diameter is 10 inches?

13. A cone whose altitude is 10 in. weighs 24 lb. At what distance from the vertex must it be cut by a plane parallel to the base, so that the frustum cut off may weigh 12 lb.?

14. If a pipe whose diameter is $2\frac{3}{4}$ in. can empty a tank in 3 hr. 44 min., how long will it take a pipe whose diameter is $3\frac{2}{3}$ in. to empty it?

15. If a sphere whose diameter is 2 ft. 1 in. weighs 3125 lb., what is the diameter of a sphere of the same material whose weight is 819.2 lb.?

XVII. PARTNERSHIP.

277. A **Partnership** is an association formed by two or more persons for the transaction of business.

The persons forming a partnership are called *partners.*

278. The **Capital** or **Stock** is the money or property which the partners invest in the business.

The **Assets** or **Resources** of a partnership comprise all property of whatever nature belonging to it.

The **Liabilities** are its debts.

SIMPLE PARTNERSHIP.

279. A **Simple Partnership** is one in which the capital of each partner is invested for the same length of time.

In such a case, the profits or losses are shared by the partners in proportion to the amounts which they have invested.

PROBLEMS.

280. **1.** A, B, and C form a partnership; A putting in \$3500, B \$3200, and C \$2800. If \$5700 is gained, what is each partner's share of the profits?

The profits are shared by A, B, and C in proportion to the numbers 3500, 3200, and 2800; or, dividing each number by 100, in proportion to the numbers 35, 32, and 28.

$35 + 32 + 28 = 95$; whence, by Art. 275,

A's share is $\frac{35}{95}$ of \$5700, or \$2100;
B's share is $\frac{32}{95}$ of \$5700, or \$1920;
C's share is $\frac{28}{95}$ of \$5700, or \$1680, *Ans.*

Note. In any case like the above, when the numbers in proportion to which the profits are shared have a *common factor*, they may be divided by this factor before applying the rule.

2. A and B form a partnership; A putting in \$3750, and B \$5375. They gained \$1460. What is each partner's share of the profits?

3. Allen, Brown, and Cole enter into partnership; Allen putting in $9000, Brown $10800, and Cole $5760. They gain $3550. What is each partner's share of the gain?

4. A and B formed a partnership; A putting in three times as much capital as B. They lost $257. What was each partner's share of the loss?

5. A bankrupt owes to A $132; to B $165; and to C $198. If his entire resources are $184.65, what is the share of each creditor?

6. A, B, and C entered into partnership; A putting in twice as much capital as B, and B putting in five times as much as C. They lost $27392. What was each partner's share of the loss?

7. Hale and Hunt formed a partnership; Hale putting in $\frac{5}{6}$ as much capital as Hunt. They gained $3795. What was each partner's share of the profits?

8. A, B, and C formed a partnership; B putting in $\frac{3}{4}$ as much capital as A, and C $\frac{5}{2}$ as much as B. They gained $6080.43. What was each partner's share of the profits?

9. Four men, A, B, C, and D, hired a pasture for $19.50 a month. A put in 7 horses, B 5 horses, C 8 horses, and D 6 horses. What was each man's share of the rent?

10. A, B, and C enter into partnership; A putting in $\frac{3}{2}$ as much capital as B, and $\frac{4}{5}$ as much as C. They lost $2531.20. What was each partner's share of the loss?

COMPOUND PARTNERSHIP.

281. A **Compound Partnership** is one in which the amounts invested by the partners are employed for unequal times.

PROBLEMS.

282. 1. A, B, and C entered into partnership. A put in $300 for 7 months, B $400 for 6 months, and C $270 for 10 months. They gained $216. What was each partner's share of the profits?

Investing \$300 for 7 months is the same as investing \$2100 for 1 month; \$400 for 6 months is the same as \$2400 for 1 month; and \$270 for 10 months is the same as \$2700 for 1 month.

Then the profits are shared by the partners in proportion to the numbers 2100, 2400, and 2700; or, dividing each number by 300, in proportion to the numbers 7, 8, and 9.

$7 + 8 + 9 = 24$; whence,

A's share is $\frac{7}{24}$ of \$216, or \$63;
B's share is $\frac{8}{24}$ of \$216, or \$72;
C's share is $\frac{9}{24}$ of \$216, or \$81, *Ans.*

From the above example, we derive the following

RULE.

Multiply the capital of each partner by the time which it is employed, and divide the profits or losses in proportion to the products.

2. A and B formed a partnership; A putting in \$840 for 5 months, and B \$720 for 7 months. They gained \$356.40. What was each partner's share of the profits?

3. A, B, and C entered into partnership. A put in \$825 for 4 months, B \$900 for 11 months, and C \$687.50 for 8 months. They lost \$467.50. What was each partner's share of the loss?

4. A, B, C, and D hired a pasture for \$116.60. A put in 4 cows for 3 months, B one cow for 6 months, C 3 cows for 5 months, and D 5 cows for 4 months. What was each man's share of the rent?

5. Adams and Burke entered into partnership. Adams put in \$3500 for 9 months, and Burke \$4900 for 6 months. They lost \$2320. What was each partner's share of the loss?

6. A, B, and C agreed to do a certain piece of work for \$3655. A furnished 25 men for 18 days, B 18 men for 20 days, and C 32 men for 15 days. What was each man's share of the amount?

7. Rand, Sears, and Thomas formed a partnership. Rand put in \$5200 for 11 months, Sears \$4500 for 13 months, and Thomas \$3900 for 16 months. They gained \$2192. What was each partner's share of the profits?

8. A and B entered into partnership for one year. A put in \$500 for 5 months, and \$500 more for the remainder of the year; B put in \$600 for 8 months, and \$200 more for the remainder of the year. They gained \$980. What was each partner's share of the profits?

9. A, B, and C hired a pasture for \$110.50. A put in 20 cows for 5 months, B 16 horses for 3 months, and C 42 sheep for 4 months. If 5 cows be considered as equivalent to 3 horses, and 6 sheep to one horse, how much should each man pay?

10. Fuller and Gray entered into partnership for one year. Fuller put in \$2300, and at the end of 8 months took out \$900; Gray put in \$3100, and at the end of 3 months took out \$800. They lost \$450. What was each partner's share of the loss?

11. A, B, and C formed a partnership for one year. A put in \$850, and at the end of 7 months withdrew \$300; B put in \$1240, and at the end of 5 months withdrew \$540; C put in \$950, and at the end of 9 months added \$200. They lost \$530. What was each partner's share of the loss?

12. A, B, and C entered into partnership. A put in $\frac{1}{3}$ of the capital for $\frac{5}{12}$ of the year; B $\frac{1}{4}$ of the capital for $\frac{2}{3}$ of the year; and C the balance for $\frac{3}{4}$ of the year. They gained \$6675. What was each partner's share of the profits?

13. On Jan. 1st, Lowe and Martin entered into partnership for one year; Lowe putting in \$5500, and Martin \$7000. On May 1st they admitted Neal with a capital of \$3500. On Sept. 1st, Lowe put in \$2000, and on Nov. 1st, Martin withdrew \$1000. They gained \$5060. What was each partner's share of the profits?

XVIII. PERCENTAGE.

283. The expression **Per Cent** signifies *hundredths.*

Thus, 7 per cent of any number is the same as .07 of the number.

284. The symbol % stands for *per cent.*

Thus, 3% means 3 per cent; $17\frac{1}{2}\%$ means $17\frac{1}{2}$ per cent; and so on.

285. The **Base** is the number of which the per cent is taken.

The **Rate Per Cent** is the fraction which denotes how many hundredths are taken.

The **Percentage** is the result.

For example, 5% of \$25 = .05 of \$25 = \$1.25.

In this case, the base is \$25, the rate per cent is 5% or .05, and the percentage is \$1.25.

286. **Percentage** is the process of computing by per cents.

EXAMPLES.

287. 1. Express $\frac{3}{4}\%$ as a decimal. *Result.* $.00\frac{3}{4}$, or .0075.

2. Express .08% as a decimal. *Result.* .0008.

Express each of the following as a decimal:

3. 9%.	6. $87\frac{1}{2}\%$.	9. $\frac{4}{5}\%$.	12. 1.6%.
4. 58%.	7. $11\frac{1}{8}\%$.	10. $\frac{3}{16}\%$.	13. .07%.
5. 136%.	8. $5\frac{3}{8}\%$.	11. $\frac{11}{25}\%$.	14. .069%.

288. To express a Rate Per Cent as a Common Fraction.

Example. Express $42\frac{6}{7}\%$ as a common fraction.

$$42\tfrac{6}{7}\% = \frac{42\frac{6}{7}}{100} = \frac{\frac{300}{7}}{100} = \frac{300}{700} = \frac{3}{7}, \quad \textit{Ans.}$$

The following table of per cents with their fractional equivalents should be carefully *committed to memory;* the results are left as exercises for the student.

Per cent.	Fraction.	Per cent.	Fraction.	Per cent.	Fraction.	Per cent.	Fraction.
$6\frac{1}{4}$	$\frac{1}{16}$	20	$\frac{1}{5}$	40	$\frac{2}{5}$	70	$\frac{7}{10}$
$8\frac{1}{3}$	$\frac{1}{12}$	25	$\frac{1}{4}$	50	$\frac{1}{2}$	75	$\frac{3}{4}$
10	$\frac{1}{10}$	30	$\frac{3}{10}$	60	$\frac{3}{5}$	80	$\frac{4}{5}$
$12\frac{1}{2}$	$\frac{1}{8}$	$33\frac{1}{3}$	$\frac{1}{3}$	$62\frac{1}{2}$	$\frac{5}{8}$	$87\frac{1}{2}$	$\frac{7}{8}$
$16\frac{2}{3}$	$\frac{1}{6}$	$37\frac{1}{2}$	$\frac{3}{8}$	$66\frac{2}{3}$	$\frac{2}{3}$	90	$\frac{9}{10}$

EXAMPLES.

Express each of the following as a common fraction in its lowest terms:

1. 48%. 4. $43\frac{3}{4}$%. 7. $88\frac{8}{9}$%. 10. $\frac{16}{5}$%. 13. 7.5%.

2. 175%. 5. $70\frac{5}{6}$%. 8. $\frac{2}{3}$%. 11. $\frac{56}{15}$%. 14. .96%.

3. $6\frac{2}{3}$%. 6. $47\frac{11}{12}$%. 9. $\frac{5}{6}$%. 12. $\frac{25}{8}$%. 15. .625%.

289. **To express a Common Fraction as a Rate Per Cent.**

1\. Express $\frac{3}{8}$ as a rate per cent.

$$\begin{array}{r} 8\underline{)3.00} \\ .37\frac{1}{2} \end{array}$$

Dividing 3 by 8 to two places of decimals, we obtain the result $.37\frac{1}{2}$.

Whence, $\frac{3}{8} = 37\frac{1}{2}$ %, *Ans.*

EXAMPLES.

Express each of the following as a per cent:

2. $\frac{4}{7}$. 6. $\frac{17}{20}$. 10. $\frac{11}{15}$. 14. $\frac{1}{125}$. 18. $\frac{401}{325}$.

3. $\frac{1}{9}$. 7. $\frac{4}{25}$. 11. $\frac{55}{32}$. 15. $\frac{4}{525}$. 19. $\frac{7}{640}$.

4. $\frac{9}{8}$. 8. $\frac{3}{40}$. 12. $\frac{13}{24}$. 16. $\frac{2}{225}$. 20. $\frac{203}{960}$.

5. $\frac{7}{12}$. 9. $\frac{21}{16}$. 13. $\frac{1}{128}$. 17. $\frac{3}{850}$. 21. $\frac{437}{875}$.

290. **To find the Percentage when the Base and Rate are given.**

1. What is 38% of \$15?

$$\begin{array}{r} \$\,15 \\ .38 \\ \hline 1\,20 \\ 4\,5 \\ \hline \$\,5.70, \end{array}\ \textit{Ans.}$$

38% is the same as .38.
Multiplying \$15 by .38, the result is \$5.70.

2. What is $77\frac{7}{9}$% of 288?

$$\frac{77\frac{7}{9}}{100}=\frac{\frac{700}{9}}{100}=\frac{7}{9}.$$

Whence, $$\frac{7}{\cancel{9}}\text{ of }\overset{32}{\cancel{288}}=224,\ \textit{Ans.}$$

Note 1. It is not easy to indicate by a rule when it is better to use the method of Ex. 1, and when that of Ex. 2.

But in general we may say that if the rate can be readily expressed as a decimal, the method of Ex. 1 is preferable, unless the base is a common fraction or mixed number which cannot be readily expressed as a decimal.

In all other cases, the method of Ex. 2 should be used.

The per cent should always be expressed *mentally* as a common fraction whenever possible. (See Table, Art. 288.)

Note 2. To find any per cent of a *compound number* (Art. 159), it is usually better to express the base in terms of the *lowest* given denomination (Art. 157).

EXAMPLES.

What is

3. 7% of 61.3?
4. 148% of \$5.25?
5. $6\frac{1}{4}$% of 992 bu.?
6. $87\frac{1}{2}$% of $\frac{32}{21}$?
7. $79\frac{1}{6}$% of $\frac{32}{57}$?
8. $71\frac{3}{7}$% of $1\frac{2}{5}$?
9. .8% of \$283.75?
10. $\frac{3}{8}$% of $6\frac{17}{18}$?
11. $58\frac{1}{3}$% of £171 12*s.*?
12. $136\frac{1}{9}$% of $\frac{27}{14}$?
13. .55% of $14\frac{17}{22}$?
14. $\frac{5}{4}$% of 283 ft. 4 in.?

15. A tradesman bought goods for \$350, and sold them at a profit of 18%. How much did he receive for them?

16. A dealer purchased goods to the amount of \$1245, and sold them at a loss of $33\frac{1}{3}$%. How much did he lose by the operation?

17. A bankrupt settled with his creditors by paying to each of them 55% of what he owed him. How much was received by a creditor to whom he owed \$1732.80?

18. A tradesman bought goods for \$586.20, and sold them at a loss of 15%. How much did he receive for them?

19. A man at his death bequeathed $66\frac{2}{3}$% of his property of \$13587 to his wife; the balance to be divided equally between his four children. How much did each child receive?

20. A man sold a horse and carriage for \$648.72, receiving $41\frac{2}{3}$% of the amount in cash. How much was still due him?

21. A merchant bought goods for \$9675.30, and sold them at a profit of $23\frac{1}{3}$%. How much did he gain by the operation?

22. If $52\frac{7}{9}$% of the population of a certain city of 29844 inhabitants are females, what is the number of males in the city?

23. My income is \$2160. I pay $43\frac{3}{4}$% of it for living expenses, 15% for life-insurance, and invest the balance in stocks. What sum do I invest in stocks?

24. A dealer bought 125 barrels of flour at \$6.72 each. After reserving 29 for his own use, he sold the rest at such a price that he gained 8% on his original outlay. At what price per barrel did he sell the flour?

25. I bought an acre of land for \$1089, and sold it at a profit of 20%. For what price per foot did I sell it?

26. If 52% of a certain ore is lead, and $1\frac{2}{8}$% of the balance is silver, how many pounds of each metal are there in a ton of the ore?

27. A man who has $2112 in the bank, draws out $8\frac{1}{3}$% of this sum, and afterwards draws out $6\frac{1}{4}$% of the balance. How much money has he still in the bank?

28. A bought goods to the amount of $315.20. He sold them to B at an advance of 25%, and B then sold them to C at $12\frac{1}{2}$% less than his outlay. How much did C pay for them?

29. The population of a certain town in 1860 was 3552. It increased $91\frac{2}{3}$% from 1860 to 1880, and decreased $37\frac{1}{2}$% from 1880 to 1890. How much greater was its population in 1890 than in 1860?

30. A certain metal, when heated, expands $\frac{1}{288}$% for each degree Fahrenheit. How much will a bar of the above metal, 30 feet long, increase in length, if its temperature be raised 80 degrees?

31. A grocer mixes 65 pounds of coffee worth 37 cents a pound, with 49 pounds worth 43 cents a pound. At what price per pound must he sell the mixture so as to gain $16\frac{2}{3}$%?

32. A gentleman has the sum of $6480 to invest; he puts $26\frac{2}{3}$% of it into railway bonds, $38\frac{8}{9}$% of what remains into stocks, and $45\frac{5}{6}$% of the balance into real estate. How much has he left?

291. **To find the Base when the Percentage and Rate are given.**

Since the Percentage is equal to the Base multiplied by the Rate, it follows that the Base may be found by *dividing the Percentage by the Rate.*

1. \$11.64 is 24% of what sum?

$$
\begin{array}{r l}
.24)\$\,11.64\,(\$\,48.50, & \textit{Ans.} \\
\underline{9\,6} & \\
2\,04 & \\
\underline{1\,92} & \\
120 &
\end{array}
$$

24% is the same as .24.
Dividing \$11.64 by .24, the result is \$48.50.

2. 85 is $83\frac{1}{3}\%$ of what number?

$$\frac{83\frac{1}{3}}{100}=\frac{\frac{250}{3}}{100}=\frac{250}{300}=\frac{5}{6}.$$

Whence, $$\frac{85}{\frac{5}{6}}=\overset{17}{\cancel{85}}\times\frac{6}{\cancel{5}}=102, \textit{ Ans.}$$

Note 1. If the rate can be readily expressed as a decimal, the method of Ex. 1 is preferable, unless the percentage is a common fraction or mixed number which cannot be readily expressed as a decimal.

In all other cases, the method of Ex. 2 should be used.

The per cent should always be expressed *mentally* as a common fraction whenever possible.

Note 2. If the percentage is a *compound number* (Art. 159), it is usually better to express it in terms of the *lowest* given denomination.

EXAMPLES.

Find the number of which

3. 2.916 is 9%.
4. \$9.99 is 54%.
5. $\frac{7}{6}$ is $37\frac{1}{2}\%$.
6. 8° 17′ is $46\frac{2}{3}\%$.
7. $1\frac{4}{21}$ is $41\frac{2}{3}\%$.
8. 35 ft. 9 in. is 75%.
9. $\frac{7}{65}$ is $\frac{4}{5}\%$.
10. $1\frac{5}{12}$ is $47\frac{2}{9}\%$.
11. $\frac{38}{225}$ is 9.5%.
12. 6 lb. 11 oz. av. is $\frac{25}{7}\%$.
13. $4\frac{1}{6}$ is $145\frac{5}{6}\%$.
14. \$1.24 is .64%.

15. A merchant sold goods for \$416.52, which was 17% more than they cost him. How much did they cost him?

If he sold the goods for .17 more than they cost him, he must have sold them for 1.17 of what they cost him.

Then, since \$416.52 is 1.17 of what the goods cost, we divide \$416.52 by 1.17 to obtain the result.

$$
\begin{array}{l}
1.17)\$416.52(\$356, \textit{Ans.} \\
\underline{351} \\
655 \\
\underline{585} \\
702
\end{array}
$$

16. The population of a certain town decreased $12\frac{1}{2}\%$ from 1880 to 1890. If the number of inhabitants in 1890 was 2877, what was the number in 1880?

If the population decreased $12\frac{1}{2}\%$, or $\frac{1}{8}$, it must be $\frac{7}{8}$ as great in 1890 as in 1880.

Then, since 2877 is $\frac{7}{8}$ of the population in 1880, we divide 2877 by $\frac{7}{8}$ to obtain the result.

$$\frac{2877}{\frac{7}{8}} = \overset{411}{\cancel{2877}} \times \frac{8}{\cancel{7}} = 3288, \textit{ Ans.}$$

17. The sum of two numbers is 96; and one of them is $66\frac{2}{3}\%$ greater than the other. What are the numbers?

The greater number is $66\frac{2}{3}\%$, or $\frac{2}{3}$, greater than the less; it is therefore $\frac{5}{3}$ of the less number.

Then the sum of the two numbers must be $\frac{3}{3} + \frac{5}{3}$, or $\frac{8}{3}$ of the less number.

$$\frac{96}{\frac{8}{3}} = \overset{12}{\cancel{96}} \times \frac{3}{\cancel{8}} = 36.$$

$$96 - 36 = 60.$$

Dividing 96 by $\frac{8}{3}$, we find the less number to be 36; and the greater number is 96 minus 36, or 60, *Ans.*

18. A man having lost 14 % of his money, finds that he has \$71.81 left. How much had he at first?

19. I spend \$1162 a year, which sum is $87\frac{1}{2}\%$ of my income. What is my income?

20. What number increased by $58\frac{1}{3}\%$ of itself is equal to 608?

21. What number diminished by 36% of itself is equal to 420?

22. A tradesman gained 16% by selling goods for \$493. How much did the goods cost him?

23. What sum of money increased by 24% of itself is equal to \$77.50?

24. A merchant lost $8\frac{1}{3}\%$ by selling goods for \$952. How much did the goods cost him?

25. What number diminished by $71\frac{3}{7}\%$ of itself is equal to $\frac{6}{35}$?

26. A dealer lost 12% by selling goods for \$260.70. How much money would he have gained by selling them for \$342?

27. Cloth, when sponged, shrinks $1\frac{7}{8}\%$ of its length. If a certain piece of cloth has lost 2 ft. 9 in. by sponging, what was its original length?

28. The number of voters in a certain city is 7925, which is $4\frac{1}{8}\%$ more than the number two years ago. How many were there two years ago?

29. A merchant gained $6\frac{2}{3}\%$ by selling goods for \$1372.16. How much did the goods cost him?

30. At a charity concert which realized \$200.20 for the poor, the expenses were 35% of the receipts. What were the expenses?

31. A brigade of soldiers lost $33\frac{1}{3}\%$ of its numbers in one battle, and $6\frac{1}{4}\%$ of the remainder in another, and had 1830 men left. How many were there at first?

32. There are 297 pupils in a certain school, and the number of boys is 20% greater than the number of girls. How many are there of each sex?

33. A merchant who owned 40% of a vessel, sold 64% of his share for \$ 4480. What was the value of the entire vessel at the same rate?

34. The difference between two numbers is 425, and one of them is $52\frac{1}{12}$% greater than the other. What are the numbers?

35. A merchant sells goods at a profit of 21%, and clears \$ 385.35. What was the selling price of the goods?

36. A gentleman invested $58\frac{1}{3}$% of a certain sum in city bonds, and $53\frac{1}{3}$% of the balance in mortgages, and had \$ 1106 left. How much had he at first?

37. A tradesman sold goods for \$ 655.50, losing $20\frac{5}{6}$% of what they cost him. At what price should the goods have been sold so as to gain $20\frac{5}{6}$%?

38. A merchant loses \$ 461.50 by selling a lot of goods at $31\frac{1}{4}$% less than cost. What was the selling price of the goods?

39. The population of a city increased 25% from 1870 to 1880, and 28% from 1880 to 1890. If the population in 1890 was 92184, what was the population in 1870?

40. A speculator bought a number of shares of railway stock. The price of the stock first advanced 24%, and then fell off 16%, and he sold for \$ 1302. What was the original cost of the shares?

292. To find what Per Cent one Number is of Another.

1. What per cent of \$ 32 is \$ 18?

\$ 32) \$ 18.00 ($.56\frac{8}{32} = 56\frac{1}{4}$%, *Ans.*	
16 0	
2 00	
1 92	
8	

Dividing \$ 18 by \$ 32, the quotient is $.56\frac{1}{4}$.

Then, \$ 18 is $.56\frac{1}{4}$ of \$ 32, or $56\frac{1}{4}$% of \$ 32.

Note 1. The division should always be carried out to *two* places of decimals.

2. What per cent of 56 is 35?

Dividing 35 by 56, we have

$$\tfrac{35}{56} = \tfrac{5}{8} = 62\tfrac{1}{2}\%, \textit{ Ans.}$$

3. What per cent of 75 is $\frac{3}{5}$?

$$\frac{\frac{3}{5}}{75} = \frac{\cancel{3}}{5} \times \frac{1}{\underset{25}{\cancel{75}}} = \frac{1}{125} = \frac{4}{500} = \frac{\frac{4}{5}}{100} = \tfrac{4}{5}\%, \textit{ Ans.}$$

In the above example, we multiply both terms of the fraction $\frac{1}{125}$ by 4, in order to make its denominator a multiple of 100.

Note 2. If the method of Ex. 2 is employed, the fraction should always be expressed *mentally* as a per cent whenever possible.

Note 3. If both the given numbers are *compound*, it is usually better to express them in terms of the *lowest* given denomination.

EXAMPLES.

What per cent of

4. 105 is 7?
5. 120 is 48?
6. \$72 is \$39?
7. 911 is 674.14?
8. 72 is 66?
9. £3 8*s*. is £2 11*s*.?
10. \$3.25 is \$3.51?
11. 94 pk. 4 qt. is 60 pk. 3 qt.?
12. $13\frac{1}{3}$ is $4\frac{1}{6}$?
13. 105 is $\frac{7}{8}$?
14. $8\frac{1}{6}$ ft. is 42 in.?
15. $83\frac{1}{3}$ is $\frac{5}{48}$?

16. If a merchant buys goods for \$420, and sells them for \$595, what does he gain per cent?

17. If a tradesman buys goods for \$352, and sells them for \$308, what does he lose per cent?

18. An article is composed of 19 parts silver, and 6 parts copper. What per cent of the article is silver? What per cent is copper? What per cent is the copper of the silver?

19. The number of voters in a certain city increased from 4644 to 5547. What was the gain per cent?

20. Air is a mixture of nitrogen and oxygen. If the nitrogen is 79% of the mixture, what per cent is the oxygen of the nitrogen?

21. If my income is $2745, and my expenses $2135, what per cent of my salary do I save?

22. From a barrel containing 24 gallons of water, 18 quarts leaked out. What per cent of the water was lost?

23. If 2048 of the 16384 inhabitants of a city are foreign-born, what per cent of the population is native?

24. If a merchant buys goods for five-sixths of what he sells them for, what per cent does he gain?

25. If from 45 tons of ore there are obtained 675 pounds of silver, what per cent of the ore is silver?

26. A miller mixes 8 barrels of flour, worth $6.25 a barrel, with 12 barrels, worth $6.50 a barrel, and sells the whole for $6.56 a barrel. What per cent does he gain?

27. A man having $203.40 in the bank, withdrew $33.90. What per cent of the original sum remained?

28. What per cent is gained by buying apples at $1.25 a bushel, and selling them at 5 cents a quart?

29. A grocer mixes 12 pounds of tea, worth 70 cents a pound, with 15 pounds, worth 84 cents a pound, and sells the mixture at 91 cents a pound. What per cent does he gain?

30. If one-seventh of the price received for an article is gain, what is the gain per cent?

31. I sold a house for $5376, which was 28% more than I gave for it. If I had sold it for $4998, what per cent should I have gained?

32. I purchased an acre of land for \$2100, and sold it at $5\frac{1}{4}$ cents a square foot. What per cent did I gain?

33. A grocer uses a false weight of 15 ounces instead of a pound. What per cent does he gain by his dishonesty? What per cent do his customers lose?

34. If $\frac{5}{87}$ of the price received for an article is gain, what is the gain per cent?

35. A merchant bought a lot of goods. The price at first declined 12%, and then advanced 16%, and he then sold out. What per cent did he gain?

36. A grocer bought 180 barrels of flour at \$6 a barrel, and sold 60 barrels at a loss of $8\frac{1}{3}$%. At what per cent above cost must he sell the remainder, so as to gain 10% on the entire transaction?

37. If $29\frac{1}{8}$% is lost by selling goods for \$275.40, what per cent would be gained by selling them for \$534.60?

APPLICATIONS OF PERCENTAGE.

293. Trade Discount.

Trade Discount is a reduction made in the price of an article. It is usually a certain per cent of the list price of the article; but in some cases, goods are sold subject to several discounts.

The **Net Price** of an article is the list price, minus the discount.

EXAMPLES.

294. 1. Find the net amount of a bill for \$563.20, $12\frac{1}{2}$% off for cash.

$$\begin{array}{r} 8)\underline{\$563.20} \\ \underline{70.40} \\ \$492.80,\ \textit{Ans.} \end{array}$$

$12\frac{1}{2}$% of \$563.20 is $\frac{1}{8}$ of \$563.20, or \$70.40. Subtracting \$70.40 from \$563.20, the net amount is \$492.80.

2. Find the net amount of a bill for \$460, subject to a discount of 15%, and 5% off for cash.

$$\begin{array}{r}
\$460 \\
.85 \\ \hline
23\,00 \\
368\,0 \\ \hline
20)\$391.00 \\
19.55 \\ \hline
\$371.45, \textit{ Ans.}
\end{array}$$

Making a discount of 15% is the same thing as taking 85% of the face of the bill, which is \$391.00.

5% of \$391 is $\frac{1}{20}$ of \$391, or \$19.55; which, subtracted from \$391, leaves \$371.45.

3. A man bought goods to the amount of \$54.90, and obtained a discount of $16\frac{2}{3}$% for cash. How much did he pay for the goods?

4. Find the net amount of a bill for \$231, subject to a discount of $8\frac{1}{3}$%, and 4% off for cash.

5. A tradesman marks a certain article \$19.60; but he makes a discount of 20% from this price, and then a discount of $6\frac{1}{4}$% for cash. What price does he receive for the article?

6. A dealer sells, at a discount of 8% from his list price, a certain article marked \$6.00. If he still makes a profit of 38%, what was the cost of the article?

7. How much must a merchant mark an article costing \$6.50, so as to be able to sell it at 9% below the list price, and still make a profit of 12%?

8. How much must a dealer mark an article costing \$3.50, so as to be able to sell it at a discount of $6\frac{2}{3}$% from the list price, and still make a profit of 16%?

9. Find the net amount of a bill for \$25.60, subject to discounts of 25%, 15%, and $12\frac{1}{2}$%.

10. A tradesman marks goods 25% above cost, and then sells them at a discount of 23% from the list price. Does he gain or lose, and what per cent?

11. How much above cost must a merchant mark an article costing him $3.24, so as to be able to sell it at a discount of 10% from the list price, and still make a profit of 25%?

12. Find the net amount of a bill for $5265, subject to discounts of $33\frac{1}{3}$%, 16%, and 10%.

13. A dealer bought silk at $1.40 a yard. What price per yard must it be marked in order that he may be able to make a discount of 20% from the list price, and still make a profit of 20%?

14. A tradesman marks an article 25% above cost, and then sells it at a discount of $12\frac{1}{2}$% from the list price. What per cent does he gain?

15. How much above cost must a merchant mark an article costing $20.79, so as to be able to sell it at a discount of $6\frac{2}{3}$% from the list price, and still make a profit of $11\frac{1}{9}$%?

16. Find the net amount of a bill for $19.20, subject to discounts of $16\frac{2}{3}$%, $7\frac{1}{2}$%, $6\frac{1}{4}$%, and 4%.

17. What per cent above cost must a merchant mark an article, in order to be able to sell it at a discount of 16% from the list price, and still make a profit of 19%?

If the article is sold at a discount of 16% from the list price, it must be sold for 84% of the list price.

If it sold for 19% above cost, it must be sold for 119% of the cost price.

Then, since $\frac{84}{100}$ of the list price is $\frac{119}{100}$ of the cost price, the list price must be $\frac{100}{84}$ of $\frac{119}{100}$, or $\frac{119}{84}$ of the cost price.

$$\frac{119}{84} = \frac{17}{12} = 141\frac{2}{3}\%.$$

That is, the list price is $141\frac{2}{3}$% of the cost price.

Then the article must be marked $41\frac{2}{3}$% above cost, *Ans.*

18. At what per cent above cost must a tradesman mark an article, in order to be able to sell it for 7% less than the list price, and still make a profit of 24%?

19. What per cent above cost must a merchant mark goods, so as to be able to make a discount of 18% from the list price, and still realize a profit of 23% ?

20. What per cent above cost must goods be marked, to allow a discount of 10% from the list price, and still make a profit of 15% ?

21. What per cent above cost must goods be marked, to allow a discount of 20%, and still make a profit of 9%?

22. What per cent above cost must goods be marked, to allow a discount of 12%, and still make a profit of 21% ?

295. Commission and Brokerage.

Commission, or **Brokerage,** is the compensation received by an agent for transacting business.

It is usually a certain per cent of the *amount received,* when selling, and of the *amount paid,* when buying goods.

The agent is variously styled a *factor, broker, collector,* or *commission merchant.*

The amount left over after paying the commission and other charges is called the *net proceeds.*

EXAMPLES.

296. 1. An agent has sold for me goods to the amount of $2300. His commission is 2½%, and the charges for carting and storage are $16.25. How much is due me?

```
 $2300
   .025
  11 500
  46 00
 $57.50
  16.25
 $73.75

$2300.
   73.75
$2226.25, Ans.
```

2½% of $2300 is $57.50, which is the amount of the commission.

Adding to this the charges, $16.25, the total amount to be deducted is $73.75.

Subtracting this from $2300, the net proceeds are $2226.25.

2. An agent receives $ 1500 to invest, after deducting a commission of 3%. What sum can he invest, and what is the amount of his commission?

```
1.03)$1500($1456.31
      103
       470
       412
        580
        515
         650
         618
          320
          309
           110
```

Since the commission is 3% of the sum invested, the amount received is 103% of the sum invested.

Then, if $ 1500 is 1.03 times the sum invested, we divide $ 1500 by 1.03, giving $ 1456.31+, which is the sum invested; and $ 1500 − $ 1456.31 = $ 43.69, the commission, *Ans.*

3. An agent sells $ 576 worth of cloth. What is his commission at $3\frac{1}{4}$%?

4. A commission merchant sells for me goods to the amount of $ 5146.50. His commission is 2%, and there are charges of $ 78.75 for storage. How much is due me?

5. A factor sells a consignment of goods for $ 254, and, after taking out his commission, remits $ 241.30 to the consignor. What is his rate of commission?

6. A broker bought for me $ 8500 worth of securities, at a brokerage of $\frac{1}{4}$%. What was the brokerage?

7. A commission merchant receives $ 551.20 to invest, after deducting a commission of 6%. What sum can he invest, and what is the amount of his commission?

8. A broker received $ 1000 to invest, after deducting a commission of 1%. What sum can he invest, and what is the amount of his commission?

9. An agent sold goods to the amount of $ 9220, receiving $ 404.25, which included his commission, and a charge for freight and storage of $ 58.50. What was his rate of commission?

10. A commission merchant receives $ 356.15 to invest in cloth at 85 cents a yard, after deducting his commission of $4\frac{3}{4}\%$. How many yards can he buy?

11. What amount must a factor receive in order that he may be able to invest $ 2400, after deducting his commission of 4%?

12. An agent charged $4\frac{1}{4}\%$ for buying goods. If his commission was $ 333.20, what did he pay for the goods?

13. A factor received a sum of money to invest, after deducting his commission of 5%. He invested $ 2251.60. What sum did he receive?

14. What sum must I send to a broker in order that he may be able to invest $ 436, after deducting his brokerage of $\frac{1}{2}\%$?

15. An agent received $ 2500 to invest, after deducting his commission of $2\frac{1}{4}\%$. What sum can he invest, and what is the amount of his commission?

16. A broker, who received $\frac{3}{4}\%$ brokerage for selling some securities, paid $ 8337 to the owner as net proceeds. For what sum were the securities sold?

17. What are the net proceeds on the sale of 368 barrels of flour at $ 6.50 a barrel, the commission being $3\frac{1}{2}\%$, and the charges for freight and storage 41 cents a barrel?

18. A merchant received $ 2194.50 to invest in flour at $ 6.25 a barrel, after deducting a commission of $4\frac{1}{2}\%$. How many barrels can he buy?

19. An agent sells $ 648 worth of goods, receiving a commission of $1\frac{1}{4}\%$. He invests the net proceeds in other goods, charging the same commission. What is the value of the goods bought?

20. My agent has sold for me 4000 yards of silk, at $ 1.69 a yard. His charges are as follows: — commission, $2\frac{1}{4}\%$; guarantee, $1\frac{1}{2}\%$; freight, $ 116.50. How much is due me?

21. An agent sells 206 yards of cloth, at $2.50 a yard, charging 2% commission. He invests the net proceeds in woollens at $1.40 a yard, charging 3% commission. How many yards can he buy?

22. A broker received $1113.84 to invest in bonds at $85 each, after deducting his brokerage of $\frac{1}{8}$%. How many bonds can he buy, and what will be the amount of his brokerage?

23. An agent sells 80 tons of coal at $8.30 a ton, receiving a commission of $1\frac{3}{4}$%. He invests the net proceeds in flour, at $6.55 a barrel, charging $3\frac{1}{4}$% commission. How many barrels can he buy?

297. Insurance.

Insurance is security against loss.

A **Policy of Insurance** is a written contract containing an agreement on the part of the Insurer, for a specified consideration, to pay a specified sum to the insured, in case of loss by fire, death, shipwreck, or otherwise.

The **Premium** is the sum paid for insurance, and is usually reckoned as a certain per cent of the amount insured.

EXAMPLES.

298. 1. What will be the annual premium for insuring $3500 on my life, at $2\frac{1}{4}$% a year?

$$\begin{array}{r} \$\,3500 \\ \underline{.02\frac{1}{4}} \\ 70\,00 \\ \underline{8\,75} \\ \$\,78.75, \text{ } \textit{Ans.} \end{array}$$

2. For what sum must goods valued at $5432 be insured at 3%, in order that, in case of loss, the owner may receive both the value of the goods and the premium paid for the insurance?

Since the premium is 3% of the sum insured, the value of the goods must be 100% – 3%, or 97%, of the sum insured.

Then, if $5432 is .97 of the sum insured, the sum insured is $5432 divided by .97.

$$
\begin{array}{l}
.97)\$5432(\$5600, \textit{Ans.} \\
\quad\quad\;\; \underline{485} \\
\quad\quad\;\; 582
\end{array}
$$

3. What is the premium for insuring $1476 on merchandise for one year at $3\frac{1}{4}\%$?

4. What is the premium for insuring $7550 on a vessel at $5\frac{1}{2}\%$?

5. If the premium for insurance at $1\frac{1}{4}\%$ is $23.25, what is the amount insured?

6. If I pay a premium of $13.24 for insuring my house for $3972, what is the rate per cent of the insurance?

7. For what sum must a cargo worth $1634 be insured at 5%, in order that, in case of loss, the owner may receive both the value of the cargo and the premium?

8. A barn was insured for $4200, at $\frac{3}{4}\%$ per annum. After 23 annual premiums had been paid, the barn was burned. What did the insurance company lose, if no allowance be made for interest?

9. A merchant insured a cargo worth $12560 for $\frac{7}{8}$ of its value, at 4%. In case of shipwreck, what would be his actual loss?

10. I paid $15.40, including $1 for the policy, for insuring $3600 on a house. What was the rate of insurance?

11. A mill was insured for $83\frac{1}{3}\%$ of its value, at $4\frac{3}{4}\%$, and the premium was $228. What was the value of the mill?

12. For what sum must a vessel worth $9168 be insured at $4\frac{1}{2}\%$, in order that, in case of loss, the owner may receive both the value of the vessel and the premium?

13. A man 44 years of age takes out a life-policy for \$ 15000, for the benefit of his wife, at the yearly rate of $2\frac{3}{4}\%$. Should his death occur at the age of 74, how much more would his widow receive than had been paid in annual premiums?

14. A merchant insures some goods for \$ 2750 at $3\frac{1}{2}\%$, the amount insured covering both the value of the goods and the premium. What was the value of the goods?

15. What premium must be paid when merchandise worth \$ 6091.50 is insured at $1\frac{3}{4}\%$, so as to cover both the value of the goods and the premium?

16. A house worth \$ 8000 was insured for $\frac{9}{10}$ of its value by three companies. The first took $\frac{1}{3}$ of the risk at $2\frac{1}{2}\%$, the second $\frac{1}{4}$ of the risk at 2%, and the third the remainder at $2\frac{1}{4}\%$. What was the total premium?

17. A shipowner insures a cargo for \$ 8400, at 5%, the amount insured covering both the value of the cargo and the premium. What was the value of the cargo?

18. If merchandise valued at \$ 6614.30 is insured for \$ 6872, the amount insured covering both the value of the goods and the premium, what is the rate per cent of the insurance?

299. Taxes.

A **Tax** is a sum of money assessed upon the person, real estate, personal property, or income of an individual, for public uses.

When assessed upon the person, it is called a **Poll Tax**, and is always a fixed amount for each male citizen of legal age, without regard to his property.

Note. A Poll Tax is not assessed in every state.

When assessed upon the property of an individual, it is called a **Property Tax**, and is usually reckoned as a certain rate per cent of his taxable property.

This rate per cent is called the **Rate of Taxation**.

It is expressed either as a rate per cent, or as so many dollars on \$ 1000.

Thus, $1\frac{1}{4}\%$ is the same as \$ 12.50 on \$ 1000.

300. To determine the rate of taxation, the whole amount of the poll taxes, if any, is deducted from the entire sum to be raised by taxation; the remainder, divided by the value of the taxable property, is the rate of taxation.

To find the tax to which any individual is liable, multiply his taxable property by the rate of taxation, and add to the product the poll tax, if any; the result will be the entire tax to which the individual is liable.

EXAMPLES.

301. 1. A certain town whose taxable property is \$ 1146000, wishes to raise \$ 14562 by taxation. There are 405 polls, each assessed \$ 2.00. What will be the rate of taxation? What will be the entire tax of an individual whose property is valued at \$ 8500?

Multiplying \$ 2.00 by 405, we have \$ 810, the whole amount of the poll taxes.

Subtracting this from \$ 14,562, the entire sum to be raised by taxation, the remainder is \$ 13,752.

Dividing this by \$1,146,000, the value of the taxable property, the rate of taxation is .012, or \$ 12 on \$ 1000.

Multiplying \$ 8500 by .012, the product is \$ 102, which is the individual's property tax.

Adding to this his poll tax, \$ 2.00, his entire tax is \$ 104.

2. If a town whose taxable property is \$ 75400, wishes to raise \$ 1017.90, what is the rate of taxation?

3. A town whose taxable property is \$ 165000, wishes to raise \$ 3419.50. There are 266 polls, each paying \$ 2.00. What is the rate of taxation?

4. A man's house is valued at \$ 3640, and he pays a poll tax of \$ 1.50. What will be his entire tax, at the rate of \$ 14.50 on \$ 1000?

5. What tax must be assessed in order that $4074 may be left, after paying a commission of 3% for collection?

6. What tax must be assessed in order that $6895 may be left, after paying a commission of 1½% for collection?

7. What tax will be paid by a man whose house is valued at $5160, and personal property at $7815, and who pays for 2 polls at $1.50 each, the rate of taxation being $16.40 on $1000?

8. The sum of $3603.66 is to be raised in a town whose taxable property is $282640. What is the rate of taxation? What will be the tax on a piece of property which is valued at $8400?

9. What tax must be assessed in order that a city may receive $34718.25, after the collector deducts his commission of 2¾%?

10. A collector turned over to the town treasurer the sum of $7968, after deducting his commission of 4%. What was the amount of his commission?

11. A town whose real estate is valued at $724600, and personal property at $561400, wishes to raise $15352.30. There are 547 polls, each paying $1.50. What is the rate of taxation?

12. At what rate must property valued at $425000 be taxed, in order that the sum of $6664 may be left, after paying a commission of 2% for collection?

13. The taxes assessed in a certain town are $24000. If 1¾% commission is paid for all taxes actually collected, and only 95% of the taxes can be collected, what are the net proceeds?

302. Duties.

Duties, or Customs, are taxes levied on imported goods.

A Specific Duty is a fixed tax upon an article without regard to its value.

An **Ad Valorem Duty** is levied at a certain rate per cent of the cost of the goods in the country from which they are imported.

Tare is an allowance made for the weight of the box, cask, etc., containing the goods.

Leqkage is an allowance for the loss of liquors in casks; *Breakage* is an allowance for the loss of liquors in bottles.

Gross weight is the entire weight of the goods and packages, before any allowances are made; *Net weight* is the weight after all allowances have been made.

EXAMPLES.

303. Note. In the following examples, the pound sterling is valued at $4.8665, and the franc at $0.193.

1. A merchant imported 550 yards of silk, invoiced at 7 francs a yard. What is the duty at 60% ad valorem?

$550 \times 7 = 3850$ francs, the value of the invoice in francs.

A franc being valued at $0.193, the value of the invoice in dollars is $3850 \times \$0.193$, or $743.05.

Then 60% of $743.05 is $445.83, *Ans.*

2. What is the duty, at 2.2 cents a pound, on 1100 pounds of tin-plate, tare 5%?

Since the tare is 5%, the number of pounds on which the duty is levied is 100% − 5%, or 95%, of 1100; that is, 1045.

At $0.022 a pound, the duty on 1045 pounds is $22.99, *Ans.*

3. What is the duty, at 8 cents a hundred weight, on 57 casks of salt, each containing 175 pounds?

4. What is the duty, at 25% ad valorem, on 20 watches, invoiced at 104 francs each?

5. What is the duty, at 45% ad valorem, on 5 dozen gold rings, invoiced at £1 4*s.* each?

6. What is the duty, at 35 cents a gallon, on 150 dozen quart bottles of olive oil, breakage 4%?

7. An importer paid $111.24, including an ad valorem duty of 35%, for a fur cape. What was the invoice price?

8. What is the duty, at 20 cents a gallon, on 10 casks of ale, each containing $31\frac{1}{2}$ gallons, leakage 3%?

9. What is the duty, at 60 cents a square yard, and 40% ad valorem, on 375 square yards of rugs, invoiced at 48 francs a square yard?

10. What is the duty, at $4.50 a pound, and 25% ad valorem, on 10000 cigars, invoiced at $6.00 a hundred, and weighing 12 pounds to the thousand?

11. A merchant imported a lot of linen clothing, invoiced at $1120, and paid $577.50 duty, tare $6\frac{1}{4}$%. What was the rate of duty?

12. What is the duty, at 60% ad valorem, on 125 yards of silk, invoiced at 5.52 francs a yard, tare 3%?

13. What is the duty, at $2.00 a dozen, and 30% ad valorem, on 600 table-knives, invoiced at 4*s.* 5*d.* each?

14. What is the duty, at 44 cents a square yard, and 35% ad valorem, on 1000 yards of carpet, $\frac{3}{4}$ of a yard wide, invoiced at 6*s.* 9*d.* a yard?

XIX. INTEREST.

304. **Interest** is money paid for the use of money.

The **Principal** is the money for the use of which interest is paid; the **Amount** is the sum of the principal and interest.

305. Interest is usually reckoned as a certain *rate per cent* of the principal for one year.

This rate per cent is called the **Rate of Interest.**

Note. It will be understood in the following examples that the rate is for one year, unless some other time is specified.

SIMPLE INTEREST.

306. **Simple Interest** is interest on the principal alone.

307. **General Method for Computing Interest at any Rate.**

In computing interest, it is customary to regard the year as consisting of 12 months of 30 days each.

EXAMPLES.

1. What is the interest of \$700 for 8 years at 5%?

$$\begin{array}{r} \$700 \\ .05 \\ \hline \$35.00 \\ 8 \\ \hline \$280.00, \textit{Ans.} \end{array}$$

The interest for one year is .05 of the principal, or \$35.00.

Multiplying this by 8, the interest for 8 years is \$280.00.

If the given time is any number of years and months, and the months can be expressed as the decimal of a year, the method of Ex. 1 may be used.

If the months cannot be expressed as the decimal of a year, it is better to *reduce the given time to months, multiply the interest for one year by the number of months, and divide the result by* 12.

2. Find the interest of \$255 for 5 y. 7 mo. at $4\frac{1}{2}\%$.

$$\begin{array}{r} \$255 \\ \underline{.04\frac{1}{2}} \\ 10\,20 \\ \underline{1\,275} \\ \$11.475 \end{array} \qquad \begin{array}{r} \$11.475 \\ \underline{67} \\ 80\,325 \\ \underline{688\,50} \\ 12\overline{)768.825} \\ \$64.068+ \\ =\$64.07, \textit{Ans.} \end{array}$$

The interest for one year is $.04\frac{1}{2}$ times \$255, or \$11.475.

5 y. 7 mo. is 67 mo.

To find the interest for 67 mo., we multiply the interest for one year by 67, and divide the result by 12.

Note 1. In dividing by 12, there is no need of carrying the quotient beyond the third place of decimals; the result should then be expressed to the *nearest cent*, the number of cents being increased by 1 if the figure in the third decimal place is 5 or more than 5. (Compare Art. 132.)

If the given time is any number of years, months, and *days*, the days should be expressed as the decimal of a month, if possible; otherwise as the fraction of a month.

3. Find the amount of \$26.25 from Dec. 20, 1886, to June 2, 1890, at 7%.

$$\begin{array}{r} \$26.25 \\ \underline{.07} \\ \$1.8375 \\ \underline{41\frac{13}{30}} \\ 1\,8375 \\ 73\,500 \\ \underline{796} \\ 12\overline{)76.1335} \\ \$6.34 = \text{Interest.} \\ \underline{26.25} = \text{Principal.} \\ \$32.59 = \text{Amount, } \textit{Ans.} \end{array} \qquad \begin{array}{r} \$1.8375 \\ \underline{13} \\ 55125 \\ \underline{1\,8375} \\ 3\overline{)2.38875} \\ \$0.796+ \end{array}$$

By the method of Art. 165, we find that the time from Dec. 20, 1886, to June 2, 1890, is 3 y. 5 mo. 13 da., or $41\frac{13}{30}$ mo.

We then multiply the interest for one year by $41\frac{13}{30}$, and divide the result by 12.

Adding the principal to the interest, the amount is \$32.59.

Note 2. The work of multiplying the interest for one year by $\frac{13}{30}$ appears in the right-hand column; we multiply \$1.8375 by 13, move the decimal point of the product one place to the left, and divide the result by 3.

Find the interest of:

4. \$725 for 6 y. at 5%.

5. \$173.25 for 3 y. 3 mo. at 6%.

6. \$476 for 1 y. 7 mo. at $2\frac{1}{4}\%$.

7. \$93.41 for 8 mo. 21 d. at 7%.

8. \$591.68 for 4 y. 10 mo. 18 d. at $3\frac{3}{4}\%$.

9. \$227.75 for 2 mo. 5 d. at $6\frac{1}{2}\%$.

10. \$81.34 for 5 y. 9 mo. 23 d. at $4\frac{1}{2}\%$.

11. \$159 from Jan. 3, 1889, to Dec. 18, 1891, at 3%.

12. \$918.37 from Nov. 20, 1883, to June 13, 1885, at $3\frac{1}{2}\%$.

13. \$78.96 from July 17, 1886, to Nov. 27, 1890, at $5\frac{1}{4}\%$.

14. \$34.70 from Aug. 29, 1891, to Aug. 7, 1892, at 8%.

15. \$209.50 from Dec. 12, 1885, to May 23, 1889, at 2%.

16. \$694.03 from May 19, 1881, to July 6, 1888, at 4%.

Find the amount of:

17. \$63.59 for 5 y. 9 mo. at $3\frac{1}{2}\%$.

18. \$245.25 for 2 y. 5 mo. at $5\frac{1}{4}\%$.

19. \$1869.84 for 6 y. 8 mo. 12 da. at $2\frac{3}{4}\%$.

20. \$463 for 3 mo. 7 d. at 4%.

21. \$3662.95 from March 7, 1891, to Oct. 13, 1891, at 6%.

22. \$851.32 from April 16, 1883, to March 26, 1890, at $4\frac{1}{4}\%$.

23. \$504.18 from June 23, 1887, to Sept. 21, 1892, at 5%.

308. The Six Per Cent Method.

The interest of any sum of money, at 6% a year,

For 2 months, or $\frac{1}{6}$ year, is .01 of the principal.
For 1 month is $\frac{1}{2}$ of .01 of the principal.
For 6 days, or $\frac{1}{5}$ month, is .001 of the principal.
For 1 day, is $\frac{1}{6}$ of .001 of the principal.

We then have the following rules for reckoning interest at 6%:

I. *Multiply* .01 *of the principal by* $\frac{1}{2}$ *the number of months.*

II. *Multiply* .001 *of the principal by* $\frac{1}{6}$ *the number of days.*

III. *Multiply* .001 *of the principal by the number of days, and divide by* 6.

If the given time is any number of years and months, Rule I. should be used.

If it is any number of months and days, use Rule II. if the days can be divided by 6; otherwise, use Rule III.

EXAMPLES.

1. Find the interest of \$926 for 3 y. 11 mo. at 6%.

$$
\begin{array}{rl}
\$9.26 & = .01 \text{ of the prin.} \\
23\frac{1}{2} & = \frac{1}{2} \text{ the no. of months.} \\
\hline
27\,78 & \\
185\,2 & \\
4\,63 & \\
\hline
\$217.61, & \textit{Ans.}
\end{array}
$$

We find .01 of the principal by moving the decimal point two places to the left.

3 y. 11 mo. is 47 mo.; and $\frac{1}{2}$ the number of months is $23\frac{1}{2}$.

2. Find the interest of \$347 for 3 mo. 18 d. at 6%.

$$
\begin{array}{rl}
\$0.347 & = .001 \text{ of the prin.} \\
18 & = \frac{1}{6} \text{ the no. of days.} \\
\hline
2\,776 & \\
3\,47 & \\
\hline
\$6.246 & = \$6.25, \textit{ Ans.}
\end{array}
$$

We find .001 of the principal by moving the decimal point three places to the left.

3 mo. 18 d. is 108 d.; and $\frac{1}{6}$ the number of days is 18.

3. Find the amount of \$152.75 for 67 days at 6%.

$$
\begin{array}{rl}
\$0.15275 & = .001 \text{ of the prin.} \\
67 & = \text{the no. of days.} \\
\hline
1\,06925 & \\
9\,1650 & \\
\hline
6)\underline{10.23425} & \\
\$1.705+ & = \text{the interest.}
\end{array}
$$

$$
\begin{array}{rl}
\$152.75 & \\
1.71 & \\
\hline
\$154.46 & = \text{the amount, } \textit{Ans.}
\end{array}
$$

Find the interest at 6% of:

4. \$461 for 5 y. 10 mo.
5. \$71.25 for 2 y. 5 mo.
6. \$338 for 3 mo. 24 d.
7. \$146.03 for 43 d.
8. \$507.57 for 96 d.
9. \$229.71 for 6 mo. 5 d.
10. \$687.40 from June 13, 1889, to Jan. 25, 1891.
11. \$4174.12 from April 9, 1892, to May 8, 1892.

Find the amount at 6% of:

12. \$28.64 for 4 y. 9 mo.
13. \$329.97 for 2 mo. 6 d.
14. \$5165.38 from Feb. 21, 1889, to March 14, 1889.
15. \$950.89 from Nov. 15, 1890, to July 26, 1892.

To find the interest at any other rate, by the six per cent method, multiply the interest at 6% by the fraction obtained by dividing the given rate by 6.

Thus, to find interest at $3\frac{3}{4}\%$, we multiply the interest at 6% by $\frac{3\frac{3}{4}}{6}$, or $\frac{5}{8}$.

In certain cases the work may be simplified; thus, for interest

At 4%, subtract from the 6% interest $\frac{1}{3}$ of itself.
At $4\frac{1}{2}\%$, subtract from the 6% interest $\frac{1}{4}$ of itself.
At 5%, subtract from the 6% interest $\frac{1}{6}$ of itself.
At 7%, add to the 6% interest $\frac{1}{6}$ of itself.
At 8%, add to the 6% interest $\frac{1}{3}$ of itself.

16. Find the interest of \$72.30 for 5 mo. 12 d. at 8%.

$$\begin{array}{r l}
\$0.0723 & = .001 \text{ of the prin.} \\
\underline{27} & = \tfrac{1}{6} \text{ the no. of days.} \\
5061 & \\
\underline{1\,446} & \\
3)\underline{\$1.9521} & = \text{int. at } 6\%. \\
\underline{.6507} & \\
\$2.6028 & = \text{int. at } 8\%, \textit{ Ans.}
\end{array}$$

After finding the interest at 6% to be \$1.9521, we add to this sum $\frac{1}{3}$ of itself, giving \$2.6028.

Find the interest of:

17. \$ 695 for 4 y. 8 mo. at 5%.

18. \$ 2926 for 24 d. at 7%.

19. \$ 807.10 for 17 d. at $3\frac{1}{4}$%.

20. \$ 948 from July 14, 1887, to May 21, 1888, at 3%.

21. \$ 472 from Nov. 22, 1890, to Sept. 9, 1892, at $4\frac{1}{2}$%.

22. \$ 565.35 from Jan. 26, 1893, to March 12, 1893, at $2\frac{1}{2}$%.

Find the amount of:

23. \$ 113.65 for 6 y. 3 mo. at 4%.

24. \$ 156.30 for 53 d. at $3\frac{1}{2}$%.

25. \$ 8789 from April 4, 1891, to Nov. 16, 1892, at $2\frac{3}{4}$%.

26. \$ 325.50 from Dec. 23, 1889, to Feb. 3, 1890, at 8%.

309. Interest for Aliquot Parts of Two Hundred Months.

Since a sum of money, at 6% interest, gains .01 of itself in 2 months, it will gain the whole of itself in 200 months.

Then the interest at 6% of any sum of money may be found by *taking that fractional part of the principal which the given time is of 200 months.*

The following table gives, for each of the stated times, the corresponding fractional part of the principal:

100 mo.	$\frac{1}{2}$	25 mo.	$\frac{1}{8}$	$8\frac{1}{3}$ mo.	$\frac{1}{24}$	2 mo.	$\frac{1}{100}$
$66\frac{2}{3}$ mo.	$\frac{1}{3}$	20 mo.	$\frac{1}{10}$	8 mo.	$\frac{1}{25}$	1 mo.	$\frac{1}{200}$
50 mo.	$\frac{1}{4}$	$16\frac{2}{3}$ mo.	$\frac{1}{12}$	5 mo.	$\frac{1}{40}$	15 d.	$\frac{1}{400}$
40 mo.	$\frac{1}{5}$	$12\frac{1}{2}$ mo.	$\frac{1}{16}$	4 mo.	$\frac{1}{50}$	12 d.	$\frac{1}{500}$
$33\frac{1}{3}$ mo.	$\frac{1}{6}$	10 mo.	$\frac{1}{20}$	$3\frac{1}{3}$ mo.	$\frac{1}{60}$	6 d.	$\frac{1}{1000}$

EXAMPLES.

1. Find the interest of \$ 436.25 for 1 y. 4 mo. 20 d. at 6 %.

1 y. 4 mo. 20 d. is $16\frac{2}{3}$ mo.

Then the interest is $\frac{1}{12}$ of \$ 436.25, or \$ 36.35, *Ans.*

Find the interest of:

2. $2763.53 for 5 y. 6 mo. 20 d. at 6%.
3. $503.76 for 15 d. at 6%.
4. $310.09 from May 22, 1887, to June 6, 1888, at 6%.
5. $417.27 for 3 y. 4 mo. at $4\frac{1}{2}$%.
6. $378.95 for 8 mo. 10 d. at 4%.
7. $821.34 from Aug. 28, 1892, to Sept. 9, 1892, at 7%.

Find the amount of:

8. $5946.41 for 2 y. 1 mo. at 6%.
9. $247.18 from Dec. 15, 1890, to March 25, 1891, at 5%.
10. $192.63 for 8 mo. at $3\frac{3}{4}$%.
11. $795.80 from Feb. 9, 1887, to Nov. 19, 1889, at $2\frac{1}{4}$%.

310. **Given the Interest, Time, and Principal or Amount, to find the Rate.**

1. At what rate must $384 be on interest to yield $41.76 in 2 y. 5 mo.?

$$\begin{array}{r} \$3.84 \\ \underline{14\frac{1}{2}} \\ 15\ 36 \\ 38\ 4 \\ \underline{1\ 92} \\ 6\overline{)55.68} \\ \$9.28 = \text{int. at } 1\%. \end{array} \qquad \begin{array}{r} \$9.28)\$41.76(4\frac{1}{2} \\ \underline{37\ 12} \\ \underline{4\ 64} \\ 9\ 28 \end{array} = \frac{1}{2}$$

We find by the six per cent method that the interest on $384 for 2 y. 5 mo., at *one per cent*, is $9.28.

Dividing the given interest, $41.76, by this result, the quotient is $4\frac{1}{2}$.

Then the required rate is $4\frac{1}{2}$%, *Ans.*

Note. If the interest, time, and *amount* are given, the principal may be found by subtracting the interest from the amount.

EXAMPLES.

At what rate per cent will

2. $480 gain $47 in 3 y. 11 mo.?
3. $51.20 amount to $52.64 in 9 mo.?

4. \$6480 gain \$72.36 in 2 mo. 7 d.?

5. \$3780 gain \$5.88 in 28 d.?

6. \$744 amount to \$764.15 in 10 mo.?

7. \$216 gain \$25.74 in 4 y. 4 mo.?

8. \$8064 gain \$415.52 in 8 mo. 25 d.?

9. \$720 amount to \$727.35 in 7 mo.?

10. \$192 amount to \$202.44 in 2 y. 5 mo.?

11. \$4320 gain \$110.88 in 7 mo. 21 d.?

12. \$960 gain \$160.56 from April 2, 1883, to Oct. 29, 1888?

13. \$384 amount to \$388.76 from March 7, 1892, to June 22, 1892?

14. \$1200 amount to \$1215.05 from July 25, 1891, to Dec. 4, 1891?

15. \$4455 gain \$147.51 from Aug. 12, 1889, to Jan. 10, 1890?

16. At what rate will a sum of money double itself in 13 y. 4 mo.?

17. At what rate will a sum of money double itself in 33 y. 4 mo.?

311. Given the Principal or Amount, Interest, and Rate, to find the Time.

1. How long must \$94 be on interest at 6% to gain \$19.27?

$$\begin{array}{l} \$\,94 \\ \underline{\;\;.06} \\ \$\,5.64)\,\$\,19.27\,(3\tfrac{5}{12}\text{ y.} \\ \qquad\quad\;\underline{16\,92} \\ \qquad\quad\;\;\frac{2\,35}{5\,64} = \frac{5}{12} \end{array}$$

3 y. 5 mo., *Ans.*

The interest of \$94 for one year at 6% is \$5.64.

Then to gain \$19.27, it will take as many years as \$5.64 is contained times in \$19.27.

2. How long must \$157 be on interest at $4\frac{1}{2}\%$ to amount to \$162.20?

```
 $157            $162.20         .736 y.
   .04½           157.             12
 ------          -------         --------
   628            $5.20          8.832 mo.
   785                              30
 ------                          --------
$7.065)$5.2000(.736 y.            24.96 d.
        4 9455
        ------
         25450       8 mo. 25 d., Ans.
         21195
        ------
          42550
```

The interest of \$157 for 1 y. at $4\frac{1}{2}\%$ is \$7.065.

Subtracting \$157 from \$162.20, the given interest is \$5.20.

Dividing \$5.20 by \$7.065, the quotient to the third place of decimals is .736 y., or 8 mo. 25 d.

Note. The quotient need never be carried beyond the third place of decimals.

To reduce .736 y. to months and days, we multiply it by 12, giving 8.832 mo., and then multiply .832 by 30, giving 24.96 d.

The final result should be expressed to the *nearest day;* the number of days being increased by 1 if the decimal is .5 or more than .5.

EXAMPLES.

Find the time in which

3. \$46 will gain \$11.73 at 6%.
4. \$250 will gain \$42.50 at 6%.
5. \$348 will amount to \$356.70 at 5%.
6. \$637.50 will amount to \$794.75 at 8%.
7. \$936 will gain \$27.30 at 6%.
8. \$147.50 will gain \$2.85 at 2%.
9. \$353.60 will amount to \$398.68 at $7\frac{1}{2}\%$.
10. \$741 will gain \$22.60 at 6%.
11. \$258.25 will amount to \$270.13 at 6%.
12. \$801.50 will amount to \$811.40 at $3\frac{1}{2}\%$.
13. \$716.84 will gain \$8.48 at 6%.

14. \$95.30 will amount to \$97.84 at 4%.

15. \$1816 will gain \$6.36 at 6%.

16. \$809.20 will gain \$30.24 at $4\frac{1}{2}$%.

17. \$264.08 will amount to \$349.91 at 6%.

18. \$56.92 will gain \$0.79 at $2\frac{1}{4}$%.

19. In what time will a sum of money double itself at 6% interest?

20. In what time will a sum of money double itself at $4\frac{1}{2}$% interest?

312. Given the Interest or Amount, Time, and Rate, to find the Principal.

1. What principal at 6% interest will gain \$6.72 in 3 y. 6 mo.?

$$\begin{array}{l} \$0.01 \\ \underline{\quad 21} \\ \$0.21)\$6.72(\$32, \textit{Ans.} \\ \qquad\quad \underline{6\,3} \\ \qquad\quad\; 42 \end{array}$$

We find the interest of \$1 for 3 y. 6 mo. at 6% by multiplying .01 of \$1 by $\frac{1}{2}$ the number of months.

Then, if one dollar gains \$0.21, to gain \$6.72 will take as many dollars as \$0.21 is contained times in \$6.72.

Dividing \$6.72 by \$0.21, the quotient is 32.

2. What principal at 5% interest will amount to \$150 in 10 mo. 18 d.?

$$\begin{array}{l} \$0.001 \\ \underline{\qquad 53} \\ \$0.053 \times \frac{5}{6} = \frac{\$0.265}{6}. \end{array}$$

$$\$1 + \frac{\$0.265}{6} = \frac{\$6.265}{6}$$

= amount of \$1 at 5%

$$\begin{array}{r} \$150 \\ \underline{6} \\ \$6.265)\$900(\$143.66, \textit{Ans.} \\ \underline{6265} \\ 27350 \\ \underline{25060} \\ 22900 \\ \underline{18795} \\ 41050 \\ \underline{37590} \\ 34600 \end{array}$$

We first find the interest of $1 for 10 mo. 18 d. at 6% by multiplying .001 of $1 by $\frac{1}{6}$ the number of days.

Multiplying the result by $\frac{5}{6}$, we find the interest at 5% to be $\frac{\$0.265}{6}$; and the *amount* of $1 for the given time and rate is $\$1 + \frac{\$0.265}{6}$, or $\frac{\$6.265}{6}$.

Then, if $1 amounts to $\frac{\$6.265}{6}$, to amount to $150 will require as many dollars as $\frac{\$6.265}{6}$ is contained times in $150.

To divide $150 by $\frac{\$6.265}{6}$, we multiply $150 by 6, and divide the product by $6.265; the result to the nearest cent is $143.66.

EXAMPLES.

What principal at interest

3. At 6% will gain $68.20 in 3 y. 8 mo.?
4. At 8% will gain $7.91 in 6 mo.?
5. At 6% will amount to $251.86 in 4 y. 9 mo.?
6. At 6% will gain $7.43 in 5 mo.?
7. At 4% will amount to $215.75 in 5 y. 2 mo.?
8. At 6% will gain $44.03 in 1 y. 1 mo. 12 d.?
9. At 7% will amount to $99.94 in 2 y. 3 mo.?
10. At 6% will gain $54.98 in 11 mo. 18 d.?
11. At 5% will amount to $77 in 9 mo. 6 d.?
12. At 6% will amount to $307.31 in 36 d.?
13. At 6% will gain $10.59 in 6 mo. 15 d.?
14. At 6% will amount to $252.04 in 27 d.?
15. At 2% will gain $1.60 in 84 d.?
16. At 4½% will gain $1.65 in 10 mo. 24 d.?
17. At 6% will amount to $594.36 in 7 mo. 11 d.?
18. At 1½% will gain $11.70 in 2 mo. 7 d.?
19. At 2½% will gain $5.61 in 4 mo. 17 d.?
20. At 3½% will amount to $138.92 in 7 mo. 5 d.?

EXACT INTEREST.

313. **Exact Interest** is interest calculated for parts of a year by considering the *actual number of days in the given time.*

It is used by the United States government on its securities, and in some business transactions.

314. In computing exact interest, the year is taken as 365 days instead of 360 days as in the common method.

Let it be required, for example, to find the interest of a sum of money for 43 days.

By the common method, the interest is $\frac{43}{360}$, and by the exact method $\frac{43}{365}$, of the interest for one year.

Now $\frac{43}{365} = \frac{43}{360} \times \frac{360}{365} = \frac{43}{360} \times \frac{72}{73}$.

That is, the exact interest is $\frac{72}{73}$ of the common interest.

We then have the following rule for computing exact interest *for parts of a year:*

Subtract from the common interest $\frac{1}{73}$ of itself.

Note 1. It must be clearly understood that this rule *does not apply to times greater than one year;* thus, to find exact interest for 2 y. 3 mo. 18 d., we should find the *common* interest for 2 y., and then the *exact* interest for 3 mo. 18 d. by the above rule, and add the results.

EXAMPLES.

1. Find the exact interest of $2500 for 2 y. 3 mo. 18 d. at 6%.

$2500	$2.50	$45.00
.12	18	.62
$300 =	20 00	44.38
int. for 2 y.	25 0	300.
	73)$45.00(.62	$344.38, *Ans.*
	43 8	
	1 20	

We find the common interest of $2500 for 2 y. by multiplying the principal by .12.

We then find the common interest of $2500 for 3 mo. 18 d. by multiplying .001 of the principal by ⅙ the number of days; the result is $45.00.

Dividing this by 73, the quotient to the nearest cent is \$0.62.

Subtracting \$0.62 from \$45.00, the exact interest for 3 mo. 18 d. is \$44.38; and adding to this the interest for 2 y., \$300, the final result is \$344.38.

Find the exact interest

2. Of \$942.50 for 3 y. 4 mo. at 6%.

3. Of \$248.25 for 2 y. 9 mo. at 4%.

4. Of \$539.84 for 10 mo. at 7%.

5. Of \$7162 for 7 mo. at $2\frac{1}{2}$%.

6. Of \$1930 from May 5, 1888, to Nov. 21, 1888, at 5%.

7. Of \$804 from Oct. 29, 1889, to July 2, 1890, at $4\frac{1}{2}$%.

8. Of \$665.40 from April 9, 1892, to March 22, 1893, at 6%.

9. Of \$3515 from Aug. 16, 1891, to Dec. 8, 1891, at $3\frac{1}{2}$%.

Note 2. To compute the times in Exs. 6 to 9, the exact number of days between the given dates must be found. See Art. 165.

PROMISSORY NOTES.

315. A **Promissory Note**, or simply a **Note**, is a written promise to pay to a specified person a specified sum of money.

316. A **Demand Note** is one which is due on demand.

If the words "with interest" are added, it draws interest from the date of the note.

FORM OF A DEMAND NOTE.

\$$215\frac{83}{100}$. *Boston, Feb.* 18, 1892.

On demand, I promise to pay to Edward Williams two hundred and fifteen $\frac{83}{100}$ dollars, with interest at 6%.

Value received.

Henry Davis.

Note. A note must always contain the words "value received."

317. A **Time Note** is one which is due at the expiration of a specified time after the date of the note.

FORM OF A TIME NOTE.

$ $500\frac{00}{100}$. *New York, July* 1, 1889.

Three months after date, we promise to pay to Martin Pratt, or order, five hundred dollars.

Value received. *A. F. Stearns & Co.*

318. The **Maker** of the note is the person who signs it.

The **Payee** is the person to whom it is payable.

The **Holder** of a note is the person who owns it.

An **Indorser** is a person who writes his name upon the back of a note, thereby becoming responsible for its payment in case the maker fails to pay it when due.

The **Face** of a note is the sum named in it.

319. If the words "or order" or "or bearer" are written after the payee's name, the note may be sold or transferred; otherwise, only the payee can collect the amount due.

A note that can be sold or transferred is called **Negotiable**.

Note. The law of Pennsylvania requires the addition of the words "without defalcation" to "value received," in a negotiable note; in New Jersey, the words "without defalcation or discount" must be added.

320. A time note **Matures**, or becomes legally payable, three days after the expiration of the time specified in the note.

These three days are called **Days of Grace**.

The date of maturity of a note is usually indicated by writing the date when nominally due and the date of maturity with a line between them; thus, Aug. $^{8}/_{11}$, 1892.

If a note is payable in a certain number of *days* after date, the date of maturity is found by counting forward from the date of the note the specified number of days, and adding three days of grace.

Thus, if a note is payable 60 days after Aug. 13, the date of maturity is Oct. $^{12}/_{15}$.

If a note is payable in a certain number of *months* after date, calendar months are understood; and the note is nominally due on the corresponding day of the month, or on the last day of the month when there is no corresponding day.

Thus, if a note is payable 3 months after Dec. 31, 1892, the date of maturity is $^{\text{March } 31}/_{\text{April } 3}$, 1893; but if it is payable 2 months after Dec. 31, 1892, the date of maturity is $^{\text{Feb. } 28}/_{\text{March } 3}$, 1893, since February has no 31st day.

If a note falls due on Sunday or on a legal holiday, it is payable on the next preceding business day.

If a note is not paid when due, a written notice, called a **Protest**, is sent by a notary public to the indorsers.

If not sent on the last day of grace, the liability of the indorsers ceases.

PARTIAL PAYMENTS.

321. **Partial Payments** are payments in part of a note or debt.

Indorsements are records of the partial payments, with their dates, made on the back of the note.

322. When partial payments have been made on an *interest-bearing* note or other obligation, various methods are employed by business men to determine the amount due at the final settlement.

323. **The Merchants' Rule.**

If the final settlement is made *within a year from the date of the note*, it is usual to employ the following, known as

THE MERCHANTS' RULE.

Find the amount of the face of the note from its date to the date of settlement; also, the amount of each partial payment from its date to the date of settlement.

Subtract the sum of the amounts of the partial payments from the amount of the face of the note.

1. What amount is due Dec. 8, 1892, on a note for $850, dated Jan. 2, 1892, and bearing interest at 6%, on which the following indorsements have been made: March 18, 1892, $200; May 5, 1892, $150; Aug. 22, 1892, $300?

Amount of $850 for	11 mo. 6 d.		$897.60
" " $200 "	8 mo. 20 d.	$208.67	
" " $150 "	7 mo. 3 d.	155.33	
" " $300 "	3 mo. 16 d.	305.30	669.30
			$228.30, *Ans.*

The amount of the face of the note from Jan. 2 to Dec. 8, or 11 mo. 6 d., at 6%, is $897.60.

The amount of the first partial payment from March 18 to Dec. 8, or 8 mo. 20 d., is $208.67.

The amount of the second partial payment from May 5 to Dec. 8, or 7 mo. 3 d., is $155.33.

The amount of the third partial payment from Aug. 22 to Dec. 8, or 3 mo. 16 d., is $305.30.

The sum of the amounts of the partial payments is $669.30; which, subtracted from the amount of the face of the note, leaves $228.30.

EXAMPLES.

2. What amount is due Sept. 20, 1891, on a note for $800, dated March 2, 1891, and bearing interest at 6%, on which the following indorsements have been made: April 30, 1891, $200; July 13, 1891, $300?

3. What amount is due Dec. 12, 1890, on a note for $960, dated Jan. 19, 1890, and bearing interest at 6%, on which the following indorsements have been made: March 27, 1890, $180; July 12, 1890, $320; Oct. 29, 1890, $250?

4. A note for $2900, dated Aug. 28, 1891, and bearing interest at $4\frac{1}{2}$%, had indorsements as follows: Nov. 15, 1891, $800; Jan. 16, 1892, $450; Feb. 27, 1892, $1100. How much was due May 3, 1892?

5. On a note for $780, given May 18, 1890, and drawing 5% interest, three payments were made: June 10, 1890, $330; Sept. 3, 1890, $290; Oct. 17, 1890, $80. What was due April 27, 1891?

6. On a note for $2000, dated Sept. 3, 1891, and bearing interest at 6%, the following indorsements were made: Nov. 28, 1891, $300; Jan. 20, 1892, $250; March 12, 1892, $725; May 18, 1892, $420. How much was due July 18, 1892?

7. A note for $698, dated Jan. 24, 1892, is indorsed as follows: Feb. 17, 1892, $115; Aug. 5, 1892, $82; Aug. 18, 1892, $129; Oct. 11, 1892, $213. At 6% interest, what is due Nov. 5, 1892?

8. On a note for $3150, dated Nov. 1, 1889, and bearing interest at 4%, the following payments were made: Dec. 27, 1889, $1080; May 15, 1890, $540; June 21, 1890, $310; Sept. 22, 1890, $770. How much was due Oct. 21, 1890?

324. The United States Rule.

If the final settlement is made *more than a year after the date of the note*, the amount due is found by the following rule, adopted by the Supreme Court of the United States, and known as

THE UNITED STATES RULE.

Find the amount of the principal to the time when the payment, or the sum of the payments, equals or exceeds the interest due.

From this amount subtract the payment, or the sum of the payments; regard the remainder as a new principal, and proceed as before to the time of settlement.

1. What amount is due Jan. 5, 1892, on a note for $750, dated Oct. 13, 1889, and bearing interest at 6%, on which the following indorsements have been made: Dec. 28, 1889, $325; Aug. 7, 1890, $10; July 1, 1891, $135?

Principal,	$750.00
Int. Oct. 13, 1889, to Dec. 28, 1889, 2 mo. 15 d.,	9.38
Amount,	$759.38
1st partial payment,	325.00
New Principal,	$434.38
Int. Dec. 28, 1889, to Aug. 7, 1890, 7 mo. 10 d.,	15.93
Int. Aug. 7, 1890, to July 1, 1891, 10 mo. 24 d.,	23.46
Amount,	$473.77
Sum of 2d and 3d partial payments,	145.00
New Principal,	$328.77
Int. July 1, 1891, to Jan. 5, 1892, 6 mo. 4 d.,	10.08
Amount due Jan. 5, 1892,	$338.85

Ans.

The amount of $750 from Oct. 13, 1889, to Dec. 28, 1889, or 2 mo. 15 d., at 6%, is $759.38.

Subtracting from this the 1st partial payment, the new principal is $434.38.

The interest of $434.38 from Dec. 28, 1889, to Aug. 7, 1890, or 7 mo. 10 d., is $15.93, *which is greater than the 2d partial payment;* we then compute interest on the principal $434.38 to the date of the next partial payment.

The interest of $434.38 from Aug. 7, 1890, to July 1, 1891, or 10 mo. 24 d., is $23.46; adding $434.38, $15.93, and $23.46, the amount is $473.77.

Subtracting from this the sum of the 2d and 3d partial payments, or $145, the new principal is $328.77.

The amount of $328.77 from July 1, 1891, to Jan. 5, 1892, or 6 mo. 4 d., is $338.85.

EXAMPLES.

2. What amount is due July 7, 1892, on a note for $395, dated Jan. 27, 1890, and bearing interest at 6%, on which the following indorsements have been made: March 5, 1891, $125; Oct. 25, 1891, $10.

3. On Nov. 9, 1889, I gave my note, on demand, for $630, with interest at 5%. On Aug. 19, 1890, I paid $20; and on Nov. 6, 1890, $400. What was due Sept. 6, 1891?

4. On a note for $1500, dated Feb. 22, 1887, and drawing interest at 6%, there was paid, Feb. 27, 1888, $250; July 4, 1889, $55; and Feb. 4, 1890, $740. What was due May 4, 1891?

5. A note for $2120, dated Aug. 17, 1891, and drawing 6% interest, is indorsed as follows: April 29, 1892, $75; July 22, 1892, $880; Nov. 22, 1892, $545. What is due Dec. 13, 1892?

6. A note for $540, dated April 6, 1886, and bearing interest at 4%, had indorsements as follows: April 30, 1886, $100; Nov. 20, 1886, $220; Aug. 23, 1888, $15. How much was due Nov. 19, 1889?

7. On a note for $806, given May 15, 1888, and bearing interest at 6%, four payments were made: Feb. 3, 1889, $30; July 28, 1889, $290; June 24, 1890, $24; Oct. 3, 1890, $156. What was due Feb. 27, 1892?

8. A note for $480, dated March 6, 1885, is indorsed as follows: Aug. 29, 1885, $15; Sept. 22, 1885, $170; Sept. 12, 1886, $20. At 7% interest, how much is due April 14, 1887?

9. On a note for $925, dated Dec. 2, 1887, and bearing 6% interest, the following payments were made: June 17, 1888, $110; Sept. 23, 1889, $30; Nov. 1, 1890, $25; Oct. 17, 1891, $460; and March 27, 1892, $175. How much was due Aug. 21, 1892?

COMPOUND INTEREST.

325. Compound Interest is interest reckoned both on the principal, and on the unpaid interest after it becomes due, which is added to the principal at regular intervals.

The unpaid interest may be added to the principal, or *compounded*, at the end of each year, half-year, quarter, or any other period of time, according to agreement.

326. 1. What will be the compound interest and amount of $ 500 for 2 y. 7 mo. 12 d., at 6%, the interest being compounded annually?

1st Principal,	$ 500.00
Int. for 1 y.,	30.00
2d Principal,	$ 530.00
Int. for 1 y.,	31.80
3d Principal,	$ 561.80
Int. for 7 mo. 12 d.,	20.79
Compound amt.,	$ 582.59
Given principal,	500.00
Compound int.,	$ 82.59, *Ans.*

The interest of $ 500 for 1 year is $ 30; which, added to $ 500, gives $ 530 as the principal for the 2d year.

The interest of $ 530 for 1 year is $ 31.80; which, added to $ 530, gives $ 561.80 as the principal for the 7 mo. 12 d.

The interest of $ 561.80 for 7 mo. 12 d. is $ 20.79; which, added to $ 561.80, gives $ 582.59 as the *compound amount* of $ 500 for 2 y. 7 mo. 12 d.

Subtracting the given principal from this, the compound interest is $ 82.59.

From the above example, we derive the following

RULE FOR COMPOUND INTEREST.

Find the amount of the given principal for the first period of time.

Using this amount as a new principal, find its amount for the second period of time; continuing in this way until the entire time has been taken.

The last amount, less the given principal, will be the compound interest required.

Note. If the interest is compounded semi-annually the rate must be taken as one-half of the yearly rate; and if compounded quarterly, the rate must be taken as one-fourth of the yearly rate.

The interest is taken as compounded annually, in the following examples, if nothing is said to the contrary.

EXAMPLES.

2. What is the compound interest of $ 820 for 3 y., at 6%?

3. What is the amount of $200 for 4 y. 5 mo., at 6% compound interest?

4. What is the compound interest of $575 for 5 y., at 2%?

5. What is the compound interest of $1050 for 2 y. 6 mo., at 6%, interest being compounded semi-annually?

6. What is the amount of $436 for 1 y. 9 mo. 24 d., at 3½% compound interest?

7. What is the compound interest of $3960 for 1 y. 6 mo., at 3%, interest being compounded semi-annually?

8. What is the amount of $758 for 11 mo. 10 d., at 6%, interest being compounded quarterly?

9. What is the compound interest of $6000 for 1 y. 8 mo., at 6%, interest being compounded every 4 months?

10. What is the compound interest of $940 for 3 y. 2 mo. 20 d., at 4½%?

11. What is the amount of $1700 for 9 mo., at 7%, interest being compounded quarterly?

327. Given the Compound Interest or Amount, Time, and Rate, to find the Principal.

1. What sum of money, at 3% compound interest, will amount to $500 in 2 y. 8 mo., interest being compounded annually?

2d principal,	$1.03	$1.082118)$500.0000($462.06,
Int. for 1 y.,	.0309	432 8472 *Ans.*
3d principal,	$1.0609	67 15280
Int. for 8 mo.,	.021218	64 92708
Compound Amt.,	$1.082118	2 225720
		2 164236
		6148400

The amount of $1 for 2 y. 8 mo., at 3% compound interest, is $1.082118.

Then, if $1 amounts to $1.082118, to amount to $500 will require as many dollars as $1.082118 is contained times in $500; the result to the nearest cent is $462.06.

Note. If the compound *interest* is given, divide the given interest by the compound *interest* of $1 for the given time and rate.

EXAMPLES.

2. What sum of money, at 6% compound interest, will produce $70 in 2 years?

3. What sum of money, at 4% interest, will amount to $1800 in 1 y. 6 mo., the interest being compounded semi-annually?

4. What principal, at 6% compound interest, will gain $105 in 3 y. 9 mo.?

5. What sum of money, at 5% compound interest, will gain $125.38 in 3 y. 8 mo.?

6. What principal, at 6% compound interest, will gain $36 in 9 months, interest being compounded quarterly?

7. What principal, at 6% compound interest, will amount to $725 in 8 mo., interest being compounded every 2 months?

8. What sum of money, at 6% compound interest, will gain $80 in 1 y. 8 mo., interest being compounded semi-annually?

328. Compound Interest Tables.

In practice, the solution of problems in compound interest may be abridged by means of the following tables, which give the amount of $1 at compound interest, for any number of years from 1 to 20 inclusive, at 1½, 2, 2½, 3, 3½, 4, 5, 6, 7, 8, 9, and 10 per cent; interest being compounded annually.

COMPOUND INTEREST TABLES.

Yrs.	1½ per cent.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.
1	1.015000	1.020000	1.025000	1.030000	1.035000	1.040000
2	1.030225	1.040400	1.050625	1.060900	1.071225	1.081600
3	1.045678	1.061208	1.076891	1.092727	1.108718	1.124864
4	1.061364	1.082432	1.103813	1.125509	1.147523	1.169859
5	1.077284	1.104081	1.131408	1.159274	1.187686	1.216653
6	1.093443	1.126162	1.159693	1.194052	1.229255	1.265319
7	1.109845	1.148686	1.188686	1.229874	1.272279	1.315932
8	1.126493	1.171660	1.218403	1.266770	1.316809	1.368569
9	1.143390	1.195093	1.248863	1.304773	1.362897	1.423312
10	1.160541	1.218994	1.280085	1.343916	1.410599	1.480244
11	1.177949	1.243374	1.312087	1.384234	1.459970	1.539454
12	1.195618	1.268242	1.344889	1.425761	1.511069	1.601032
13	1.213552	1.293607	1.378511	1.468534	1.563956	1.665074
14	1.231756	1.319479	1.412974	1.512590	1.618695	1.731676
15	1.250232	1.345868	1.448298	1.557967	1.675349	1.800944
16	1.268985	1.372786	1.484506	1.604706	1.733986	1.872981
17	1.288020	1.400241	1.521618	1.652848	1.794676	1.947901
18	1.307341	1.428246	1.559659	1.702433	1.857489	2.025817
19	1.326951	1.456811	1.598650	1.753506	1.922501	2.106849
20	1.346855	1.485947	1.638616	1.806111	1.989789	2.191123

Yrs.	5 per cent.	6 per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	1.050000	1.060000	1.070000	1.080000	1.090000	1.100000
2	1.102500	1.123600	1.144900	1.166400	1.188100	1.210000
3	1.157625	1.191016	1.225043	1.259712	1.295029	1.331000
4	1.215506	1.262477	1.310796	1.360489	1.411582	1.464100
5	1.276282	1.338226	1.402552	1.469328	1.538624	1.610510
6	1.340096	1.418519	1.500730	1.586874	1.677100	1.771561
7	1.407100	1.503630	1.605781	1.713824	1.828039	1.948717
8	1.477455	1.593848	1.718186	1.850930	1.992563	2.143589
9	1.551328	1.689479	1.838459	1.999005	2.171893	2.357948
10	1.628895	1.790848	1.967151	2.158925	2.367364	2.593742
11	1.710339	1.898299	2.104852	2.331639	2.580426	2.853117
12	1.795856	2.012197	2.252192	2.518170	2.812665	3.138428
13	1.885649	2.132928	2.409845	2.719624	3.065805	3.452271
14	1.979932	2.260904	2.578534	2.937194	3.341727	3.797498
15	2.078928	2.396558	2.759031	3.172169	3.642482	4.177248
16	2.182875	2.540352	2.952164	3.425943	3.970306	4.594973
17	2.292018	2.692773	3.158815	3.700018	4.327633	5.054470
18	2.406619	2.854339	3.379932	3.996019	4.717120	5.559917
19	2.526950	3.025600	3.616527	4.315701	5.141661	6.115909
20	2.653298	3.207136	3.869684	4.660957	5.604411	6.727500

1. What is the compound interest of \$400 for 15 y. 6 mo., at 6%, interest being compounded annually?

Amount of \$1 for 15 y. at 6%,	\$2.396558
	400
Amount of \$400 for 15 y.,	\$958.6232
Interest of \$958.6232 for 6 mo.,	28.759
Compound Amount,	\$987.38
	400.00
Compound Interest,	\$587.38, *Ans.*

Note 1. If the given time extends beyond the limits of the table, find the amount for any convenient length of time, and then use this amount for a new principal.

Note 2. If the interest is compounded *semi-annually*, take *one-half* the given rate, and *twice* the given time.

Thus, to find the compound interest of any sum of money for 8 y. at 6%, interest being compounded semi-annually, we find by the table the compound interest of the sum for 16 y. at 3%.

If the interest is compounded *quarterly*, take *one-fourth* the given rate, and *four times* the given time.

EXAMPLES.

2. What is the amount of \$95 for 5 y. 10 mo., at 6%, interest being compounded annually?

3. What is the amount of \$1250 for 3 y. 9 mo., at 6%, interest being compounded quarterly?

4. What is the compound interest of \$800 for 8 y. 7 mo., at 5%, interest being compounded semi-annually?

5. What is the compound interest of \$480 for 11 y. 5 mo. 10 d., at 4%, interest being compounded annually?

6. What principal, at 5% compound interest, will amount to \$500 in 13 y., interest being compounded annually?

7. What sum of money, at 6% compound interest, will gain \$230 in 9 y. 8 mo., interest being compounded semi-annually?

ANNUAL INTEREST.

329. If, when interest is payable annually on a note or other obligation, the payments are not made when due, the amount due at the time of settlement may in certain cases be found by *reckoning simple interest on the principal, and on each annual interest after it becomes due.*

This is called **Annual Interest.**

1. What amount is due at the end of 3 y. 6 mo. 12 d., on a note for $500, with 6% interest payable annually, on which no payments have been made?

Principal,	$500.00
Int. of $500 for 3 y. 6 mo. 12 d., at 6%,	106.00
Int. of $30 for 4 y. 7 mo. 6 d., at 6%,	8.28
Amount due,	$614.28, *Ans.*

The interest of the principal for 3 y. 6 mo. 12 d., is $106.

Each annual interest is $30; the first draws interest for 2 y. 6 mo. 12 d.; the second for 1 y. 6 mo. 12 d.; and the third for 6 mo. 12 d.

In all, this is equivalent to $30 drawing interest for 4 y. 7 mo. 6 d., which is $8.28.

Then the amount due is $614.28.

EXAMPLES.

2. What amount is due at the end of 2 y. 9 mo. 18 d., on a note for $900, with 6% interest payable annually, on which no payments have been made?

3 What amount is due June 27, 1888, on a note for $385, dated Feb. 5, 1885, with 6% interest payable annually, on which no payments have been made?

4. What is the annual interest of $763 for 2 y. 5 mo. 10 d., at 5%?

5. What interest is due on a note for $6450, with 4% interest payable annually, at the end of 3 y. 8 mo. 24 d.?

6. What amount is due Nov. 12, 1892, on a note for $898, dated Nov. 29, 1887, with 6% interest payable annually, on which no payments have been made?

XX. DISCOUNT.

TRUE DISCOUNT.

330. **Discount** is a reduction made from a debt that is paid before it becomes due.

The **Present Worth** of a sum of money due at some future date without interest, is that sum which, if put at interest for the given time, will amount to the given sum.

The **True Discount** is the difference between the given sum and its present worth; that is, it is the interest of the present worth for the given time.

EXAMPLES.

331. 1. What is the present worth and true discount of \$ 300, due 1 y. 5 mo. hence, at 5% ?

$$\begin{array}{r}\$1.00\\ .05\\ \hline\end{array}$$

$$\$0.05\times\frac{17}{12}=\frac{\$0.85}{12}.$$

$$\$1+\frac{\$0.85}{12}=\frac{\$12.85}{12}.$$

$$\begin{array}{rl}
 & \$300\\
 & \underline{12}\ \ \$300\\
\$12.85) & \$3600(\underline{\$280.16}, \text{ present worth.}\\
 & \underline{2570}\ \ \ \$19.84, \text{ true discount.}\\
 & 10300\\
 & \underline{10280}\\
 & 2000\\
 & \underline{1285}\\
 & 7150
\end{array}$$

This is the same as finding what principal will amount to \$ 300 in 1 y. 5 mo. at 5% (Art. 312).

We find the interest of \$1 for 1 y. 5 mo. at 5% by multiplying .05 of \$1 by $\frac{17}{12}$; the result is $\frac{\$0.85}{12}$.

Then the *amount* of \$1 for the given time and rate is

$\$1+\frac{\$0.85}{12}$, or $\frac{\$12.85}{12}$.

To divide \$300 by $\frac{\$12.85}{12}$, we multiply it by 12, and divide the product by \$ 12.85; the result to the nearest cent is \$280.16, which is the *present worth*.

Subtracting the present worth from the given sum, \$300, the *true discount* is \$ 19.84.

EXAMPLES.

Find the present worth and true discount of:

2. \$400 due 2 y. 11 mo. hence at 6%.
3. \$890 due 6 mo. hence at 5%.
4. \$725 due 4 y. 10 mo. hence at $4\frac{1}{2}$%.
5. \$1730 due 3 y. 4 mo. hence at $3\frac{1}{2}$%.
6. \$682 due 5 mo. 27 d. hence at 6%.
7. \$269.20 due 2 mo. 18 d. hence at 3%.
8. \$2500 due 3 mo. 6 d. hence at 7%.
9. \$950 due 9 mo. 11 d. hence at 6%.
10. \$135.75 due 1 y. 8 mo. 15 d. hence at 4%.
11. \$347.68 due 7 mo. 20 d. hence at $2\frac{1}{2}$%.

BANK DISCOUNT.

332. **Bank Discount** is a sum of money charged by a bank for the payment of a negotiable note (Art. 319) before it becomes due.

It is reckoned as *the simple interest of the face of the note from the day of discount to the day of maturity* (Art. 320).

The time from the day of discount to the day of maturity is called the **Term of Discount**, and the rate of interest is called the **Rate of Discount**.

The **Proceeds** or **Avails** of a discounted note is its face less the bank discount.

333. If a note is discounted on the day of its date, the term of discount is the time specified in the note, plus *three days of grace* (Art. 320).

If a note is due a certain number of *months* after date, the term of discount is found in *months and days* (Art. 165); if it is due a certain number of *days* after date, the term of discount is found in *exact days*.

Note. It will be understood, in the examples of the present chapter, that the note is discounted on the day of its date if nothing is said to the contrary.

EXAMPLES.

334. 1. Find the proceeds of a 4-months' note for $485, discounted on the day of its date at 6%.

Face of note,	$485.00
Int. 4 mo. 3 d., at 6%,	9.94
Proceeds,	$475.06, *Ans.*

The term of discount is 4 mo. 3 d. (Art. 333).

By Art. 332, the bank discount is the interest of $485 for 4 mo. 3 d., at 6%, which is $9.94.

Subtracting this from the face of the note, the proceeds is $475.06.

If an *interest-bearing* note is discounted, the discount is reckoned on the *amount due at maturity.*

2. Find the proceeds of a note for $1050, dated May 12, 1892, payable 90 days after date, and bearing interest at 4%, if discounted June 23, 1892, at 5%.

Face of note,	$1050.00
Interest of $1050 for 93 d., at 4%,	10.85
Amount due at maturity,	$1060.85
Interest of $1060.85 for 51 d., at 5%,	7.51
Proceeds,	$1053.34, *Ans.*

The day of maturity is 93 d. after May 12, 1892, or Aug. 13, 1892.

The interest of $1050 for 93 d., at 4%, is $10.85; whence the amount due at maturity is $1060.85.

The term of discount is the exact number of days from June 23, 1892, to Aug. 13, 1892, or 51 d.

Then the bank discount is the interest of $1060.85 for 51 d. at 5%; that is, $7.51.

Subtracting $7.51 from $1060.85, the proceeds is $1053.34.

Note. The term of discount in Ex. 2 may be found without obtaining the day of maturity, by counting the exact number of days from May 12, 1892, to June 23, 1892, and subtracting the result from 93 days.

3. Find the proceeds of a 3-months' note for $500, discounted at 6%.

4. What is the proceeds of a 60-day note for $950, discounted at 5%?

5. What is the bank discount on a note for $2000, due 90 days hence, at 7%?

6. Find the bank discount on a note for $620, payable in 4 months, and discounted 2 mo. 9 d. after date, at 6%.

7. How much money should a bank pay to the holder of a note for $1000, due in 30 days, if discounted at 4%?

8. What is the bank discount on a note for $700, dated July 13, 1891, and payable 90 days after date, if discounted Aug. 25, 1891, at $5\frac{1}{2}$%?

9. Find the proceeds of a note for $6000, payable 2 months after date, and bearing interest at 6%, if discounted at 5%.

10. A note for $425.30, dated Oct. 4, 1890, and payable 6 months after date, was discounted Jan. 24, 1891, at 6%. What was the proceeds?

11. What is the proceeds of a 60-day note for $800, dated March 16, 1892, and bearing interest at 4%, if discounted April 22, 1892, at $4\frac{1}{2}$%?

12. What charge will be made by a bank for discounting, at 4%, a note for $384.50, due 75 days hence?

13. A note for $275, payable 5 months after date, was discounted at $5\frac{1}{2}$%. What was the bank discount?

14. What sum will be realized by discounting a 30-day note for $3000; the rate of discount being 7%?

15. Find the bank discount on a note for $190, dated Aug. 26, 1890, payable 90 days after date, with interest at $4\frac{1}{2}$%, and discounted Nov. 3, 1890, at 6%.

16. A 15-day note for $503.70 was discounted at a bank at $4\frac{1}{2}$%; how much did the bank receive?

17. A note for $1100, dated Sept. 10, 1892, and payable 6 months after date, with interest at 6%, was discounted Feb. 7, 1893, at $4\frac{1}{4}$%. What was the proceeds?

18. Find the bank discount on a note for $9000, due in 4 months, the rate of discount being $6\frac{1}{2}\%$.

19. A 60-day note for $750, dated June 28, 1889, was discounted July 22, 1889, at $5\frac{3}{4}\%$. What was the bank discount?

20. Find the proceeds of a note for $280, dated May 21, 1888, payable 45 days after date, with interest at $3\frac{1}{4}\%$, and discounted June 7, 1888, at 6%.

21. A 3-months' note for $400, dated Nov. 19, 1892, was discounted Jan. 3, 1893, at $6\frac{1}{4}\%$. What sum was received by the holder?

22. How much should a bank receive for discounting a note for $847.25, payable 30 days after date, with interest at 5%; the rate of discount being $4\frac{3}{4}\%$?

335. To find the Face of a Note to yield a given Proceeds.

1. What must be the face of a note, due 90 days hence, which, when discounted at 4%, will yield $500?

$$\begin{array}{r} \$\,0.001 \\ 93 \\ \hline \end{array}$$

$$\frac{\$\,0.093}{6} = \frac{\$\,0.031}{2} \times \frac{2}{3} = \frac{\$\,0.031}{3} = \text{int. of } \$\,1 \text{ for } 93\,\text{d. at } 4\%.$$

$$\$1 - \frac{\$0.031}{3} = \frac{\$3 - \$0.031}{3} = \frac{\$2.969}{3} = \text{proceeds of a } \$1 \text{ note.}$$

$$\begin{array}{r} \$\,500 \\ .3 \\ \hline \$\,2.969)\$\,1500.00(\$\,505.22,\ \textit{Ans.} \\ 1484\,5 \\ \hline 15\,500 \\ 14\,845 \\ \hline 6550 \\ 5938 \\ \hline 6120 \end{array}$$

We first find the interest of $1 for 93 d. at 6 % by multiplying .001 of $1 by 93, and dividing the product by 6; the result is $\frac{\$0.031}{2}$.

Multiplying this by $\frac{2}{3}$, the interest of \$1 for 93 d. at 4% is $\frac{\$0.031}{3}$.

Subtracting this from \$1, the remainder is $\frac{\$2.969}{3}$, which is the *proceeds of a* \$1 *note for* 90 *days.*

Then to yield \$500, the face of the note must be as many dollars as $\frac{\$2.969}{3}$ is contained times in \$500.

To divide \$500 by $\frac{\$2.969}{3}$, we multiply \$500 by 3, and divide the product by \$2.969; the result to the nearest cent is \$505.22, which is the face of the note required.

EXAMPLES.

2. What must be the face of a note, due 3 months hence, which, when discounted at 6%, will yield \$600?

3. The proceeds of a 30-day note, discounted at 5%, was \$340. What was the face of the note?

4. Wishing to borrow \$225 at a bank, for what sum must my note be drawn at 60 days to obtain that amount, the rate of discount being 6%?

5. For what amount must a 5-months' note be drawn, so that, when discounted at 7%, the proceeds may be \$8000?

6. The holder of a 75-day note received \$550 as proceeds, when the note was discounted at 4%. What was the face of the note?

7. For what sum must a note, payable in 6 months, be drawn, to yield \$1500 when discounted at $4\frac{1}{2}$%?

8. If the rate of discount is 7%, what must be the face of a note, payable 2 months hence, to yield \$425 when discounted?

9. For what amount must my note, due in 90 days, be drawn, in order that I may receive \$908.70 when the note is discounted at $5\frac{1}{2}$%?

10. A merchant received a 60-day note in payment for goods sold; he at once had it discounted at $3\frac{3}{4}$%, and realized \$375. For what amount were the goods sold?

XXI. EXCHANGE.

336. A **Draft** is a written order from one person to another, directing him to pay a specified sum of money to a third person.

The **Drawer** of a draft is the person who signs it.

The **Drawee** is the person to whom it is addressed.

The **Payee** is the person to whom it is payable.

The **Face** of a draft is the sum named in it.

337. A **Sight Draft** is one which is payable on presentation to the drawee.

FORM OF A SIGHT DRAFT.

$ 600$\frac{00}{100}$. *Philadelphia, Feb.* 12, 1892.

At sight, pay to the order of Henry F. Sears six hundred dollars, value received, and charge the same to the account of

E. B. Hart & Co.

To Stone & Morison, Cleveland, Ohio.

The above draft may be supposed to have been drawn under the following circumstances:

A merchant in Philadelphia wishes to pay $ 600 to Henry F. Sears, in Cleveland.

He proceeds to a banking-firm, E. B. Hart & Co., who have an account with Stone & Morison, of Cleveland.

They sell him a draft for the above amount, which he forwards to Mr. Sears at Cleveland.

When Mr. Sears receives it, he carries it to Stone & Morison, who pay it on presentation.

338. A **Time Draft** is one which is payable at the expiration of some specified time after presentation, or after the date of the draft.

It is usual to allow *three days of grace.*

FORM OF A TIME DRAFT.

$ 845$\frac{00}{100}$. Chicago, Nov. 23, 1892.

Sixty days after date, pay to the order of Charles H. Jackson eight hundred and forty-five dollars, value received, and charge the same to the account of

William Rogers.

To the Erie National Bank,
Buffalo, N. Y.

339. The **Acceptance** of a time draft by the drawee is an agreement on his part to pay it.

To accept a draft, the drawee writes the word "Accepted" across its face, with the date, and his signature.

The draft is then called an *Acceptance*, and the drawee an *Acceptor*.

An acceptance may be negotiated in the same manner as a promissory note; if discounted at a bank, the *term of discount* is the time specified in the draft, plus three days of grace (Art. 333).

340. **Exchange** is the system of making payments by remitting drafts.

341. Exchange is said to be at a certain per cent **Premium,** or **Above Par,** when a draft sells for the specified per cent *more* than its face value.

Thus, if exchange is at 1% premium, a draft for $ 100 will sell for $100, plus 1% of $ 100, or $ 101.

Exchange is said to be at a certain per cent **Discount,** or **Below Par,** when a draft sells for the specified per cent *less* than its face value.

Thus, if exchange is at 1% discount, a draft for $ 100 will sell for $ 100, less 1% of $ 100, or $ 99.

The **Rate of Exchange** is the per cent which a draft costs more or less than its face value.

DOMESTIC EXCHANGE.

342. **Domestic** or **Inland Exchange** is exchange between persons in the *same country.*

EXAMPLES.

343. 1. Find the cost of a sight draft for \$452, when exchange is at $1\frac{1}{2}\%$ premium.

$$\begin{array}{r} \$\,452 \\ .01\frac{1}{2} \\ \hline 4\,52 \\ 2\,26 \\ \hline \$\,6.78 \end{array} \qquad \begin{array}{r} \$\,452.00 \\ 6.78 \\ \hline \$\,458.78, \textit{ Ans.} \end{array}$$

$1\frac{1}{2}\%$ of \$452 is \$6.78.

Then, since the draft sells for \$6.78 more than its face, the cost is \$458.78.

In finding the cost of a *time* draft, the *interest of the face of the draft for the specified time, plus three days of grace, must be deducted from the cost;* for the drawer has the use of the money for that length of time before the drawee pays the draft.

2. What will be the cost of a draft for \$1000, due 30 days after sight, with interest at 5%, exchange being at $\frac{3}{4}\%$ discount?

$$\$\,10.00 \times \frac{3}{4} = \frac{\$\,30.00}{4} = \$\,7.50.$$

$$\begin{array}{r} \$\,1.00 \\ 33 \\ \hline 6)\underline{\$\,33.00} \\ 6)\underline{\$\,5.50} \\ .92 \\ \hline \$\,4.58 \end{array} \qquad \begin{array}{r} \$\,1000.00 \\ 7.50 \\ \hline \$\,992.50 \\ 4.58 \\ \hline \$\,987.92, \textit{ Ans.} \end{array}$$

We find $\frac{3}{4}\%$ of \$1000 by multiplying .01 of \$1000 by $\frac{3}{4}$; the result is \$7.50.

Then, since exchange is at a discount, a *sight* draft for \$1000 will cost \$1000 – \$7.50, or \$992.50.

The interest of the face of the draft for 33 days, at 5%, is \$4.58.

Deducting this from the cost of the sight draft, the required cost is \$987.92.

3. Find the cost of a sight draft for \$500, when exchange is at $1\frac{1}{4}\%$ premium.

4. What will be the cost of a sight draft for \$ 280, if exchange is at $\frac{1}{2}\%$ discount?

5. What will be the cost of a draft for \$ 8000, due 60 days after sight, with interest at 4%, exchange being at $\frac{7}{8}\%$ discount?

6. What must be paid for a draft on New York for \$ 472, due 30 days after sight, with interest at 5%, if exchange is at 1% premium?

7. I wish to purchase a sight draft on San Francisco for \$1965. If exchange is at $1\frac{7}{8}\%$ discount, how much must I pay for it?

8. A merchant purchased a draft for \$ 700, due 90 days after sight, with interest at $4\frac{1}{2}\%$, exchange being at $\frac{1}{4}\%$ premium. What was the cost?

9. How much must be paid for a draft for \$ 344, due 30 days after sight, with interest at $3\frac{3}{4}\%$, exchange being at $\frac{5}{8}\%$ premium?

10. What will be the cost of a draft for \$ 632, due 60 days after sight, with interest at $3\frac{1}{2}\%$, if exchange is at $3\frac{3}{8}\%$ discount?

11. How much must be paid for a draft on Cincinnati for \$ 1378, due 3 months after sight, with interest at $4\frac{1}{4}\%$, if exchange is at a discount of $2\frac{1}{8}\%$?

344. To find the Face of a Draft when the Cost is given.

1. What is the face of a sight draft which can be bought for \$ 248.85, when exchange is at $1\frac{1}{4}\%$ discount?

\$ 1.00	\$ 1.00	\$ .9875) \$ 248.85(\$ 252, *Ans.*
$.01\frac{1}{4}$	.0125	197 50
1 00	\$ .9875	51 350
25		49 375
\$ .0125		1 9750
		1 9750

$1\frac{1}{4}\%$ of \$1 is \$.0125.

Subtracting this from \$1, the cost of a sight draft for \$1 is \$.9875.

Then if the cost of the given draft is \$248.85, its face will be as many dollars as \$.9875 is contained times in \$248.85.

The result is \$252.00.

2. What will be the face of a draft, due 60 days after sight, with interest at 6%, which can be bought for \$1000, when exchange is at a premium of $\frac{5}{8}\%$?

$$\$0.01 \times \frac{5}{8} = \frac{\$0.05}{8} = \$0.00625$$

$$\begin{array}{r} \$0.00625 \\ 1. \\ \hline \$1.00625 \\ .0105 \\ \hline \$0.99575 \end{array}$$

$$\begin{array}{r} \$.001 \\ 63 \\ \hline 6)\$.063 \\ \hline \$.0105 \end{array}$$

$$\$0.99575)\$1000.00(\$1004.27,\ \textit{Ans.}$$

$$\begin{array}{r} 99575 \\ \hline 425000 \\ 398300 \\ \hline 267000 \\ 199150 \\ \hline 678500 \end{array}$$

We find $\frac{5}{8}\%$ of \$1 by multiplying .01 of \$1 by $\frac{5}{8}$; the result is \$.00625.

Then, since exchange is at a premium, a *sight* draft for \$1 will cost \$1 + \$.00625, or \$1.00625.

The interest of \$1 for 63 days at 6% is \$.0105.

Deducting this from \$1.00625, the cost of a draft for \$1, due 60 days after sight, with interest at 6%, will be \$0.99575.

Dividing the given cost by this, the result to the nearest cent is \$1004.27.

3. What is the face of a sight draft which can be bought for \$435.60, if exchange is at 1% discount?

4. A merchant bought a sight draft on Pittsburg for \$657.72, when exchange was at $1\frac{1}{4}\%$ premium. What was the face of the draft?

5. What will be the face of a draft, due 30 days after sight, with interest at 6%, which can be bought for $ 918, when exchange is at a premium of $\frac{1}{2}$% ?

6. A sight draft on Philadelphia was bought for $ 240, when exchange was at $\frac{3}{4}$% discount. What was the face of the draft?

7. Find the face of a draft on Detroit, due 60 days after sight, with interest at 4%, which can be bought for $ 760, exchange being at $1\frac{3}{4}$% premium.

8. I purchased a sight draft on St. Louis for $ 189.84, when exchange was at $1\frac{1}{8}$% discount. What was the face of the draft?

9. How large a draft, due 90 days after sight, with interest at 3%, can be bought for $ 575, when exchange is at a discount of $\frac{1}{8}$% ?

10. A merchant paid $ 3000 for a 60-day draft, with interest at 5%, exchange being at $1\frac{3}{8}$% premium. What was the face of the draft?

11. How large a draft on Baltimore, due one month after sight, with interest at $2\frac{1}{4}$%, can be purchased for $ 2380, when exchange is at a premium of $1\frac{5}{8}$% ?

FOREIGN EXCHANGE.

345. **Foreign Exchange** is exchange between persons in *different countries.*

346. A draft, in foreign exchange, is usually called a **Bill of Exchange.**

A **Set of Exchange** is a series of three bills, all of the same date and tenor, called the *First, Second,* and *Third* of exchange, respectively.

They are sent by different mails to avoid the delay which might arise from the loss of a single draft.

If any one of the three is paid, the others become void.

FORM OF A FOREIGN BILL OF EXCHANGE.

£200. *Boston, Dec. 3, 1892.*

At sight of this First of Exchange, second and third of the same date and tenor unpaid, pay to the order of George Lewis two hundred pounds sterling, value received, and charge the same to the account of

Kidder, Peabody, & Co.

To Messrs. Baring Brothers,
London, England.

347. Exchange on Great Britain or Ireland is quoted at the value of a pound sterling in United States dollars; exchange on France, at the value of a dollar in francs; exchange on Germany, at the value of 4 reichsmarks in cents.

Thus the statement

"Bankers' Sterling, sight, 4.86¼; Commercial bills, 60 days, 4.82½; Francs, sight, 5.16¼; Reichsmarks, sight, 95¾," would be interpreted as follows:

Sight drafts on a bank or banker in Great Britain or Ireland, $4.86¼ to the pound sterling; drafts drawn on merchants, due 60 days after sight, $4.82½ to the pound sterling; sight drafts on France, 5.16¼ francs to the dollar; sight drafts on Germany, 95¾ cents per 4 marks.

Sterling Exchange is exchange on Great Britain or Ireland.

EXAMPLES.

348. 1. What will be the cost of a bill of exchange on London for £160 8*s*., when exchange is quoted at 4.85?

$$
\begin{array}{r}
£160\ 8s. = £160.4 \\
4.85 \\
\hline
80\,20 \\
1283\,2 \\
6416 \\
\hline
\$777.94,\ \textit{Ans.}
\end{array}
$$

£160 8*s*. is the same as £160.4.

Since each pound sterling costs $4.85, 160.4 pounds will cost 160.4 times $4.85, or $777.94.

2. Find the cost of a bill of exchange on Paris for 632 francs, exchange being quoted at $5.16\frac{1}{4}$.

```
5.1625)632.00($122.42, Ans.
       516 25
       115 750
       103 250
        12 5000
        10 3250
         2 17500
         2 06500
          110000
```

Since 5.1625 francs can be bought for $1, to buy 632 francs will take as many dollars as 5.1625 is contained times in 632.

The result to the nearest cent is $122.42.

3. How large a draft on Berlin can be bought for $100, if exchange on Germany is quoted at $95\frac{1}{2}$?

```
       $100
          4
$.955)$400.00(418.85 marks, Ans.
       382 0
        18 00
         9 55
         8 450
         7 640
           8100
           7640
            4600
```

Since 4 marks cost $.955, as many marks can be bought for $100 as 955 is contained times in 4 times $100, or $400.

4. A merchant bought a bill of exchange on London for £348, when exchange was quoted at $4.86\frac{3}{4}$. What was the cost?

5. Find the cost of a draft on Berlin for 848 marks, exchange on Germany being quoted at $95\frac{1}{4}$.

6. How much must be paid for a bill of exchange on Paris for 3000 francs, exchange at 5.16?

7. How large a draft on London can be bought for $190.12, exchange at 4.85?

8. How large a draft on Paris can be bought for $5786, exchange at $5.17\frac{1}{2}$?

9. Find the cost of a bill of exchange on Liverpool for £16 5*s.*, exchange at 4.85½.

10. If exchange on Germany is quoted at 94¾, how much must be paid for a draft on Bremen for 2175 marks?

11. If exchange on Paris is quoted at 5.18½, how much must be paid for a bill of exchange for 725 francs?

12. A merchant paid $7085.03 for a draft on Berlin, exchange on Germany at 96½. What was the face of the draft?

13. How much must be paid for a bill of exchange on Bristol for £523 17*s.*, exchange at 4.88¼?

14. What is the face of a bill of exchange on Glasgow costing $858, exchange at 4.87½?

15. If exchange on France is quoted at 5.19¼, how large a draft on Marseilles can be bought for $946.50?

16. Find the cost of a bill of exchange on Belfast for £95 12*s.* 6*d.*, exchange at 4.87¾.

17. How large a draft on London can be bought for $646.29½, exchange at 4.88?

18. If exchange on Germany is 95½, what will be the face of a draft on Hamburg costing $358.37?

19. A merchant imported 2650 yards of silk, invoiced at 7.20 francs a yard, and 1200 yards of woollens, invoiced at 6.70 francs a yard. Find the cost of a draft on Paris for the amount of the bill, exchange at 5.16¾.

XXII. EQUATION OF PAYMENTS.

349. **Equation of Payments** is the process of finding at what time several payments, due at different times, may all be paid at once, without injustice to either debtor or creditor.

The time thus found is called the **Equated Time.**

EXAMPLES.

350. 1. A owes B \$150, of which \$25 is due in 3 months, \$50 in 4 months, \$35 in 5 months, and \$40 in 7 months. What is the equated time of payment?

$$\begin{array}{rl} 25 \times 3 = & 75 \\ 50 \times 4 = & 200 \\ 35 \times 5 = & 175 \\ \underline{40} \times 7 = & \underline{280} \\ 150 &)730(4\tfrac{13}{15} \text{ mo.} = 4 \text{ mo. } 26 \text{ d., } \textit{Ans.} \\ & \underline{600} \\ & 130 \end{array}$$

A is entitled to 3 months' use of \$25, or 25×3 months' use of \$1;
" " " 4 " " " \$50, or 50×4 " " " \$1;
" " " 5 " " " \$35, or 35×5 " " " \$1;
" " " 7 " " " \$40, or 40×7 " " " \$1.

In all, A is entitled to 730 months' use of \$1.

Then he is entitled to the use of \$150 for as many months as 150 is contained times in 730, which is $4\frac{13}{15}$ months, or 4 mo. 26 d.

2. What is the average time of paying \$200 due May 4, \$300 due June 12, and \$400 due July 24?

$$\begin{array}{rl} 200 \times 0 = & 0 \\ 300 \times 39 = & 11700 \\ \underline{400} \times 81 = & \underline{32400} \\ 900 &)\underline{44100} \\ & 49 \text{ d.} \end{array}$$

May 4 + 49 d. = June 22, *Ans.*

We select the earliest date, May 4, as a convenient date from which to reckon times.

From May 4 to June 12 is 39 d., and from May 4 to July 24 is 81 d.

Proceeding as in Ex. 1, we find the equated time to be 49 d. after May 4, or June 22.

Note 1. The date from which times are reckoned is called the *Focal Date*.

Note 2. If there is a common term of credit, we may find the average time without regard to that term, and add it to the result.

Thus, if goods are bought on 60 days' credit as follows: July 5, $600; Aug. 15, $400; Sept. 10, $500; we find the equated time of payment without regard to the term of credit, and add 60 days to the result.

Note 3. If, in any result, the fraction of a day is $\frac{1}{2}$, or more than $\frac{1}{2}$, it is reckoned as one day; but if it is less than $\frac{1}{2}$, it is disregarded.

3. A owes $250 due in 9 months; if he pays $75 in 4 months, and $55 in 8 months, when should he pay the balance?

$$75 \times 5 = 375$$
$$55 \times 1 = \underline{\ \ 55}$$
$$120)430(3\tfrac{7}{12} \text{ mo.}$$
$$\underline{360}$$
$$70$$

3 mo. 18 d., *Ans.*

By paying $75 5 months before it is due, and $55 1 month before it is due, A loses the use of $1 for 75 × 5 + 55 × 1, or 430 months.

Hence, he is entitled to the use of the balance, $120, long enough after it becomes due to be equivalent to 430 months' use of $1.

Then he is entitled to the use of the balance for as many months after it becomes due as 120 is contained times in 430; that is, $3\frac{7}{12}$ months, or 3 mo. 18 d.

4. What is the average time of paying $55 due in 3 months, $170 due in 9 months, and $135 due in 7 months?

5. A owes B $300, of which $45 is due in 6 months, $85 in 8 months, $75 in 9 months, and $95 in 11 months. What is the equated time of payment?

6. Find the equated time of paying $420 due in 30 days, $720 due in 60 days, $120 due in 90 days, and $540 due in 120 days.

7. What is the equated time of paying $15 due March 15, $135 due April 6, and $90 due May 25?

8. Find the average time of paying $175 due Aug. 29, $340 due Sept. 23, $225 due Oct. 5, and $410 due Nov. 13.

9. On May 23, I bought a piece of land for $1300, on four months' credit. If I pay $625 on July 23, when should I pay the balance?

10. If goods are bought on three months' credit as follows: Sept. 19, 1889, $1150; Oct. 3, 1889, $925; Nov. 12, 1889, $775; what is the equated time of payment?

11. Four sixty-day notes bear date as follows: Jan. 4, 1891, $565; Feb. 27, 1891, $350; June 18, 1891, $495; July 30, 1891, $210. What is the average date of payment?

Note. A sixty-day note falls due 63 days after its date.

12. On Oct. 21, Henry Williams owed $157.25 due in 40 days, $223.75 due in 60 days, and $186 due in 90 days. What is the equated date of payment?

13. A bill of $1200 is due in 5 months from Jan. 13, 1892. If payments are made as follows: $259, March 20, 1892; $248, May 5, 1892; when is the balance due?

14. A tradesman owes $300 due in 5 months, and $750 due in 9 months; if at the end of 5 mo. 20 d. he pays $450, when should the balance be paid?

15. Hooker and Ingalls bought merchandise on 60 days' credit as follows: June 28, $72.30; Aug. 1, $156.75; Sept. 4, $95.10. What is the average date of payment?

16. A merchant owes $2350 due in 10 months. If he pays $400 in 2 months, $350 in 4 months, and $525 in 8 months, when should he pay the balance?

17. Four ninety-day notes bear date as follows: March 9, 1892, $388.05; May 24, 1892, $254.75; Aug. 13, 1892, $525; Oct. 30, 1892, $409.20. What is the average date of payment?

AVERAGE OF ACCOUNTS.

351. **Average of Accounts** is the process of finding at what time the balance of an account may be paid, without injustice to either debtor or creditor.

EXAMPLES.

352. 1. Find the equated time for paying the balance of the following account:

Dr. CHARLES STUART. *Cr.*

1892.			1892.		
May 28	To Mdse. 30 d.	$350	June 8	By Draft, 30 d.	$275
June 16	" "	125	July 15	" Cash,	250
July 7	" " 2 mo.	275	Aug. 21	" "	125

Solution.

June 27,	$350 \times 11 =$	3850	July 11,	$275 \times 25 =$	6875
June 16,	$125 \times 0 =$	0	July 15,	$250 \times 29 =$	7250
Sept. 7,	$275 \times 83 =$	22825	Aug. 21,	$125 \times 66 =$	8250
	$750	26675		$650	22375
	650	22375			
	$100 bal.	4300 bal.			

$4300 \div 100 = 43$ d.

June 16 + 43 d. = July 29, *Ans.*

The sum of the items on the debit side of the account is $750, and on the credit side $650.

Subtracting $650 from $750, there is a balance of $100 on the debit side of the account, showing the amount still due from Charles Stuart.

Payment for the merchandise sold May 28 is due 30 days after May 28, or June 27; and payment for that sold July 7 is due 2 months after July 7, or Sept. 7.

The 30-day draft dated June 8 is due 33 days after June 8, or July 11.

Thus, June 16 is the earliest date at which any item becomes due.

Using June 16 as the focal date, we find that the sum of the products on the debit side of the account is equivalent to the use of $1 for 26675 days.

Also, the sum of the products on the credit side is equivalent to the use of \$1 for 22375 days.

Subtracting 22375 from 26675, there is a balance on the debit side of the account, equivalent to the use of \$1 for 4300 days.

Now in order to make the sum of the products on the credit side equal to the sum of the products on the debit side, it is evident that the balance of \$100 must be paid at some date *after* the focal date.

That is, Mr. Stuart can settle the account equitably by paying \$100 as many days after June 16 as is equivalent to the use of \$1 for 4300 days.

But the use of \$1 for 4300 days is equivalent to the use of \$100 for as many days as 100 is contained times in 4300; that is, 43 days.

Hence, he can settle the account equitably by paying \$100 43 days after June 16; that is, on July 29.

If, in the above example, the sum of the products on the *credit* side of the account had been greater by 4300 than the sum of the products on the debit side, an *earlier focal date* could have been found, which would have made the sum of the products on the credit side equal to the sum of the products on the debit side.

If, for example, the focal date had been 43 days earlier than June 16, the sum of the products on the debit side would have been greater by 750×43, while the sum of the products on the credit side would have been greater by 650×43; and since 750×43 exceeds 650×43 by 4300, this would have made the sum of the products on the credit side equal to the sum of the products on the debit side.

This earlier focal date would then have been the equated time of payment.

It is evident from the above that if the two balances are on the same side of the account, the equated time is after the focal date; but if they are on opposite sides of the account, the equated time is before the focal date.

From the above example we derive the following

RULE.

Write to the left of each item of the account its date of maturity; and select as a focal date the earliest date at which any item becomes due.

Multiply each item by the number of days between its date of maturity and the focal date, and add the products on each side of the account.

Divide the balance of the sums of the products by the balance of the account, giving the number of days between the focal date and the equated time of payment.

If the balances are on the same side of the account, the equated time is after the focal date; if they are on opposite sides, the equated time is before the focal date.

2. Find the equated time for paying the balance of the following account:

Dr. GEORGE ADAMS. *Cr.*

1891.			1891.		
Sept. 16	To Mdse. 30 d.	$550	Oct. 4	By Cash,	$500
Oct. 28	" "	375	Nov. 9	" "	325

3. Find the equated time for paying the balance of the following account:

Dr. HENRY COLE. *Cr.*

1890.			1890.		
Nov. 12	To Mdse. 30 d.	$620	Dec. 24	By Cash,	$840
Dec. 8	" " 2 mo.	475	1891.		
1891.			Feb. 7	" Mdse.	750
Jan. 25	" "	745			

4. Find the equated time for paying the balance of the following account:

Dr. WILLIAM BLAKE. *Cr.*

1892.			1892.		
Jan. 18	To Mdse. 1 mo.	$450	Jan. 31	By Draft, 30 d.	$300
Feb. 6	" " 60 d.	600	April 4	" Cash,	550

5. At what date should the balance of the following account begin to draw interest?

Dr. EDWARD DODGE. *Cr.*

1892.			1892.		
May 5	To Mdse. 1 mo.	$165	May 18	By Cash,	$115
June 20	" "	280	June 29	" "	150
July 11	" " 60 d.	105	July 10	" Draft, 60 d.	175

6. Find the equated time for paying the balance of the following account:

Dr. RICHARD HAYES. *Cr.*

1891.			1891.		
Feb. 23	To Mdse. 30 d.	$275	Mar. 19	By Draft, 3 mo.	$200
Mar. 15	" "	420	April 6	" Cash,	175
May 9	" " 2 mo.	355	May 31	" Draft, 30 d.	450

7. Find the equated time for paying the balance of the following account:

Dr. JOHN EVANS. *Cr.*

1892.			1892.		
Aug. 23	To Mdse. 90 d.	$345	Sept. 2	By Draft, 90 d.	$275
Sept. 16	" " 2 mo.	775	Nov. 18	" Cash,	450
Nov. 13	" " 3 mo.	530	Dec. 7	" Draft, 2 mo.	325
			Dec. 28	" Cash,	210

8. When should the balance of the following account begin to draw interest?

Dr. JAMES FRENCH. *Cr.*

1890.			1890.		
Dec. 21	To Mdse. 60 d.	$380	Dec. 19	By Draft, 2 mo.	$350
1891.			1891.		
Jan. 15	" "	560	Jan. 3	" Mdse.	1045
Feb. 27	" " 2 mo.	420	Mar. 16	" Draft, 60 d.	1000
April 1	" " 30 d.	700			

Note 1. The *latest* date at which any item becomes due may be taken as a focal date; in such a case, if the balances are on the same side of the account, the equated time is *before* the focal date; if they are on opposite sides, the equated time is *after* the focal date.

Note 2. In settling an account in which the sum of the items on the debit side equals the sum of the items on the credit side, it may happen that the sum of the products on one side of the account is greater than the sum of the products on the other side.

Thus, in settling the following account:

Dr. DANIEL GREEN. *Cr.*

1892.			1892.		
May 5	To Mdse.	$130	May 17	By Cash,	$150
June 12	" "	210	June 25	" "	190

with May 5 as the focal date, we find:

$$\begin{array}{rcl|rcl}
130 \times 0 = & & 0 & 150 \times 12 = & & 1800 \\
\underline{210} \times 38 = & & \underline{7980} & \underline{190} \times 51 = & & \underline{9690} \\
\$340 & & 7980 & \$340 & & 11490 \\
\underline{340} & & & & & \underline{7980} \\
0 \text{ bal.} & & & & & 3510 \text{ bal.}
\end{array}$$

It appears from the above that there is still due from Mr. Green the use, or *interest*, of $1 for 3510 days; which, at 6%, is $0.59.

If the balance of 3510 had been on the *debit* side of the account, settlement would be made by paying Mr. Green $0.59.

THE INTEREST METHOD.

353. Another method of averaging accounts is known as the **Interest Method.**

We will select for illustration the example which is solved by the product method on page 277.

We will suppose that the account is settled on Sept. 7, the latest date at which any item becomes due.

Solution.

Int. of $350 for 72 d. =	$4.20		Int. of $275 for 58 d. =	$2.6583
" " 125 " 83 d. =	1.7292		" " 250 " 54 d. =	2.25
" " 275 " 0 d. =	0		" " 125 " 17 d. =	.3542
$750	$5.9292		$650	$5.2625
650	5.2625			
$100 bal.	$0.6667 bal.			

Int. of $100 for 1 d. = $.0167.

.6667 ÷ .0167 = 40 d.

Sept. 7 − 40 d. = July 29, *Ans.*

The balance of the account is $100 on the debit side.

Payment for the merchandise bought May 28 is due June 27; if it is not made until Sept. 7, Mr. Stuart should pay interest on the amount for 72 days; which, at 6%, is $4.20.

In like manner, he should pay interest on $125 for 83 days, which is $1.7292.

Hence, if the account is settled Sept. 7, he should pay $5.9292 interest in addition to the sum of the items on the debit side.

Now on June 8 he gave his draft, at 30 days, for $275, which became due July 11; in paying this sum 58 days before Sept. 7, he is entitled to interest on the amount for 58 days, which is $2.6583.

In like manner, he is entitled to interest on $250 for 54 days, which is $2.25, and on $125 for 17 days, which is $0.3542.

Hence, if the account is settled Sept. 7, he is entitled to interest to the amount of \$ 5.2625.

This leaves a balance of interest due from him of \$ 5.9292 – \$ 5.2625, or \$ .6667; that is, if he settles the account Sept. 7, he must pay \$ .6667 in addition to the balance of the account.

But by paying the \$ 100 a little earlier than Sept. 7, he can offset the interest charge; and the question now is, how many days will it take \$ 100 to gain \$ .6667 at 6%?

In one day, \$ 100 gains \$ .0167.

Then, to gain \$ .6667 will take as many days as .0167 is contained times in .6667; which is 40.

Then, by paying the \$ 100 40 days before Sept. 7, that is, on July 29, he can settle the account with equity.

It is evident that if the balances had been on opposite sides of the account, in the above example, the equated time of payment would have been *after* Sept. 7.

From the above example, we derive the following

RULE.

Select as a focal date the latest date at which any item becomes due.

Compute interest at 6% on each item for the number of days from the date when it becomes due to the focal date.

Divide the balance of interest by the interest of the balance of the account for one day, giving the number of days from the focal date to the equated time of payment.

If the balances are on the same side of the account, the equated time is earlier than the focal date; if they are on opposite sides, it is later than the focal date.

The teacher may have the examples of Art. 352 performed by the interest method.

XXIII. STOCKS AND BONDS.

354. **Stock** is the capital of a corporation; it is divided into a certain number of equal parts called **Shares.**

355. The original value of a share is usually $100; it will be so considered in the present chapter, unless the contrary is stated.

356. The original value of a share of stock is called its **Par Value,** and the price at which it sells is called its **Market Value.**

357. If a share of stock sells for *more* than its par value, the stock is said to be *above par,* or *at a premium;* if it sells for *less* than its par value, the stock is said to be *below par,* or *at a discount.*

358. The market value of a stock is usually quoted at a certain *per cent* of the par value.

Thus, if a stock is quoted at 105, it is selling for 5% above its par value; if it is quoted at 95, it is selling for 5% below its par value.

359. A **Certificate of Stock** is a document issued by a corporation, specifying the number of shares owned by the holder, and the par value of each share.

A **Dividend** is a sum divided among the stockholders from the profits of the business.

An **Assessment** is a sum required of the stockholders to meet the losses or expenses of the business.

Dividends and Assessments are generally reckoned at a certain per cent of the par value of the stock.

360. A **Bond** is the interest-bearing note of a government or corporation.

The interest on bonds is usually paid semi-annually.

A *Coupon* is a certificate of interest attached to a bond.

361. Bonds are usually named according to their rate of interest and date of maturity.

Thus, "U. S. $4\frac{1}{2}$'s. '91" signifies Bonds issued by the United States government, bearing $4\frac{1}{2}\%$ interest, the principal payable in 1891.

362. Brokerage is the commission received by a broker for buying or selling stocks and bonds.

It is usually $\frac{1}{8}\%$ of the par value of the stock or bond.

Note. It will be understood, in the following examples, that the brokerage is not included in the quoted price of a stock.

Thus, if a man buys stock at $112\frac{1}{2}$, and the brokerage is $\frac{1}{8}\%$, he pays the broker $112\frac{5}{8}$; if he sells stock at $112\frac{1}{2}$, and the brokerage is $\frac{1}{8}\%$, he receives only $112\frac{3}{8}$ from the broker.

EXAMPLES.

363. 1. Find the cost of 20 shares New York Central stock, at $112\frac{3}{4}$, brokerage $\frac{1}{8}\%$.

$112\frac{3}{4} + \frac{1}{8} = 112\frac{7}{8}$.

$\$112\frac{7}{8} \times 20 = \$2257.50$, *Ans.*

Since the cost of one share, including brokerage, is $\$112\frac{7}{8}$, the cost of 20 shares will be $20 \times \$112\frac{7}{8}$, or $\$2257.50$.

2. How much will be received from the sale of 16 shares Chicago, Burlington, and Quincy stock at $103\frac{5}{8}$, brokerage $\frac{1}{8}\%$?

$103\frac{5}{8} - \frac{1}{8} = 103\frac{1}{2}$.

$\$103\frac{1}{2} \times 16 = \$1656$, *Ans.*

Since the price received for one share is $\$103\frac{1}{2}$, the price received for 16 shares will be $16 \times \$103\frac{1}{2}$, or $\$1656$.

3. What amount of Mexican Central 4's, at $64\frac{1}{8}$, can be bought for $\$16062.50$, including a brokerage of $\frac{1}{8}\%$?

$64\frac{1}{8} + \frac{1}{8} = 64\frac{1}{4} = 64.25$.

$16062.5 \div .6425 = \$25000$, *Ans.*

Since the cost of one dollar's worth, including brokerage, is $\$0.6425$, as many dollars' worth can be bought for $\$16062.50$ as .6425 is contained times in 16062.5.

Dividing 16062.5 by .6425, the quotient is 25000.

Hence, $\$25000$ worth of bonds can be bought for $\$16062.50$.

4. If 48 shares Union Pacific stock are sold for $1992, brokerage $\frac{1}{4}$%, what is the quoted price of the stock?

$\$1992 \div 48 = \$41.50 = \$41\frac{1}{2}$.
$41\frac{1}{2} + \frac{1}{4} = 41\frac{3}{4}$, *Ans.*

Dividing $1992 by 48, the price received for one share is $41½.

Then the quoted price is 41½, plus the brokerage of ¼, or 41¾.

5. Find the cost of 95 shares Old Colony stock, at $186\frac{5}{8}$, brokerage $\frac{1}{8}$%.

6. Find the cost of 84 shares of telegraph stock at $95\frac{1}{4}$, brokerage $\frac{1}{8}$%.

7. How much will be received from the sale of 37 shares of bank stock at $147\frac{7}{8}$, brokerage $\frac{1}{8}$%?

8. A man sold 58 shares of New York and New England stock at $44\frac{3}{4}$, brokerage $\frac{1}{8}$%. How much did he receive?

9. Find the cost of $12000 U. S. 4's, when at a premium of $12\frac{3}{4}$%, brokerage $\frac{1}{8}$%.

10. Find the cost of 72 shares of railway stock, when $31\frac{1}{8}$% below par, brokerage $\frac{1}{4}$%.

11. How many shares of Missouri Pacific stock, at $56\frac{1}{2}$, can be bought for $2604.75, including a brokerage of $\frac{1}{8}$%?

12. A gentleman sold bonds at $88\frac{3}{4}$, brokerage $\frac{1}{4}$%, receiving the sum of $6637.50. What amount of bonds did he sell?

13. I sold mining stock at $126\frac{7}{8}$, brokerage $\frac{1}{8}$%, receiving the sum of $5450.25. How many shares did I sell?

14. How many shares of railway stock, at $151.75 a share, can be bought for $95073.75, including a brokerage of $\frac{1}{8}$%?

15. What amount of Missouri, Kansas, and Texas 5's, at a discount of $51\frac{1}{8}$%, can be bought for $7237.50, including a brokerage of $\frac{1}{8}$%?

16. How many shares of bank stock, at $27\frac{1}{2}$% above par, can be bought for $9581.25, including a brokerage of $\frac{1}{4}$%?

17. If 12 shares of Elevated Railway stock are sold for \$1837.50, including a brokerage of $\frac{1}{8}\%$, what is the quoted price of the stock?

18. If 45 shares Boston and Maine stock can be bought for \$7627.50, including a brokerage of $\frac{1}{8}\%$, what is the quoted price of the stock?

19. If \$6000 worth of bonds are sold for \$5497.50, including a brokerage of $\frac{1}{8}\%$, how much below par are the bonds quoted?

20. If 18 shares of railway stock can be bought for \$1460.25, including a brokerage of $\frac{1}{4}\%$, what is the quoted price of the stock?

21. If \$7500 Iowa Central bonds can be bought for \$6581.25, including a brokerage of $\frac{1}{8}\%$, at what per cent discount are the bonds quoted?

22. If 298 shares Delaware and Hudson stock can be bought for \$40788.75, including a brokerage of $\frac{1}{8}\%$, how much above par is the stock quoted?

23. If the brokerage, at $\frac{1}{8}\%$, for selling stock is \$23.25, how many shares were sold?

24. What annual income is received from mining stocks whose par value is \$11300, paying $1\frac{3}{4}\%$ dividends semi-annually?

25. A corporation declared a dividend of $7\frac{1}{4}\%$, paying to the stockholders a total amount of \$5437.50. What was its capital stock?

26. A man purchased 358 shares of a certain stock at $3\frac{5}{8}\%$ below par, and sold it at a premium of $7\frac{3}{4}\%$; if he paid $\frac{1}{8}\%$ brokerage on each transaction, how much did he gain?

27. A corporation whose capital is \$225000, levies an assessment of \$4218.75 on its stockholders. What is the rate per cent of the assessment?

28. What par value of stocks paying $1\frac{5}{8}\%$ dividends quarterly, will produce an annual income of \$513.50?

29. If $441.75 is lost by buying stocks at $5\frac{1}{4}\%$ premium, and selling them at $6\frac{1}{8}\%$ below par, in each case paying a brokerage of $\frac{1}{8}\%$, how many shares were bought?

30. A man sold 204 shares of stock at 80, and with the proceeds bought stock at $106\frac{3}{8}$. If the brokerage on each transaction was $\frac{1}{8}\%$, how many shares did he buy?

31. A man sold 285 shares of railway stock at $103\frac{1}{4}$, and invested the proceeds in bank stock at $78\frac{1}{4}$. If the brokerage on each transaction was $\frac{1}{8}\%$, how many shares of bank stock did he receive?

32. What annual income will be realized from investing $2145 in a 5% stock at $107\frac{1}{8}$, brokerage $\frac{1}{8}\%$?

$$107\tfrac{1}{8} + \tfrac{1}{8} = 107\tfrac{1}{4} = 107.25.$$
$$\$2145 \div 1.0725 = \$2000.$$
$$\$2000 \times .05 = \$100, \textit{ Ans.}$$

The cost of the stock, including brokerage, is 107.25.

Then stock to the par value of $2145 ÷ 1.0725, or $2000, can be bought for $2145.

The annual income from $2000 at 5% is $2000 × .05, or $100.

33. What amount must be invested in a $5\frac{1}{2}\%$ stock, at $93\frac{3}{4}$, no allowance being made for brokerage, to realize an annual income of $374?

$$\$374 \div .055 = \$6800.$$
$$\$6800 \times .93\tfrac{3}{4} = \$6375, \textit{ Ans.}$$

If the annual income is to be $374, the par value of the stock must be $\$374 \div .05\frac{1}{2}$, or $6800.

To purchase $6800 worth of stock at $93\frac{3}{4}$, the amount to be invested is $\$6800 \times .93\frac{3}{4}$, or $6375.

34. If I purchase at $95\frac{3}{4}$, brokerage $\frac{1}{4}\%$, a stock paying 6% dividends annually, what per cent does the investment yield?

$$95\tfrac{3}{4} + \tfrac{1}{4} = 96.$$
$$\tfrac{6}{96} = \tfrac{1}{16} = 6\tfrac{1}{4}\%, \textit{ Ans.}$$

The cost of the stock, including brokerage, is $96 a share.

Then, if $96 produces an annual income of $6, the rate per cent of the investment is $\frac{6}{96}$, or $6\frac{1}{4}\%$.

35. What must be paid for a $5\frac{3}{4}\%$ stock, no allowance being made for brokerage, in order that the investment may yield 5%?

$5.75 \div .05 = 115$, *Ans.* The annual income from one share is \$5.75.

Then if \$5.75 is 5% of the cost of a share, the cost of a share must be \$5.75 ÷ .05, or \$115.

Hence, the stock must be bought at 115.

36. What annual income will be realized from investing \$3455 in 4% bonds at $86\frac{3}{8}$, no allowance being made for brokerage?

37. What annual income will be realized from investing \$2805 in a $6\frac{1}{4}\%$ stock at $116\frac{7}{8}$, brokerage $\frac{1}{8}\%$?

38. What annual income will be realized by investing \$4331.25 in a $7\frac{3}{4}\%$ stock, at a premium of $23\frac{1}{2}\%$, brokerage $\frac{1}{4}\%$?

39. What annual income will be realized by investing \$7879.50 in a $4\frac{3}{4}\%$ stock, at $22\frac{3}{4}\%$ below par, no allowance being made for brokerage?

40. What sum must be invested in a $5\frac{1}{4}\%$ stock at $105\frac{1}{4}$, no allowance being made for brokerage, to realize an annual income of \$220.50?

41. What sum must be invested in 3% bonds at $96\frac{1}{2}$, brokerage $\frac{1}{8}\%$, to yield an annual income of \$180?

42. What sum must be invested in a $4\frac{1}{2}\%$ stock at a discount of $16\frac{1}{2}\%$, brokerage $\frac{1}{4}\%$, to yield an annual income of \$648?

43. A man realized an annual income of \$425.25 by investing in a $6\frac{3}{4}\%$ stock, at $18\frac{1}{4}\%$ above par, no allowance being made for brokerage. What was the sum invested?

44. If a $5\frac{1}{4}\%$ stock is purchased at 105, no allowance being made for brokerage, what per cent does the investment yield?

45. If a $7\frac{7}{8}\%$ stock is bought at $174\frac{7}{8}$, brokerage $\frac{1}{8}\%$, what per cent does the investment yield?

46. If 3% bonds are bought at $46\frac{5}{8}$, brokerage $\frac{1}{4}\%$, what per cent does the investment yield?

47. If $4\frac{1}{8}\%$ stock is bought at $123\frac{3}{4}$, no allowance being made for brokerage, what per cent does the investment yield?

48. What must be paid for a 7% stock, no allowance being made for brokerage, in order that the investment may yield $3\frac{1}{8}\%$?

49. What must be paid for a $4\frac{5}{8}\%$ stock, no allowance being made for brokerage, in order that the investment may yield $6\frac{1}{4}\%$?

50. What must be paid for a $5\frac{7}{8}\%$ stock, no allowance being made for brokerage, in order that the investment may yield 4%?

51. At what price must $3\frac{3}{4}\%$ bonds be quoted, in order that a person investing in them, brokerage $\frac{1}{4}\%$, may realize $5\frac{5}{8}\%$ for his money?

52. At what price must a 7% bond be quoted to yield the same per cent on the investment as a 5% bond at $107\frac{1}{2}$?

53. Which will yield the greater per cent on the investment, a 6% stock at 125, or a $4\frac{1}{2}\%$ stock at 90, no allowance being made for brokerage?

54. A man sells 6% stock to the par value of \$7400 at $113\frac{3}{4}$, and invests the proceeds in 5% stock at $92\frac{1}{2}$, no allowance being made for brokerage. Is his income increased or diminished, and how much?

55. At what price must a $3\frac{1}{2}\%$ stock be bought, to yield the same per cent on the investment as a $4\frac{1}{2}\%$ stock at $21\frac{1}{4}\%$ below par?

56. Which is the better investment, a 7% stock at $143\frac{3}{8}$, or a $5\frac{1}{2}\%$ stock at $112\frac{5}{8}$, brokerage $\frac{1}{8}\%$ on each transaction?

57. A man sells $4\frac{1}{2}\%$ stock to the par value of \$9400 at $82\frac{1}{8}$, and invests the proceeds in $8\frac{1}{2}\%$ stock at $163\frac{7}{8}$, brokerage on each transaction $\frac{1}{8}\%$. Is his income increased or diminished, and how much?

58. Which will yield the greater income, \$6757.50 invested in 6% bonds at $112\frac{3}{8}$, or \$6632.50 invested in 5% bonds at $94\frac{1}{2}$, brokerage in each case $\frac{1}{4}\%$?

59. A man sold \$12000 $5\frac{1}{4}\%$ state bonds at $110\frac{3}{4}$, and invested the proceeds in city bonds at $84\frac{3}{4}$, brokerage in each case $\frac{1}{4}\%$. If his income was increased \$33, what per cent were the city bonds?

60. Which will yield the greater per cent on the investment, a $5\frac{1}{4}\%$ stock at a premium of $3\frac{3}{4}\%$, or a $3\frac{7}{8}\%$ stock at a discount of $28\frac{1}{2}\%$, brokerage in each case $\frac{1}{8}\%$?

61. A man sold $5\frac{1}{2}\%$ stock to the par value of \$8600 at $104\frac{1}{4}$, and invested the proceeds in $6\frac{1}{2}\%$ stock, decreasing his income by \$21.25. If no allowance be made for brokerage, what price did he pay for the $6\frac{1}{2}\%$ stock?

62. A man sold 78 shares of $5\frac{7}{8}\%$ stock at $131\frac{1}{2}$, and 43 shares of $3\frac{3}{4}\%$ stock at $81\frac{7}{8}$, and invested the proceeds in $4\frac{3}{8}\%$ stock at $91\frac{5}{8}$; brokerage on each transaction $\frac{1}{8}\%$. Was his income increased or diminished, and how much?

XXIV. PROGRESSIONS.

ARITHMETICAL PROGRESSION.

364. An **Arithmetical Progression** is a series of numbers which increase or decrease by a constant difference, called the **Common Difference.**

Thus, 1, 3, 5, 7, 9, 11 is an *increasing* arithmetical progression, in which the common difference is 2.

Again, 14, 11, 8, 5, 2, is a *decreasing* arithmetical progression, in which the common difference is 3.

The numbers which compose the progression are called its **Terms.**

365. To find any Term of an Arithmetical Progression.

Example. Find the 18th term of the arithmetical progression 3, 9, 15, etc.

Com. dif. $= 9 - 3 = 6$.
18th term $= 3 + (17 \times 6)$
$= 3 + 102$
$= 105$, *Ans.*

In this case, the common difference is 6.

Now the *second* term is equal to the first term plus *once* the common difference; the *third* term is equal to the first term plus *twice* the common difference; etc.

Hence, the *eighteenth* term will be equal to the first term plus *seventeen* times the common difference; that is, $3 + (17 \times 6)$, or 105.

If the progression had been a *decreasing* one, we should have *subtracted* seventeen times the common difference from the first term.

From the above example, we derive the following

RULE.

To find any term of an arithmetical progression, add to or subtract from the first term, according as the progression is increasing or decreasing, the common difference multiplied by a number less by 1 than the number of the required term.

366. **To find the Sum of the Terms of an Arithmetical Progression.**

Example. Find the sum of the terms of the arithmetical progression 5, 9, 13, 17, 21, 25, 29.

$$\begin{array}{r}5+\ \ 9+13+17+21+25+29\\ 29+25+21+17+13+\ \ 9+\ \ 5\\ \hline 34+34+34+34+34+34+34\end{array}$$

The sum of the terms of the progression is $5+9+13+17+21+25+29$.

We write underneath this the sum of the terms of the progression in *reverse order*; that is, $29+25+21+17+13+9+5$.

Adding each term of the second line to the term directly above it, the sum is 34.

Hence, *twice* the sum of the terms is 7×34, or $7 \times (5+29)$.

Therefore the sum of the terms is $\frac{7 \times (5+29)}{2}$, or 119, *Ans.*

Observing that 7 is the number of terms, 5 the first term, and 29 the last term, we have the following

RULE.

To find the sum of the terms of an arithmetical progression, multiply the sum of the first and last terms by the number of terms, and divide the result by 2.

EXAMPLES.

367. **1.** Find the last term and the sum of the terms of the arithmetical progression 19, $18\frac{1}{3}$, $17\frac{2}{3}$, etc., to 24 terms.

The common difference is $19 - 18\frac{1}{3}$, or $\frac{2}{3}$.

Since the number of terms is 24, the last term is the 24th term.

By Art. 365, the 24th term is $19 - (23 \times \frac{2}{3})$, or $3\frac{2}{3}$.

By Art. 366, the sum of the terms is $\frac{24 \times (19 + 3\frac{2}{3})}{2}$, or 272.

2. Find the 11th term of the progression 2, 9, 16, etc.

3. Find the 38th term of the progression $4\frac{3}{4}$, $5\frac{1}{2}$, $6\frac{1}{4}$, etc.

4. Find the 23d term of the progression 327, 316, 305, etc.

5. Find the 20th term of the progression 120, 116.4, 112.8, etc.

6. Find the 47th term of the progression $\frac{3}{10}$, $\frac{5}{12}$, $\frac{8}{15}$, etc.

Find the last term and the sum of the terms of:

7. 4, 10, 16, etc., to 12 terms.

8. 9, 23, 37, etc., to 21 terms.

9. 293, 285, 277, etc., to 29 terms.

10. 486, 473, 460, etc., to 36 terms.

11. $3\frac{1}{3}$, 5, $6\frac{2}{3}$, etc., to 48 terms.

12. 97, 90.3, 83.6, etc., to 14 terms.

13. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, etc., to 59 terms.

14. $\frac{7}{8}$, $\frac{3}{4}$, $\frac{1}{1}\frac{3}{8}$, etc., to 23 terms.

15. Find the sum of the integers beginning with 1, and ending with 99.

16. Find the sum of the even integers beginning with 2, and ending with 100.

17. Continue the progression $\frac{7}{18}$, $\frac{7}{12}$, $\frac{7}{9}$, etc., to four more terms.

18. Find the sum of the first 18 integers which are multiples of 7.

19. Find the sum of all the multiples of 11, from 110 to 990, inclusive.

20. A body falls $16\frac{1}{12}$ feet the first second, and in each succeeding second $32\frac{1}{6}$ feet more than in the next preceding one. How far will it fall in the 16th second? How far will it fall in 16 seconds?

21. A man travelled $43\frac{1}{3}$ miles the first day, and on each succeeding day $2\frac{3}{4}$ miles less than on the next preceding. How far did he travel on the 11th day? How far did he travel in 11 days?

22. If a person saves \$100 a year, and puts this sum at simple interest at $4\frac{1}{2}\%$ at the end of each year, to how much will his property amount at the end of 25 years?

GEOMETRICAL PROGRESSION.

368. A **Geometrical Progression** is a series of numbers which increase or decrease by a constant multiplier, called the **Ratio.**

Thus, 1, 3, 9, 27, 81 is an *increasing* geometrical progression, in which the ratio is 3.

Again, 64, 32, 16, 8, 4, is a *decreasing* geometrical progression, in which the ratio is $\frac{1}{2}$.

The numbers which compose the progression are called its **Terms.**

369. To find any Term of a Geometrical Progression.

Example. Find the 6th term of the geometrical progression 2, 6, 18, etc.

$$\text{Ratio} = \tfrac{6}{2} = 3.$$
$$\text{6th term} = 2 \times 3^5$$
$$= 2 \times 243$$
$$= 486, \textit{Ans.}$$

Dividing the second term, 6, by the first term, 2, the ratio is 3.

Now the *second* term is equal to the first term times the *first* power of the ratio; the *third* term is equal to the first term times the *second* power of the ratio; etc.

Hence, the *sixth* term will be equal to the first term times the *fifth* power of the ratio; that is, 2×3^5, or 486.

From the above example, we derive the following

RULE.

To find any term of a geometrical progression, multiply the first term by that power of the ratio whose exponent is less by 1 *than the number of the required term.*

370. To find the Sum of the Terms of a Geometrical Progression.

1. Find the sum of the terms of the geometrical progression 2, 6, 18, 54, 162.

The ratio is 3.

The sum of the terms $= 2 + 6 + 18 + 54 + 162.$ (1)

Multiplying each term of the result by the ratio, 3, we have

$3 \times$ the sum of the terms $= 6 + 18 + 54 + 162 + (162 \times 3).$ (2)

Subtracting (1) from (2), we have

$2 \times$ the sum of the terms $= (162 \times 3) - 2.$

Then, the sum of the terms $= \dfrac{(162 \times 3) - 2}{2} = \dfrac{(162 \times 3) - 2}{3 - 1}.$

Observing that 162 is the last term, 3 the ratio, and 2 the first term, we have the following rule:

The sum of the terms of an increasing geometrical progression is equal to the product of the last term by the ratio, minus the first term, divided by the ratio minus 1.

2. Find the sum of the terms of the geometrical progression 768, 192, 48, 12, 3.

The ratio is $\frac{1}{4}$.

The sum of the terms $= 768 + 192 + 48 + 12 + 3.$ (1)

Multiplying each term of the result by the ratio, $\frac{1}{4}$, we have

$\frac{1}{4} \times$ the sum of the terms $= 192 + 48 + 12 + 3 + (3 \times \frac{1}{4}).$ (2)

Subtracting (2) from (1), we have

$\frac{3}{4} \times$ the sum of the terms $= 768 - (3 \times \frac{1}{4}).$

Then, the sum of the terms $= \dfrac{768 - (3 \times \frac{1}{4})}{\frac{3}{4}} = \dfrac{768 - (3 \times \frac{1}{4})}{1 - \frac{1}{4}}.$

Observing that 768 is the first term, 3 the last term, and $\frac{1}{4}$ the ratio, we have the following rule:

The sum of the terms of a decreasing geometrical progression is equal to the first term, minus the product of the last term by the ratio, divided by 1 *minus the ratio.*

371. In Ex. 1, Art. 370, we have

$$162 \times 3 = (2 \times 3^4) \times 3 = 2 \times 3^5.$$

That is, the product of the last term of a geometrical progression by the ratio is equal to the first term, multiplied by that power of the ratio whose exponent is equal to the number of terms.

Then the rules of Art. 370 may be stated as follows:

The sum of the terms of an increasing geometrical progression is equal to the first term, multiplied by that power of the ratio whose exponent is equal to the number of terms, minus the first term, divided by the ratio minus 1.

The sum of the terms of a decreasing geometrical progression is equal to the first term, minus the first term multiplied by that power of the ratio whose exponent is equal to the number of terms, divided by 1 *minus the ratio.*

EXAMPLES.

372. **1.** Find the last term and the sum of the terms of the geometrical progression 3, 12, 48, etc., to 6 terms.

The ratio is $\frac{12}{3}$, or 4.
By Art. 369, the last or 6th term is 3×4^5, or 3072.
By the first rule of Art. 370, the sum of the terms is

$$\frac{4 \times 3072 - 3}{4 - 1}, \text{ or } 4095.$$

2. Find the sum of the terms of the geometrical progression 2, $\frac{2}{3}$, $\frac{2}{9}$, etc., to 7 terms.

The ratio is $\frac{2}{3} \div 2$, or $\frac{1}{3}$.
By the second rule of Art. 371, the sum of the terms is

$$\frac{2 - 2 \times (\frac{1}{3})^7}{1 - \frac{1}{3}} = \frac{2 - \frac{2}{2187}}{\frac{2}{3}} = \frac{\frac{4372}{2187}}{\frac{2}{3}} = \frac{2186}{729}, \textit{ Ans.}$$

3. Find the 5th term of the progression 4, 20, 100, etc.

4. Find the 6th term of the progression $\frac{3}{8}$, $\frac{1}{2}$, $\frac{2}{3}$, etc.

5. Find the 9th term of the progression $6\frac{3}{4}$, $4\frac{1}{2}$, 3, etc.

6. Find the 7th term of the progression 600, 150, $37\frac{1}{2}$, etc.

Find the last term and the sum of the terms of:

7. 1, 2, 4, etc., to 11 terms.

8. 6, 18, 54, etc., to 8 terms.

9. 20, 10, 5, etc., to 10 terms.

10. $\frac{3}{4}$, $\frac{9}{16}$, $\frac{27}{64}$, etc., to 6 terms.

11. $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, etc., to 7 terms.

12. $12\frac{1}{2}$, 5, 2, etc., to 5 terms.

13. Find the sum of the terms of the progression $12\frac{4}{5}$, 16, 20, etc., to 5 terms.

14. Find the sum of the terms of the progression 15, 10, $6\frac{2}{3}$, etc., to 8 terms.

15. A man agreed to work for 14 days on condition that he should receive 1 cent the first day, 2 cents the second day, 4 cents the third day, and so on. How much did he receive in all?

16. A man travelled 384 miles the first day, and on each succeeding day one-half as many miles as on the next preceding. How far did he travel on the 10th day? How far did he travel in 10 days?

17. Continue the progression $4\frac{20}{27}$, $3\frac{5}{9}$, $2\frac{2}{3}$, etc., to three more terms.

18. The population of a certain city at the end of each year is 1.04 times as great as at the beginning of the year. If the population on Jan. 1, 1890, was 15625, what will it be on Jan. 1, 1893?

19. If the first term is $100, the ratio 1.05, and the number of terms 5, what is the last term?

COMPOUND INTEREST.

373. Problems in compound interest may be solved by aid of the principles of geometrical progression.

Thus, let $100 be put at compound interest at 6%.

The amount at the end of one year is $\$100 \times 1.06$.

The amount at the end of two years is $\$100 \times 1.06 \times 1.06$, or $\$100 \times (1.06)^2$; and so on.

Hence, *the amount at the end of any number of years is equal to the principal, multiplied by that power of* 1 *plus the rate, whose exponent is equal to the number of years.*

EXAMPLES.

1. Find the amount of $1600 for 4 years, at 5% compound interest.

By the above rule, the required amount is

$$\$1600 \times (1.05)^4, \text{ or } \$1944.81, \textit{ Ans.}$$

2. What principal will gain $15.608 in 3 years at 4% compound interest?

The amount of $1 for 3 years at 4% compound interest is $(1.04)^3$ dollars, or $1.124864.

Then, $1 will gain $.124864 in 3 years at 4% compound interest.

Then, to gain $15.608 will take as many dollars as .124864 is contained times in 15.608, which is $125, *Ans.*

3. Find the amount of $6400 for 4 years, at 3% compound interest.

4. Find the amount of $8000 for 5 years, at 4% compound interest.

5. Find the compound interest of $300 for 4 years, at 6%.

6. Find the compound interest of $760 for 3 years, at $5\frac{1}{2}$%.

7. What principal will amount to $25585.35 in 2 years, at $3\frac{1}{4}$% compound interest?

8. What principal will amount to $5477.5974 in 3 years, at $4\frac{1}{2}$% compound interest?

9. What principal will gain $274.07131 in 5 years, at 5% compound interest?

ANNUITIES.

374. An **Annuity** is a specified sum of money payable at equal intervals of time.

Note. We shall consider in the present chapter those cases only in which the payments are annual.

The **Amount** or **Final Value** of an annuity is the sum of all the payments, together with the interest on each payment from the time it becomes due until the annuity ceases.

The **Present Worth** of an annuity is that sum of money which, at the specified rate of interest, will amount to the final value.

375. Annuities at Simple Interest.

Problems in annuities at simple interest may be solved by aid of the principles of arithmetical progression.

1. Find the amount of an annuity of $400 for 6 years, at 5% simple interest.

The first payment draws interest for five years; the second payment for four years; etc.

Now the amount of the first payment at the end of five years is $400 × 1.25, or $500.

The amount of the second payment at the end of four years is $400 × 1.20, or $480; etc.

We then have an arithmetical progression, whose first term is $500, last term $400, and number of terms 6.

Therefore, by Art. 366, the sum of the terms is

$$\tfrac{6}{2} \times (\$500 + \$400), \text{ or } \$2700, \textit{ Ans.}$$

From the above example, we derive the following

RULE.

Find the amount of the annual payment for a number of years less by 1 than the given time.

Add this amount to the annual payment, and multiply the result by one-half the given number of years.

2. Find the present worth of the annuity of Ex. 1.

By Art. 331, the present worth of $2700 due 6 years hence, at 5%, is $\frac{\$2700}{1.3}$, or $2076.92+, *Ans.*

3. What annuity to continue for 4 years, at 6% simple interest, can be purchased for $ 1090?

An annuity of $1 to continue for 4 years, at 6% simple interest, will amount to $\frac{4}{2}$ × ($1.18 + $1), or $4.36.

The amount of $1090 for 4 years, at 6%, is $1351.60.

Then, an annuity of as many dollars can be purchased for $1090 as 4.36 is contained times in 1351.60; which is $310, *Ans.*

EXAMPLES.

4. Find the amount and present worth of an annuity of $ 300 for 5 years, at 4% simple interest.

5. Find the amount and present worth of an annuity of $ 250 for 3 years, at $4\frac{1}{2}$% simple interest.

6. Find the amount and present worth of an annuity of $ 800 for 8 years, at 5% simple interest.

7. Find the amount and present worth of an annuity of $ 720 for 7 years, at $3\frac{3}{4}$% simple interest.

8. What annuity to continue for 6 years, at 6% simple interest, can be purchased for $ 2070?

9. What annuity to continue for 8 years, at 3% simple interest, can be purchased for $ 4420?

10. What annuity to continue for 9 years, at 4% simple interest, can be purchased for $ 2088?

11. What annuity to continue for 11 years, at $5\frac{1}{2}$% simple interest, can be purchased for $ 5610?

376. Annuities at Compound Interest.

Problems in annuities at compound interest may be solved by aid of the principles of geometrical progression.

1. Find the amount of an annuity of $ 500 for 4 years, at 3% compound interest.

The fourth or last payment draws no interest.

The amount of the third payment at the end of one year, at 3%, is $500 × 1.03.

The amount of the second payment at the end of 2 years at 3% compound interest is \$500 × $(1.03)^2$ (Art. 373); etc.

We then have an increasing geometrical progression, whose first term is \$500, ratio 1.03, and number of terms 4.

Therefore, by Art. 371, the sum of the terms is

$$\frac{\$500 \times [(1.03)^4 - 1]}{.03}, \text{ or } \$2091.81+, \textit{Ans.}$$

2. Find the present worth of the annuity of Ex. 1.

By Art. 373, the amount of \$1 for 4 years, at 3% compound interest, is $(1.03)^4$ dollars, or \$1.12550881.

Then to amount to \$2091.81+ will take as many dollars as 1.12550881 is contained times in 2091.81+, which is \$1858.54+, *Ans.*

3. What annuity to continue for 3 years, at 5% compound interest, can be purchased for \$2522?

An annuity of \$1 to continue for 3 years, at 5% compound interest, will amount to $\frac{(1.05)^3 - 1}{.05}$ dollars, or \$3.1525.

The amount of \$2522 for 3 years, at 5% compound interest, is \$2522 × $(1.05)^3$, or \$2919.53025.

Then an annuity of as many dollars can be purchased for \$2522, as 3.1525 is contained times in 2919.53025; which is \$926.10, *Ans.*

EXAMPLES.

4. Find the amount and present worth of an annuity of \$200 for 3 years, at 5% compound interest.

5. Find the amount and present worth of an annuity of \$300 for 4 years, at 6% compound interest.

6. Find the amount and present worth of an annuity of \$400 for 5 years, at 4% compound interest.

7. What annuity, to continue for 2 years, at 6% compound interest, can be purchased for \$515?

8. What annuity to continue for 3 years, at 4% compound interest, can be purchased for \$975.50?

9. What annuity to continue for 4 years, at 5% compound interest, can be purchased for \$600?

XXV. MISCELLANEOUS EXAMPLES.

377. **1.** Find the value of

$$(7263 - \overline{34242 \div 439}) \times (61143 \div \overline{837 - 748}).$$

2. Divide $166\frac{11}{14}$ by 17, and reduce the result to a mixed number.

3. Reduce $\frac{287}{543}$ to 91767ths.

4. A man spent $\frac{2}{7}$ of his money for provisions, $\frac{5}{8}$ of the remainder for clothing, $\frac{2}{15}$ of the remainder for charity, and had \$9.10 left. How much had he at first?

5. Find the interest of \$3528.75 from Nov. 25, 1887, to Sept. 11, 1891, at $4\frac{1}{2}\%$.

6. Find the proceeds of a 3-months' note for \$576, discounted on the day of its date at $3\frac{3}{4}\%$.

7. The area of a square field is 2 A. 77 sq. rd. 17 sq. yd. $4\frac{3}{4}$ sq. ft.; find its side in rods, yards, and feet.

8. Simplify $\dfrac{396 \times 425 \times 1274}{4896 \times 325 \times 1078}$.

9. The circumference of the hind-wheel of a carriage is 9 ft. 2 in., and of the fore-wheel 7 ft. 9 in. How many times does each wheel turn in travelling 9 mi. 220 rd.?

10. Extract the square root of .729275008576.

11. A bin 8 ft. 3 in. long, 5 ft. 8 in. wide, and 4 ft. 2 in. deep is filled with wheat. If a bushel is equal to $1\frac{1}{4}$ cu. ft., how much are the contents worth, at 96 cents a bushel?

12. Multiply 7 mi. 113 rd. 4 yd. 2 ft. 11 in. by 27.

13. When it is 3.08 P.M. at St. Petersburg, lon. 30° 19′ 48″ E., it is 4 hr. 56 min. $51\frac{2}{3}$ sec. A.M. at San Francisco. What is the longitude of San Francisco?

14. Simplify $\dfrac{6\frac{5}{8} - 5\frac{1}{5} + 4\frac{3}{4} - 3\frac{1}{2}}{8\frac{9}{10} - 7\frac{7}{8} + 6\frac{1}{5} - 5\frac{1}{4}}$.

15. A can do a piece of work in $8\frac{4}{5}$ hr.; A and B together can do it in $4\frac{8}{31}$ hr.; and A and C together can do it in 4 hr. How many hours will it take B and C together to do the work?

16. Subtract $\frac{86}{119}$ from $\frac{125}{136}$, and reduce the result to its lowest terms.

17. Find the exact number of days from May 15, 1873, to March 12, 1892.

18. A gentleman bequeathed $37\frac{1}{2}\%$ of his property of \$15120 to his wife, $44\frac{4}{9}\%$ of what remained to his son, $71\frac{3}{7}\%$ of the balance to his daughter, and the remainder to charity. How much did he leave to charity?

19. How long must \$487 be on interest at $3\frac{1}{4}\%$ to gain \$99.43?

20. Express $\frac{128}{175}$ as a circulating decimal.

21. Find the side, and the number of board feet, in the squared timber that can be sawed from a log whose length is 19 ft. 7 in., and diameter at the smaller end 16 in.

22. Multiply $83\frac{483}{767}$ by 59.

23. A wheel revolves 5000 times in travelling 8 mi. 296 rd. What is its radius in inches?

24. Express .008171875 as a common fraction in its lowest terms.

25. If it costs \$98.55 to plaster a hemispherical dome whose diameter is 34 ft. 9 in., how much will it cost to plaster a hemispherical dome whose diameter is 57 ft. 11 in.?

26. What per cent above cost must a merchant mark an article, in order to be able to sell it at a discount of 16% from the list price, and still make a profit of 11%?

27. Find the L. C. M. of 1656, 3087, and 8316.

28. Divide 31 sq. mi. 114 A. 132 sq. rd. 21 sq. yd. 3 sq. ft. 90 sq. in. by 18.

29. A tradesman sold merchandise for \$1101.75, and gained $\frac{26}{87}$ of what it cost him. How much did he gain by the operation?

30. When it is 7.36 A.M. at Washington, lon. 77° 3′ 37″ W., what time is it at Calcutta, lon. 88° 19′ 2″ E.?

31. Divide $298\frac{15}{38}$ by 23.

32. A train of 54 cars is loaded with coal, each car containing 4 long tons, 18 long hundred-weight. What is the value of the coal, at \$4.95 a short ton?

33. Find the amount of \$2893.40 from March 27, 1885, to March 18, 1889, at $2\frac{1}{4}\%$.

34. I paid \$65, including \$1.75 for the policy, for insuring \$4600 on a house. What was the rate of insurance?

35. Express 8 mi. 289 rd. 3 yd. 2 ft. 11 in. in inches.

36. If a train performs a certain journey in 5 h. 36 min., travelling at the rate of 56 feet a second, how long will it take it, travelling at the rate of 1085 yards a minute?

37. Find the present worth and true discount of \$478.95 due 1 yr. 9 mo. 10 d. hence, at 4%.

38. How large a draft on Bremen can be purchased for \$500, if exchange on Germany is quoted at $95\frac{3}{4}$?

39. Divide $2\frac{103}{456}$ by $1\frac{101}{264}$, and reduce the result to a mixed number.

40. Find the lateral area and volume of a pyramid with a square base, each side of whose base is 26 in., and whose altitude is 84 in.

41. A merchant sold goods for \$17212.50, and lost $\frac{13}{67}$ of what they cost him. How much did the goods cost him?

42. A rectangular field is 15 rd. 4 yd. $1\frac{1}{2}$ ft. long, and 12 rd. 3 yd. 1 ft. wide. How much is it worth at \$592.90 an acre?

43. Find the G. C. D. of 54432, 63504, and 98784.

44. Express £84 9*s.* 2*d.* 1 far. as a fraction of £123 8*s.* 9*d.* 3 far.

45. The population of a town decreased $6\frac{1}{4}\%$ from 1870 to 1880, and increased $13\frac{8}{9}\%$ from 1880 to 1890. If the population in 1890 was 4305, what was the population in 1870?

46. A merchant imported 875 sq. yd. of rugs, invoiced at 139.2 francs a square yard. What was the duty, at 55 cents a square yard, and 45% ad valorem?

47. Prove that 4057 is a prime number.

48. Express $\frac{119}{25600}$ and $\frac{17}{320}$ as decimals, and divide the first result by the second.

49. How much will it cost to cover a floor 20 ft. 3 in. long and 15 ft. 7 in. wide, with carpeting 28 in. wide, at \$1.16 a yard, if the strips run lengthwise of the floor? How much if the strips run across the room?

50. What number is that $\frac{18}{35}$ of $\frac{49}{27}$ of which exceeds $\frac{26}{25}$ of $\frac{55}{91}$ of it by $1\frac{79}{385}$?

51. A broker receives \$1000 to invest, after deducting a brokerage of $1\frac{1}{4}\%$. What sum can he invest, and what is the amount of his commission?

52. Divide 1081 lb. 1 oz. 13 pwt. 9 gr. by 2 lb. 0 oz. 16 pwt. 3 gr.

53. At what rate per cent will \$540 amount to \$732.39 in 6 y. 5 mo. 22 d.?

54. Reduce $\frac{72765}{102375}$ to its lowest terms.

55. Find the cost of a pile of wood 29 ft. 7 in. long, 5 ft. 3 in. high, and 4 ft. wide, at $\$7.16\frac{4}{5}$ a cord.

56. Simplify $\left(1.375 + 1\frac{1}{4} \times \frac{.07}{.028} + .\dot{6}\right) \div 2\frac{19}{57}$, and express the result as a decimal.

57. If a bushel of wheat weighs 59 lb., what is the value of nine carloads of wheat, each weighing 8 T. 17 cwt., at $\$1.03\frac{3}{4}$ a bushel?

58. Simplify $(8\frac{11}{14} - 4\frac{23}{27}) - (7\frac{6}{7} - 5\frac{8}{9})$.

59. Extract the cube root of 456.266246971625.

60. A bankrupt owes to A, \$398.75; to B, \$508.75; to C, \$316.25; and to D, \$563.75. If his resources are \$1115.40, what is each creditor's share?

61. Find the G. C. D. of 7429, 11339, and 12673.

62. The hypotenuse of a right triangle is $28\frac{1}{4}$ ft., and one of the sides about the right angle is $1\frac{1}{4}$ yd.; find the other side in inches.

63. If a tank whose length is 6 ft. 5 in. contains $578\frac{13}{16}$ gallons of water, what is the length of a similar tank which contains $2246\frac{1}{16}$ gallons?

64. Express 158324 in. in terms of higher denominations.

65. An agent sells 415 yards of woollens, at \$1.52 a yard, charging $2\frac{1}{2}\%$ commission. He invests the net proceeds in silks at \$1.95 a yard, charging $3\frac{3}{4}\%$ commission. How many yards can he buy?

66. A can do a piece of work in 15 days, B in 18 days, C in 21 days, and D in 24 days. How many days will it take all of them together to do the work? If they receive the sum of \$79.95 for the work, how should the money be divided?

67. Find the cost of a draft on Chicago for \$1500, due 60 days after sight, with interest at $3\frac{1}{2}\%$, if exchange is at $1\frac{1}{4}\%$ discount.

68. If five men can do a piece of work in 4 d. 5 h. 21 min. 45 sec., how long will it take nine men to do the work?

69. Find the exact interest of \$769.50 from June 11, 1891, to March 23, 1892, at $3\frac{1}{4}\%$.

70. Express 0.79839 lb. troy in lower denominations.

71. If \$14513.75 is realized from the sale of \$17000 worth of bonds, including a brokerage of $\frac{1}{8}\%$, what is the quoted price of the bonds?

72. Find the last term and the sum of the terms of the arithmetical progression $\frac{4}{9}$, $\frac{7}{12}$, $\frac{13}{18}$, etc., to 63 terms.

73. Reduce $\frac{80783}{415661}$ to its lowest terms.

74. Find the last term and the sum of the terms of the geometrical progression $1\frac{7}{9}$, $2\frac{2}{3}$, 4, etc., to 10 terms.

75. Add 6 sq. mi. 313 A. 152 sq. rd. 21 sq. yd. 8 sq. ft. 137 sq. in., 13 sq. mi. 602 A. 67 sq. rd. 14 sq. yd. 3 sq. ft. 122 sq. in., 34 sq. mi. 447 A. 112 sq. rd. 9 sq. yd. 5 sq. ft. 64 sq. in., and 19 sq. mi. 296 A. 89 sq. rd. 28 sq. yd. 7 sq. ft. 98 sq. in.

76. What sum must be invested in a $6\frac{1}{4}\%$ stock at 117, brokerage $\frac{1}{8}\%$, to yield an annual income of \$487.50?

77. Find the price in pounds, shillings, pence, and farthings, of an article worth \$21.84, if the sovereign be worth \$$4.87\frac{1}{2}$.

78. A sum of money was divided between A, B, C, and D, in such a way that A received $\frac{5}{21}$, B $\frac{7}{30}$, C $\frac{11}{35}$, and D the remainder, which was \$35.55. What was the sum divided, and how much did each receive?

79. If a bushel $= 1\frac{1}{4}$ cu. ft., what must be the depth of a bin 5 ft. 4 in. long, and 4 ft. 9 in. wide, to hold 98 bushels of grain?

80. Express $\frac{473}{625}$ and $\frac{109}{1280}$ as decimals, and find the product of the results.

81. Find the proceeds of a note for \$875, dated Nov. 25, 1892, payable 60 days after date, and bearing interest at 4%, if discounted Dec. 11, 1892, at $5\frac{1}{2}\%$.

82. Find the cost of a draft on Boston for \$ 2875, due 90 days after sight, with interest at 5%, if exchange is at $2\frac{3}{8}$% premium.

83. If a man can do a piece of work in $8\frac{3}{5}$ days, working 10 h. 55 min. a day, how many days will it take him, working 8 h. 36 min. a day?

84. At what rate per cent will \$ 1125 gain \$ 198.75 from Sept. 20, 1885, to June 5, 1890?

85. A merchant sold goods for \$ 658.35, losing $21\frac{3}{7}$% of what they cost him. At what price should the goods have been sold so as to gain $13\frac{1}{3}$%?

86. What is the duty, at 40% ad valorem, on an importation of crockery, invoiced at £ 896 5*s.* 6*d.*, if the pound sterling be valued at \$ 4.8665?

87. Express 225 rd. 4 yd. 2 ft. $6\frac{13}{17}$ in. as a fraction of a mile.

88. A gentleman left .3 of his property to his wife, .4 of the remainder to his son, .65 of the remainder to his daughter, and the balance, \$ 1543.50, to charitable institutions. How much did each receive?

89. How many shares of stock, at a premium of $13\frac{1}{2}$%, can be bought for \$ 93854.25, including a brokerage of $\frac{1}{8}$%?

90. What is the equated time of paying \$ 519 due in 30 days, \$ 348 due in 60 days, \$ 497 due in 90 days, and \$ 286 due in 120 days?

91. Arrange in order of magnitude $\frac{29}{42}$, $\frac{53}{78}$, and $\frac{187}{273}$.

92. How long must \$ 926.50 be on interest at $2\frac{1}{2}$% to amount to \$ 1200?

93. Divide 35 d. 17 h. 41 min. 59 sec. by 2.84.

94. How many cubic feet are there in a tapering column 17 ft. in height, whose lower base is a square 16 in. on a side, and upper base 14 in. on a side?

95. Subtract $39\frac{109}{117}$ from $57\frac{55}{104}$, and reduce the result to its lowest terms.

96. Find the cost of 384 boards, each 12 ft. 10 in. long, 7 in. wide, and $\frac{7}{8}$ in. thick, at \$18.75 per M.

97. The local time at two places on the equator differs by 9 h. 37 min. 33 sec. What is the distance between them in miles, if a degree of the equator be taken as $69\frac{1}{6}$ miles?

98. Find the L. C. M. of 4199, 7429, and 12673.

99. If a cubic foot of water weighs 1000 oz., and a cubic inch of gold weighs $\frac{17}{24}$ lb., what is the specific gravity of gold?

100. Find the cost of 938 shares of railway stock at $125\frac{3}{4}$, brokerage $\frac{1}{8}\%$.

101. A tank contains 8 cu. yd. 21 cu. ft. 1048 cu. in. of water. If it can be emptied by a pipe in 1 h. $24\frac{2}{5}$ min., how many cubic inches pass through the pipe in one second?

102. Divide 87843 into parts proportional to $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$, $4\frac{1}{5}$, and $5\frac{1}{6}$.

103. Find the amount and present worth of an annuity of \$540 for 8 years, at $4\frac{3}{4}\%$ simple interest.

104. Extract the square root of $\frac{83}{112}$ to six places of decimals.

105. How much above cost must a tradesman mark an article costing \$24.64, so as to be able to sell it at a discount of $8\frac{1}{3}\%$ from the list price, and still make a profit of $15\frac{5}{8}\%$?

106. $\frac{65}{171}$ of $3\frac{185}{148}$ is $1\frac{29}{209}$ times what number?

107. What is the length of the longest straight line that can be drawn on a square floor whose area is 237 sq. ft. 97 sq. in.?

108. Express $83\frac{167}{768}$ lb. troy in grains.

109. What will be the face of a draft, due 30 days after sight, with interest at $4\frac{1}{2}\%$, which can be bought for \$1000, when exchange is at a discount of $3\frac{1}{2}\%$?

110. Simplify $\dfrac{5.\dot{3} - 2.45 + 7.8\dot{3}}{1.61\dot{3} \div 6.05}$.

111. Subtract 7 times £136 19*s.* 10*d.* 1 far. from 12 times £87 6*s.* 3*d.* $2\frac{1}{2}$ far.

112. What is the average time of paying \$186 due March 12, \$155.25 due April 7, \$414 due May 29, and \$258.75 due July 20?

113. What principal at 5% interest will amount to \$600 in 1 y. 11 mo. 5 d.?

114. If $18\frac{3}{4}\%$ is gained by selling goods for \$126.54, what per cent would be lost by selling them for \$76.96?

115. What must be the face of a note due 90 days hence, which, when discounted at $5\frac{1}{4}\%$, will yield \$850?

116. Express 3 lb. 9 oz. 17 pwt. 14 gr. in avoirdupois weight.

117. Simplify $\dfrac{19}{5 + \dfrac{17}{9 + \dfrac{11}{13}}}$.

118. What is the compound interest of \$1260 for 9 mo. at 7%, interest being compounded quarterly?

119. What is the face of a sight draft which can be bought for \$869.35, when exchange is at a premium of $\frac{5}{8}\%$?

120. If a tank 6 ft. 9 in. long, 3 ft. 4 in. wide, and 2 ft. 2 in. deep, holds $365\frac{5}{8}$ gallons of water, how deep must a tank be that is 5 ft. 10 in. long, and 3 ft. 9 in. wide, to hold $437\frac{1}{2}$ gallons of water?

121. Simplify $\dfrac{.095 \div .0005}{.0087 \div .3} \div \dfrac{3.42 \div .006}{.000812 \div .04}$.

122. I bought 5 A. 136 sq. rd. of land at the rate of \$1200 an acre, and sold it at a profit of 21%. At what price per square foot did I sell it?

123. Find the G. C. D. of $62\frac{2}{65}$, $55\frac{32}{85}$, $47\frac{187}{221}$, and $63\frac{63}{143}$.

124. A town whose taxable property is valued at \$975400, wishes to raise \$14130.25. There are 479 polls, each paying \$1.50. What is the rate of taxation? What tax will be paid by an individual who pays for 3 polls, and has taxable property to the amount of \$8600?

125. If a bushel contains 2150.42 cu. in., what must be the depth of a cylindrical measure 12 in. in diameter, to hold a peck?

126. What principal, at 6% compound interest, will gain \$325 in 2 y. 2 mo., interest being compounded semi-annually?

127. Multiply together $2\frac{117}{294}$, $2\frac{98}{275}$, $2\frac{164}{243}$, and $2\frac{125}{256}$.

128. Express $\frac{8}{18}$ sq. mi. in lower denominations.

129. Find the cost of a draft on Paris for 2016.80 francs, exchange on France being quoted at $5.17\frac{1}{4}$.

130. If the weight of a cubic foot of water is $62\frac{1}{2}$ lb., how many cubic inches of mercury (specific gravity 13.596) does it take to weigh 103 lb.?

131. Express .07593 cd. in cubic inches.

132. Extract the cube root of $\frac{49}{72}$ to four places of decimals.

133. A triangular plot of ground contains 2.875 A. If its altitude is $25\frac{1}{11}$ rods, what is its base in feet?

134. Express 11 oz. 13 dr. as a decimal of a pound troy.

135. Express 5 lb. 13 oz. 7 dr. in troy weight.

136. What principal at $7\frac{1}{2}\%$ interest will gain \$52.75 from Feb. 8, 1888, to Jan. 4, 1892?

137. What common fraction will produce $.3\dot{8}07692\dot{3}$?

138. Express 9 cu. ft. 771.3792 cu. in. as a decimal of a cord.

139. How many spherical bullets, each $\frac{1}{2}$ in. in diameter, can be formed from five pieces of lead, each in the form of a cone, whose altitude is 21 in., and radius of base 4 in.?

140. Find the L. C. M. of $\frac{208}{1155}$, $\frac{646}{1575}$, $\frac{988}{2625}$, and $\frac{1768}{5145}$.

141. A provision dealer uses a false measure of 3 pk. 7 qt. instead of a bushel. What per cent does he gain by his dishonesty? What per cent do his customers lose?

142. How much will be realized from the sale of 354 shares of mining stock at a discount of $27\frac{1}{8}\%$, brokerage $\frac{1}{4}\%$?

143. A man travelled $69\frac{4}{5}$ miles the first day, and on each succeeding day three-fifths as many miles as on the next preceding. How far did he travel on the 8th day? How far did he travel in 8 days?

144. If 69.84 pounds of coffee can be bought for \$30.55½, how many pounds can be bought for \$76.89½?

145. A merchant owes \$2143.50 due in 10 months. If he pays \$425 in 3 months, \$580 in 5 months, and \$278.50 in 8 months, when should he pay the balance?

146. If $\frac{7}{81}$ of the price received for an article is gain, what is the gain per cent?

147. Multiply 11 T. 17 cwt. 81 lb. 9 oz. 13 dr. by $\frac{38}{23}$.

148. If a gallon contains 231 cu. in., what must be the diameter of a cistern whose depth is 9 ft., to hold 1080 gallons?

149. Find the cost of a bill of exchange on Liverpool for £224 16*s*., when exchange on England is quoted at $4.86\frac{1}{4}$.

150. If the pint liquid measure contains 28.875 cu. in., how many cubic feet are there in a barrel of $31\frac{1}{2}$ gallons?

151. Extract the sixth root of 177210755.074809.

152. A circular garden, 85 ft. in diameter, is surrounded by a walk 6 ft. wide. How many square yards are there in the walk?

153. If 30 men can dig a trench 108 feet long, $8\frac{3}{4}$ feet wide, and 9 feet deep, in $10\frac{4}{5}$ days of $6\frac{2}{3}$ hours each, how many days of 8 hours each will it take 24 men to dig a trench 96 feet long, $12\frac{5}{6}$ feet wide, and 12 feet deep?

154. For what amount must a vessel worth \$ 12325, and her cargo worth \$ 8709.36, be insured at $4\frac{1}{4}\%$, in order that, in case of loss, the owner may recover the value of the vessel and cargo, and the premium?

155. A, B, and C formed a partnership for one year. A put in \$ 925, and at the end of 5 months added \$ 250; B put in \$ 1075, and at the end of 10 months withdrew \$ 325; C put in \$ 1250, and at the end of 8 months withdrew \$ 475. They gained \$ 1337. What was each partner's share of the gain?

156. What annuity to continue for 13 years, at $3\frac{1}{2}\%$ simple interest, can be purchased for \$ 6292?

157. What amount is due April 2, 1891, on a note for \$ 458, dated April 24, 1886, with $4\frac{1}{2}\%$ interest payable annually, on which no payments have been made?

158. Find the lower base in rods of a trapezoid whose area is 66885 sq. in., altitude $16\frac{1}{4}$ ft., and upper base $7\frac{3}{4}$ yd.

159. Express 2 sq mi. in square inches.

160. Simplify $\dfrac{\frac{3}{8} \text{ of } 22\frac{1}{2}}{(1\frac{5}{6} \div 2\frac{1}{5}) - \frac{1}{3}} \times \dfrac{8\frac{2}{9} \div \frac{2}{3}}{69\frac{3}{8} \div \frac{3}{16}} - \frac{23}{54}$.

161. Add together $\frac{129}{286}$, $\frac{151}{462}$, $\frac{277}{546}$, and $\frac{443}{2002}$, and reduce the result to its lowest terms.

162. At what per cent below par must a $4\frac{1}{4}\%$ stock be quoted, to yield the same per cent on the investment as a $5\frac{3}{4}\%$ stock at a premium of $23\frac{1}{2}\%$, brokerage in each case $\frac{1}{8}\%$?

163. Find the amount and present worth of an annuity of \$500 for 4 years, at 5% compound interest.

164. Simplify $\dfrac{.\dot{8}-.\dot{3}\dot{9}}{.\dot{6}\dot{9}+.\dot{2}}+\dfrac{.\dot{3}+.1\dot{8}\dot{1}}{.\dot{5}-.24\dot{6}}$, and reduce the result to a mixed number.

165. A grocer bought 36 bu. 0 pk. 3 qt. of nuts, at \$3.20 a bushel, and sold them at 12 cents a liquid quart. If a quart liquid measure contains $57\frac{3}{4}$ cu. in., and a quart dry measure $67\frac{1}{5}$ cu. in., did he gain or lose, and how much?

166. Express 6 sq. rd. 19 sq. yd. 3 sq. ft. 57 sq. in. as a decimal of 9 sq. rd. 7 sq yd. 2 sq. ft. 12 sq. in.

167. Simplify $\dfrac{(\frac{18}{85}\text{ of }18\frac{8}{9})+8\frac{5}{6}-4\frac{7}{16}}{4\frac{1}{3}-(3\frac{15}{16}\div 14\frac{7}{40})+1\frac{1}{9}}$.

168. If the specific gravity of lead is 11.4, and a cubic foot of water weighs 1000 oz., find the weight in pounds of a sphere of lead 11 in. in diameter.

169. What amount is due Oct. 7, 1892, on a note for \$2725, dated Nov. 18, 1891, and bearing interest at 4%, on which the following payments have been made: Jan. 11, 1892, \$520; April 17, 1892, \$790; June 25, 1892, \$655; and Aug. 12, 1892, \$480?

170. A man sells $4\frac{1}{2}\%$ stock to the par value of \$65400 at $92\frac{1}{4}$, and invests the proceeds in $3\frac{3}{4}\%$ stock at $81\frac{5}{8}$, brokerage $\frac{1}{8}\%$ on each transaction. Is his income increased or diminished, and how much?

171. A tapering hollow iron column, one inch thick, is 24 ft. long, 10. in. in outside diameter at the larger end, and 8 in. in diameter at the smaller. Find its weight, if a cubic inch of the metal weighs .27 lb.

172. Simplify $\dfrac{5\frac{1}{3} \times 2\frac{7}{12}}{8\frac{6}{7} \times 1\frac{1}{2}} + \dfrac{1\frac{5}{12}}{\frac{3}{5} \text{ of } 1\frac{8}{9}} - (2\frac{1}{12} \div 2\frac{11}{17})$.

173. A cistern can be filled by three pipes in 9 h. 20 min., 8 h. 45 min., and 12 h. 36 min., respectively. How many hours and minutes will it take to fill the cistern if all the pipes are opened together?

174. A railway embankment, 226 rods in length, is 8 ft. 6 in. wide at the top, 21 ft. 6 in. wide at the bottom, and 6 ft. 4 in. high. How many cubic yards of earthwork does it contain?

175. On a note for $1000, dated Feb. 24, 1887, and bearing interest at 6%, the following indorsements were made: Jan. 5, 1888, $175; May 16, 1889, $30; Dec. 8, 1889, $250; Aug. 28, 1890, $200. How much was due March 19, 1891?

176. If a rail weighs 75 pounds to the yard, how many tons of rails will be required to lay a piece of double track railway 11 mi. 272 rd. in length?

177. A train leaves A for B, 44 miles distant, travelling at the rate of a chain in $1\frac{1}{5}$ seconds. Eighteen minutes later a train leaves B for A, travelling at the rate of $\frac{2}{3}$ of a chain a second. How many miles from B will they meet?

178. Simplify $\dfrac{(\frac{44}{63} \div \frac{40}{369}) + (5\frac{13}{15} \div 1\frac{1}{27})}{(6\frac{17}{18} \times 3\frac{3}{20}) - (4\frac{11}{16} \times 2\frac{2}{9})}$.

179. A room 22 ft. 4 in. long, 15 ft. 9 in. wide, and 9 ft. 6 in. high, has three doors, each 3 ft. wide and 6 ft. 9 in. high, four windows, each 3 ft. wide and 5 ft. $5\frac{1}{3}$ in. high, and is surrounded by a base-board 9 in. wide. How much will it cost to plaster it, at 42 cents a square yard? How much will it cost to paper it with paper 21 in. wide, 10 yards to a roll, at $1.12 a roll?

180. What annuity to continue for 5 years, at 3% compound interest, can be purchased for $1000?

181. Find the equated time for paying the balance of the following account:

Dr. WILLIAM LEWIS. *Cr.*

1889			1889		
Oct. 29	To Mdse. 30 d.	$500	Dec. 20	By Cash.	$750
Nov. 24	" "	250	1890		
1890			Jan. 19	" Draft, 90 d.	600
Jan. 6	" " 2 mo.	600	Feb. 9	" " 1 mo.	450
Feb. 27	" " 60 d.	1000	Mar. 28	" Cash.	200

MISCELLANEOUS EXAMPLES INVOLVING THE METRIC SYSTEM.

378. **1.** Express $.00527^{dl}$ of water in cubic centimeters, and find its weight in dekagrams.

2. Divide $.0072321273^{cu\ dm}$ by 7.489, and express the result as a decimal of a cubic millimeter.

3. Find the altitude in dekameters of a trapezoid whose area is $149878^{sq\ cm}$, and bases $.05787^{Hm}$ and 29.65^{dm}, respectively.

4. A wood-pile is 32 ft. long, 4 ft. wide, and 5 ft. 6 in. high. How much is it worth, at $2.50 a ster?

5. If a train travels at the rate of 36 miles an hour, what is its rate in hektometers a minute?

6. Find the weight in dekagrams of a ream of paper, each sheet 24^{cm} long and 15^{cm} wide, if a sheet of the same thickness 125^{mm} long and 8^{cm} wide weighs 8.2^{dg}.

7. Express 2 mi. 223 rd. 4 yd. 1 ft. 9 in. in kilometers.

8. How much do I lose by buying 5 A. 135 sq. rd. of land at $300 an acre, and selling it at $675 a hektar?

9. Express 2 gal. 3 qt. 1 pt. 3 gi. in deciliters.

10. How many chains are there in a hektometer?

11. How many decimeters are there in a link?

12. A rectangular garden, 35.4^{m} long and $.289^{Hm}$ wide, is surrounded by a walk 19.6^{dm} in width. Find the area of the walk in ars.

13. Express $.874^{Kl}$ in liquid measure, and also in dry measure.

14. If it costs $9.54 to travel 397½ miles by rail, what is the rate of fare in cents per kilometer?

15. How many hektoliters of grain can be put into a receptacle 4.38^{m} long, 27.9^{dm} wide, and 185^{cm} deep? How many myriagrams of water can be put into it?

16. How many bricks, each 2.3^{dm} long, 86^{mm} wide, and 5.04^{cm} thick, will be required to build a wall $.645^{Hm}$ long, $.46^{m}$ wide, and 1.89^{Dm} high?

17. Find the value, at 12 cents a square meter, of a circular piece of land, whose circumference is one hektometer.

18. How many silver half-dollars can be coined from ten bars of silver, each 55^{cm} long, 36^{mm} wide, and 25^{mm} thick, if each coin weighs 12.5^{g}, and the specific gravity of silver is 10.5?

19. A merchant imports 3500^{m} of silk, invoiced at 6.5 francs a meter, and sells it at $1.55 a yard. If the franc is valued at 19.3 cents, how much does he gain?

20. If a cubic foot of granite weighs 167 lb., what is the weight of a cubic meter in metric tons?

21. How much will it cost to cover a floor 6.68^{m} long and 5.84^{m} wide, with carpeting 78^{cm} wide, at 75 cents a meter, if the strips run lengthwise of the room? How much if the strips run across the room?

22. Find the length in hektometers of the longest straight line that can be drawn in a rectangular field, whose area is $14.44908^{sq\,Dm}$, and width $.02776^{Km}$.

23. Find the volume in cubic decimeters of a frustum of a square pyramid, whose altitude is 825^{mm}, lower base $.28^{m}$ on a side, and upper base 22.6^{cm} on a side.

24. How many yards of fence will be required to enclose a circular grass-plot whose area is 6.157336^{a}?

25. A wood-pile is 8.2^{m} long, 14^{dm} wide, and 165^{cm} high. Find its value at $6.70 a cord.

26. A cubical tank holds 563975.2^{Dg} of water; what is its depth in dekameters?

27. How many dekaliters of petroleum (specific gravity .8778) does it take to weigh 84663.81^{Hg}?

28. What is the depth in decimeters of a cylindrical tank, 83.5^{cm} in diameter, which holds 9856809.27^{dg} of water?

29. Express 9.38^{Ha} in ordinary square measure.

30. A ditch is 142^{dm} long, 18.3^{cm} wide, and 876^{mm} deep. How many metric tons of water will it contain?

31. A rod of steel (specific gravity 7.8) is 5.5^{m} long, and 18^{mm} in diameter. Find its weight in hektograms.

32. Express 15 lb. 9 oz. 17 pwt. 21 gr. in dekagrams.

33. A tank is 2.28^{m} long, 1.75^{m} wide, and 1.625^{m} deep. How many kilograms of oil (specific gravity .898) will it contain?

34. What is the surface in square decimeters of a sphere whose volume is $2572446.8^{cu\ mm}$?

35. A cubical piece of lead, whose entire surface is $13.5^{sq\ dm}$, is melted, and formed into a cone the radius of whose base is 15^{cm}. Find the altitude of the cone in meters.

36. What is the diameter in dekameters of a cylindrical cistern 32^{dm} deep, which holds 769.692^{Dl} of water?

37. The volume of a cone is $33250.6944^{cu\ cm}$, and its altitude is 7.2^{dm}. Find the radius of its base in meters, and its lateral area in square dekameters.

38. What is the depth in millimeters of a tank that is 188^{cm} long and 12.5^{dm} wide, and holds 2.5949546^{T} of alcohol (specific gravity .791)?

39. How many cubic centimeters of metal are there in a hollow iron tube of uniform diameter, whose length is 3.2^{m}, thickness 2^{mm}, and outside diameter 1.2^{cm}?

40. A bar of aluminum, 9.2^{dm} long and 2.5^{cm} in diameter, weighs 11.6062485^{Hg}. What is its specific gravity?

41. Express $.378^{Mg}$ in avoirdupois, and in troy weight.

42. If a cubic decimeter of silver weighs 104.93^{Hg}, what is the weight of a cubic foot in pounds avoirdupois?

43. A river, whose current flows at the rate of 6.4^{Km} an hour, is $.16^{Hm}$ deep and 84.6^{Dm} wide. How many metric tons of water pass a given point in 43 min. 20 sec.?

44. If a rail weighs 36^{Kg} to the meter, how many pounds does it weigh to the yard?

45. A tank containing 20457.2^{Hg} of water has two taps. One fills it at the rate of $342^{cu\ cm}$ a second, and the other empties the contents at the rate of a dekaliter in .9765625 minutes. How many hours and minutes will it take to fill the tank if both taps are opened?

46. The cross-section of a tunnel 2.5^{Km} in length, is in the form of a rectangle 4.2^{m} wide and 3.8^{m} high, surmounted by a semicircle, whose diameter is equal to the width of the rectangle. How much did it cost to excavate it, at $ 6 a cubic meter?

47. A cannon is in the form of a frustum of a cone, joined to a hemisphere whose diameter is equal to that of the larger end of the cannon. The diameter of the larger end of the cannon is 4.2^{dm}, and of the smaller end 32^{cm}; and its entire length is 2.1^{m}. The interior bore of the cannon is 24^{cm} in diameter, and 2^{m} deep. How many cubic decimeters of metal were used in its construction?

APPENDIX.

MEASURES.

Measures of Length.

A furlong	= 40 rods.
A league	= 3 miles.
A fathom	= 6 feet.
A cable-length	= 120 fathoms.

A *nautical mile, geographical mile,* or *knot,* is equal to 1.15 miles.

A *marine league* is equal to 3 nautical miles.

A line	= $\frac{1}{12}$ inch.
A hand	= 4 inches.
A palm	= 3 inches.
A span	= 9 inches.
A cubit	= 18 inches.

Cloth Measure.

$2\frac{1}{4}$ inches	= 1 nail.
4 nails	= 1 quarter of a yard.
4 quarters	= 1 yard.
3 quarters	= 1 Ell Flemish.
5 quarters	= 1 Ell English.

Measures of Area.

A rood	= 40 square rods.
A square	= 100 square feet.

Measures of Volume.

A cord foot	= 16 cubic feet.
A perch	= $24\frac{3}{4}$ cubic feet.

A perch of masonry is $16\frac{1}{2}$ ft., or 1 rod long, $1\frac{1}{2}$ feet wide, and 1 foot high.

Measures of Weight.

A quarter	$= 25$	pounds av.
A stone	$= 14$	pounds av.
A pig	$= 21\frac{1}{2}$	stone.
A fother	$= 8$	pigs.

Diamond Weight.

4 quarters	$=1$	grain.
4 grains	$=1$	carat.

A grain, diamond weight, is equal to four-fifths of a grain troy, and a carat is equal to $3\frac{1}{5}$ grains troy.

Measures of Time.

A *solar year* is the time required for the sun, after leaving either equinox, to return to it again; and is 365 days, 5 hours, 48 minutes, and 49.7 seconds.

A *sidereal year* is the time required for the earth to make a complete revolution about the sun; and is 365 days, 6 hours, 9 minutes, and 9.6 seconds.

A *common, civil,* or *legal year* is one of 365 days; and a leap, or bissextile year is one of 366 days.

By the *Julian Calendar,* instituted by Julius Cæsar, every fourth year is made a leap year; which produces an error, as compared with the solar year, of 44 minutes and 41.2 seconds in four years, or about one day in 129 years.

By the *Gregorian Calendar,* instituted by Pope Gregory XIII., in 1582, those years only are leap years whose numbers are divisible by 4, and not by 100, unless they are also divisible by 400.

The error of the Gregorian Calendar amounts to about one day in 3866 years.

The reckoning of time by the Julian Calendar is termed *Old Style*, and by the Gregorian Calendar *New Style*.

The latter is used by most civilized nations, and was adopted by England in 1752; in that year, by Act of Parliament, the day following Sept. 2d was counted as Sept. 14.

The present difference between the two styles is 12 days.

To find the Difference in Time between Two Dates.

The following method is largely used by business men:

The year is regarded as consisting of 12 months of 30 days each.

The months are represented by their *numbers;* thus, January is the first month, February the second, etc.

The difference in time is then found as in subtraction of denominate numbers.

Example. Find the difference in time between Oct. 15, 1883, and June 7, 1892.

1892	6	7
1883	10	15
8 y.	7 mo.	22 d., *Ans.*

June is the sixth month of the year, and October the tenth month.
15 d. from 37 d. leaves 22 d.
10 mo. from 17 mo. leaves 7 mo.
1883 from 1891 leaves 8 y.

Then the required result is 8 y. 7 mo. 22 d.

In some States, *the actual number of days* in the preceding month must be used, when the number of days in the subtrahend time is greater than the number in the minuend time.

Thus, in the above example, the month preceding June has 31 days.

We should then say, 15 d. from 38 d. leaves 23 d.; and the time would be 8 y. 7 mo. 23 d.

Comparison of Thermometers.

In the **Fahrenheit Scale** (F.), the temperature of the freezing point of water is marked 32°, and the temperature of the boiling point 212°; the difference being 180°.

In the **Centigrade Scale** (C.), the freezing point is marked 0°, and the boiling point 100°.

In the **Réaumur Scale** (R.), the freezing point is marked 0°, and the boiling point 80°.

Thus, $1°$ F. $= \frac{100}{180}$, or $\frac{5}{9}°$ C., and $\frac{80}{180}$, or $\frac{4}{9}°$ R.
$1°$ C. $= \frac{180}{100}$, or $\frac{9}{5}°$ F., and $\frac{80}{100}$, or $\frac{4}{5}°$ R.
$1°$ R. $= \frac{180}{80}$, or $\frac{9}{4}°$ F., and $\frac{100}{80}$, or $\frac{5}{4}°$ C.

1. Express 59° F. in the Centigrade scale, and in the Réaumur scale.

59° F. is 27° above the freezing point.
But 27° F. is equal to $27 \times \frac{5}{9}$, or 15° C., or to $27 \times \frac{4}{9}$, or 12° R.
Hence, 59° F. is equivalent to 15° C., or to 12° R., *Ans.*

2. Express 35° C. in the Fahrenheit scale, and in the Réaumur scale.

35° C. is 35° above the freezing point.
But 35° C. is equal to $35 \times \frac{9}{5}$, or 63° F., or to $35 \times \frac{4}{5}$, or 28° R.
Hence, 35° C. is equivalent to 95° F., or to 28° R., *Ans.*

3. Express −18° R. in the Fahrenheit scale, and in the Centigrade scale.

−18° R. is 18° below the freezing point.
But 18° R. is equal to $18 \times \frac{9}{4}$, or $40\frac{1}{2}°$ F., or to $18 \times \frac{5}{4}$, or $22\frac{1}{2}°$ C.
Hence, −18° R. is equivalent to $-8\frac{1}{2}°$ F., or to $-22\frac{1}{2}°$ C., *Ans.*

EXAMPLES.

Express each of the following in the Centigrade, and in the Réaumur scale:

4. 77° F. **5.** 140° F. **6.** 8° F. **7.** −34° F.

Express each of the following in the Fahrenheit, and in the Réaumur scale:

8. 55° C. **9.** 70° C. **10.** −12°. **11.** −25° C.

Express each of the following in the Fahrenheit, and in the Centigrade scale:

12. 52° R. **13.** 25° R. **14.** −10° R. **15.** −22° R.

Miscellaneous Terms.

A sheet of paper folded into:

2 leaves, forms a *folio;*

4 leaves, forms a *quarto,* or 4to;

8 leaves, forms an *octavo,* or 8vo;

12 leaves, forms a *duodecimo,* or 12mo;

18 leaves, forms an 18mo;

24 leaves, forms a 24mo.

MONEY AND COINS.

United States Money.

For the denominations of United States money, see Art. 142.

The *coins* of the United States are as follows:

Gold; the *quarter-eagle, half-eagle, eagle,* and *double-eagle.*

Silver; the *dime, quarter-dollar, half-dollar,* and *dollar.*

Nickel; the *five-cent piece.*

Bronze; the *cent.*

Note. The coinage of gold dollars, gold three-dollar pieces, and nickel three-cent pieces, was suspended by act of Congress, approved Sept. 26, 1890.

The monetary system of Canada is the same as that of the United States.

English Money.

For the denominations of English money, see Art. 154.

The coins of Great Britain are:

Gold; the *half-sovereign* and *sovereign.*

Silver; the *three-penny piece, six-pence, shilling, florin, half-crown,* and *crown.*

Copper; the *half-penny* and *penny.*

Note. The silver four-penny piece is no longer coined.

The value of the pound sterling in United States money is $4.8665.

French Money.

100 centimes (*c.*) = 1 franc. (*f.*)

The coins of France are:

Gold; the *five-franc, ten-franc, twenty-franc, forty-franc,* and *hundred-franc* pieces.

Silver; the *franc, two-franc,* and *five-franc* pieces, and the *twenty-five centime* and *fifty-centime* pieces.

Bronze; the *one-centime, two-centime, five-centime,* and *ten-centime* pieces.

Note. The gold hundred-franc piece is called a *Napoleon.*

The monetary systems of Belgium and Switzerland are the same as that of France.

The value of the franc in United States money is 19.3 cents.

German Money.

100 pfennigs (*pf.*) = 1 mark. (*m.*)

The coins of Germany are:

Gold; the *five-mark, ten-mark,* and *twenty-mark* pieces.

Silver; the *one-mark, two-mark,* and *three-mark* pieces; and the *twenty-pfennig* and *fifty-pfennig* pieces.

Nickel; the *five-pfennig* and *ten-pfennig* pieces.

Bronze; the *one-pfennig* and *two-pfennig* pieces.

Note. The silver three-mark piece is called a Thaler.

The value of the mark in United States money is 23.8 cents.

The following table gives the values, in United States money, of *Foreign Coins,* as proclaimed by the Secretary of the Treasury, Oct. 1, 1891:

Country.	Monetary Unit.	Value in U. S. Money.
Argentine Republic.	Peso.	$0.965.
Austria-Hungary.	Florin.	.357.
Belgium.	Franc.	.193.
Bolivia.	Boliviano.	.723.
Brazil.	Milreis.	.546.
British Possessions, N. A., except Newfoundland.	Dollar.	1.00.
Central American States; Costa Rica, Guatemala, Honduras, Nicaragua, Salvador.	Peso.	.723.
Chili.	Peso.	.912.
China.	Tael, Shanghai.	1.068.
	" Haikwan (customs).	1.189.
Colombia.	Peso.	.723.
Cuba.	Peso.	.926.
Denmark.	Crown.	.268.
Ecuador.	Sucre.	.723.
Egypt.	Pound (100 piasters).	4.943.
Finland.	Mark.	.193.
France.	Franc.	.193.
German Empire.	Mark.	.238.
Great Britain.	Pound sterling.	4.8665.
Greece.	Drachma.	.193.
Hayti.	Gourde.	.965.
India.	Rupee.	.343.
Italy.	Lira.	.193.
Japan.	Yen (gold).	.997.
	" (silver).	.779.
Liberia.	Dollar.	1.00.
Mexico.	Dollar.	.785.
Netherlands.	Florin.	.402.
Newfoundland.	Dollar.	1.014.
Norway.	Crown.	.268.
Peru.	Sol.	.723.
Portugal.	Milreis.	1.08.
Russia.	Rouble.	.578.
Spain.	Peseta.	.193.
Sweden.	Crown.	.268.
Switzerland.	Franc.	.193.
Tripoli.	Mahbub of 20 piasters.	.652.
Turkey.	Piaster.	.044.
Venezuela.	Bolivar.	.145.

Note. The *francs* of Belgium, France, and Switzerland, the *mark* of Finland, the *drachma* of Greece, the *lira* of Italy, and the *peseta* of Spain, have all the same value.

The *crowns* of Denmark, Norway, and Sweden have all the same value.

The *boliviano* of Bolivia, the *sucre* of Ecuador, the *sol* of Peru, and the *pesos* of Costa Rica, Guatemala, Honduras, Nicaragua, Salvador, and Colombia, have all the same value.

LEGAL RATES OF INTEREST.

The **Legal Rate** of interest is the rate which is established by law.

The following table gives the legal rate of interest in each state and territory of the Union.

When no rate is mentioned, the legal rate is that given in the left-hand column; if specified in writing, any rate not exceeding that in the right-hand column is legal.

STATE.	RATE.		STATE.	RATE.		STATE.	RATE.	
Alabama.	8	8	Kentucky.	6	6	Nevada.	7	Any.
Arkansas.	6	10	Louisiana.	5	8	Ohio.	6	8
Arizona.	10	Any.	Maine.	6	Any.	Oregon.	10	12
California.	10	Any.	Maryland.	6	6	Pennsylvania.	6	6
Connecticut.	6	6	Massachusetts.	6	Any.	Rhode Island.	6	Any.
Colorado.	8	Any.	Michigan.	7	10	South Carolina.	7	8
Dakota.	7	12	Minnesota.	7	10	Tennessee.	6	6
Delaware.	6	6	Mississippi.	6	10	Texas.	6	10
Florida.	8	10	Missouri.	6	8	Utah.	10	Any.
Georgia.	7	8	Montana.	7	Any.	Vermont.	6	6
Idaho.	10	18	N. Hampshire.	6	6	Virginia.	6	6
Illinois.	7	7	New Jersey.	6	6	West Virginia.	6	8
Indian Ter.	6	Any.	New Mexico.	6	Any.	Washington.	8	Any.
Indiana.	6	8	New York.	6	6	Wisconsin.	6	10
Iowa.	6	8	North Carolina.	6	8	Wyoming.	12	Any.
Kansas.	6	8	Nebraska.	7	10	Dist. of Col.	6	10

SPECIAL STATE RULES FOR PARTIAL PAYMENTS.

The Connecticut Rule.

When at least one year's interest has accrued at the time of a payment, and in the case of the last payment, follow the United States Rule (Art. 324).

When less than a year's interest has accrued at the time of a payment, except the last, find the amount of the principal for an entire year, and the amount of the payment for the remainder of the year after it is made, and subtract the amount of the payment from the amount of the principal for a new principal; but if the payment is less than the interest which is due at the time that it is made, no interest is allowed on the payment.

Example. A note for $2000, dated Oct. 8, 1888, and bearing interest at 6%, had the following indorsements: March 2, 1889, $500; Aug. 18, 1890, $20. What was due Dec. 23, 1890?

Solution.

Principal,	$2000.00
Int. for 1 yr.,	120.00
Amount, Oct. 8, 1889,	$2120.00
Amount of 1st payment to Oct. 8, 1889, 7 mo. 6 d.,	518.00
New Principal, Oct. 8, 1889,	$1602.00
Int. for 1 yr.,	96.12
Amount, Oct. 8, 1890,	$1698.12
2d payment,	20.00
New principal, Oct. 8, 1890,	$1678.12
Int. to Dec. 23, 1890, 2 mo. 15 d.,	20.98
Amount due, Dec. 23, 1890,	$1699.10

In the above example, the first payment is made less than a year after the date of the note.

We find the amount of the principal for an entire year to be $2120.

The remainder of the year after the first payment is made is 7 mo. 6 d.; and the amount of the first payment for this time is $518.

Subtracting $518 from $2120, the new principal is $1602.

The amount of this principal for one year is $1698.12.

The second payment, being less than the interest which is due at the time that it is made, draws no interest; then subtracting $20 from $1698.12, the new principal is $1678.12.

The amount of this principal, to Dec. 23, 1890, is $1699.10.

The New Hampshire Rule for Partial Payments on a Note, or other Obligation, drawing Annual Interest.

If in any year, reckoning from the time when the annual interest began to accrue, payments have been made, compute interest on them to the end of the year.

Find also the accrued annual interest on the principal, and any simple interest which may be due upon unpaid annual interest, at the end of the year.

Then the amount of the payment, or payments, is subtracted from the amount due on the note at the end of the year.

But if the payment, or payments, are less than the sum of the simple and accrued annual interests due at the end of the year, no interest is allowed on the payments.

In such a case, simple interest is computed on the balance of interest due, unless the payment, or payments, are less than the simple interest due on unpaid annual interest, in which case the balance of simple interest draws no interest.

Example. A note for $ 2500, dated July 10, 1887, and bearing interest at 6%, had the following indorsements: April 4, 1889, $ 600; May 26, 1890, $ 100. What was due Nov. 25, 1891?

Solution.

Principal,		$ 2500.00
1st Ann. Int., to July 10, 1888,		150.00
2d Ann. Int., to July 10, 1889,		150.00
Int. on 1st Ann. Int. to July 10, 1889,		9.00
Amount due, July 10, 1889,		$ 2809.00
1st payment, April 4, 1889,	$ 600.00	
Int. on 1st payment to July 10, 1889,	9.60	609.60
New principal, July 10, 1889,		$ 2199.40
3d Ann. Int., to July 10, 1890,		131.96
2d payment,		100.00
Bal. of Int. due July 10, 1890,		$ 31.96

Bal. of Int. due July 10, 1890,	$ 31.96
Int. to Nov. 25, 1891,	2.64
New principal, July 10, 1890,	2199.40
4th Ann. Int., to July 10, 1891,	131.96
Int. on principal from July 10, 1891, to Nov. 25, 1891,	49.49
Int. on 4th Ann. Int., to Nov. 25, 1891,	2.97
Amount due, Nov. 25, 1891,	$ 2418.42

In the above example, the first payment is made during the second year after the date of the note.

The accrued annual interest on the principal, at the end of the second year, is $ 300; and the simple interest due upon the unpaid annual interest of the first year is $ 9.

Hence, the amount due on the note, at the end of the second year, is $ 2809.

The first payment is made 3 mo. 6 d. before the end of the second year.

The amount of $ 600, for 3 mo. 6 d., is $ 609.60.

Subtracting this from $ 2809, the new principal at the end of the second year is $ 2199.40.

The second payment is made during the third year after the date of the note.

The annual interest due on the principal at the end of the third year is $ 131.96.

The second payment being less than this, draws no interest; then subtracting $ 100 from $ 131.96, the balance of interest due is $ 31.96.

Simple interest is then reckoned on this balance to Nov. 25, 1891, amounting to $ 2.64.

The annual interest due on the principal at the end of the fourth year is $131.96; and the simple interest due on this, Nov. 25, 1891, is $ 2.97.

The interest due on the principal from July 10, 1891, to Nov. 25, 1891, is $ 49.49.

Adding the last five sums to the new principal, July 10, 1890, the amount due Nov. 25, 1891, is $ 2418.42.

The Vermont Rule.

If in any year, reckoning from the time when the annual interest began to accrue, payments have been made, compute interest on them to the end of the year.

The amount of the payments is then applied:

First, to cancel any simple interest which may be due upon unpaid annual interest.

Second, to cancel the accrued annual interest.

Third, to reduce the principal.

The Vermont Rule is the same as the first three paragraphs of the New Hampshire Rule.

Note. At the option of the teacher, the examples given under the United States Rule (Art. 324) may be performed by the Connecticut Rule, the New Hampshire Rule, or the Vermont Rule.

TO COMPUTE INTEREST ON ENGLISH MONEY.

To compute interest on English money, reduce the shillings, pence, and farthings, if any, to the decimal of a pound, and then proceed as in United States money.

The decimal of a pound in the result should be reduced to shillings, pence, and farthings.

1. Find the interest of £83 13*s.* 9*d.* for 3 y. 6 mo., at 5%.

We have, £83 13*s.* 9*d.* = £83.6875.

The interest of £83.6875 for 3 y. 6 mo. at 5% is £14.6453125.

Reducing £.6453125 to lower denominations, the result is 12*s.* 10*d.* 3.5 far.

Hence, the required interest is £14 12*s.* 10*d.* 3.5 far., *Ans.*

EXAMPLES.

Find the interest and amount:

2. Of £56 5*s.* for 3 y. 11 mo., at $3\frac{1}{2}$%.

3. Of £31 14*s.* 6*d.* for 8 mo. 18 d., at 4%.

4. Of £27 8*s.* 3*d.* for 5 mo. 25 d., at 6%.

5. Of £40 19*s.* 2*d.* 1 far. for 1 y. 1 mo. 10 d., at $4\frac{1}{2}$%.

6. At what rate per cent per annum will £36 17*s.* 4*d.* gain £3 9*s.* 1*d.* 2 far. in 1 y. 6 mo.?

7. In what time will £190 gain £12 2*s.* 3*d.*, at 3%?

8. What principal will gain 17*s.* 7*d.* 2 far. in 1 y. 2 mo. 12 d., at 5%?

BUSINESS FORMS.

Receipt in Full.

$\$271\frac{64}{100}$. *Cincinnati, July 3, 1891.*

Received from Henry Clark two hundred and seventy-one $\frac{64}{100}$ dollars, in full of all demands to date.

Edward H. Perry.

Receipt on Account.

$\$100$. *Buffalo, Nov. 12, 1890.*

Received from James E. Hoyt one hundred dollars on account.

William G. Faxon.

Bank Checks.

A *Check* is a written order addressed to a bank by a person having money on deposit, requesting the payment, on presentation, of a certain sum to the person named therein, or his order.

Form of a Bank Check.

$\$73\frac{25}{100}$. *New York, Feb. 21, 1892.*

The National Park Bank.

Pay to J. H. Crocker, or order, seventy-three $\frac{25}{100}$ dollars.

No. 815. *W. E. Martin & Co.*

A *Certified Check* is one on the face of which the Cashier or Paying Teller of the bank has written the word "Certified," and under it his signature; the bank in this way guarantees the payment of the check.

Form of a Certified Check.

$\$153\frac{80}{100}$. *Boston, Sept. 17, 1891.*

The Market National Bank.

Pay to George F. Jones, or order, one hundred and fifty-three $\frac{80}{100}$ dollars.

Certified.
C. F. Holden,
Cashier.

No. 349. *William Breck.*

Certificates of Deposit.

A *Certificate of Deposit* is a statement made by a bank, certifying that the person named therein has deposited in the bank a specified sum of money.

It is often used in place of a draft in making a remittance.

Form of a Certificate of Deposit.

$\$250\frac{00}{100}$. *Philadelphia, May* 4, 1892.

The National Bank of Commerce.

David A. King has deposited in this bank two hundred and fifty $\frac{00}{100}$ *dollars, to the credit of himself, payable on the return of this certificate, properly indorsed.*

No. 1047. *F. F. Hill, Cashier.*

SAVINGS BANK ACCOUNTS.

A **Savings Bank** receives small sums of money on deposit, paying interest therefor.

Money deposited on or before certain specified dates draws interest from those dates.

Interest is computed either monthly, quarterly, or semi-annually, on the *smallest balance that has been on deposit during the entire term of interest;* but no interest is allowed on any sum which is withdrawn, for the time between the date of its withdrawal and the date of the last dividend; nor is interest allowed on fractional parts of a dollar.

If interest is not drawn when due, it is added to the principal, and draws interest as a new deposit; thus, savings banks pay compound interest.

A depositor in a savings bank receives a bank-book, in which all deposits and amounts withdrawn are entered.

He may usually withdraw his entire deposit, or any portion of it, at any time when he sees fit; but some banks require a week's notice before paying money to a depositor.

Example. In a certain savings bank, money deposited on or before the first days of January, April, July, and October, draws interest from those dates at 4%; the interest being payable on the above dates.

A depositor, whose balance Jan. 1, 1891, was $152.43, deposited on Feb. 3, 1891, $75, on May 20, 1891, $30, and on Aug. 12, 1891, $100.

He drew on April 27, 1891, $46, and on Oct. 7, 1891, $151. What was his balance on Jan. 1, 1892?

Solution.

DATES.	DEPOSITS.	DRAFTS.	INTEREST.	BALANCES.
1891				
Jan. 1				$152.43
Feb. 3	$75.00			227.43
April 1			$1.52	228.95
" 27		$46.00		182.95
May 20	30.00			212.95
July 1			1.82	214.77
Aug. 12	100.00			314.77
Oct. 1			2.14	316.91
" 7		151.00		165.91
1892				
Jan. 1.			1.65	167.56

On making the deposit Feb. 3, the balance becomes $227.43.

On April 1, interest is paid on the smallest balance that has been on deposit since Jan. 1.

No interest being paid on fractional parts of a dollar, the interest due April 1 is 1% of $152, or $1.52; and the balance becomes $228.95.

On April 27, $46 is withdrawn, and on May 20, $30 is deposited, making the balance $212.95.

On July 1, interest is paid on the smallest balance that has been on deposit since April 1, which is $182.95.

1% of $182 is $1.82; which makes the balance July 1 $214.77.

Aug. 12, $ 100 is deposited, and the balance becomes $ 314.77.

On Oct. 1, interest is paid on a balance of $ 214, amounting to $ 2.14; which makes the balance Oct. 1 $ 316.91.

Oct. 7, $ 151 is withdrawn, leaving $ 165.91.

On Jan. 1, 1892, interest is paid on $ 165, amounting to $ 1.65; and the balance due on that date is $ 167.56, *Ans.*

SCALES OF NOTATION.

In the ordinary method of expressing whole numbers, a figure in any place represents a number *ten* times as great as if it stood in the next place to the right.

This method of representing numbers is called the *Common*, or *Decimal Scale of Notation;* and the multiplier 10 is called the *Radix.*

It is possible, however, to represent numbers by taking as a radix any whole number except 1.

The following table gives the name and radix of each of the first eleven scales:

Scale.	Radix.	Scale.	Radix.	Scale.	Radix.
Binary	2	Senary	6	Decimal	10
Ternary	3	Septenary	7	Undenary	11
Quaternary	4	Octary	8	Duodecimal	12
Quinary	5	Nonary	9		

To express a number in any uniform scale, as many distinct symbols are required as there are units in the radix of the given scale.

Thus, in the decimal scale, 10 symbols are required; in the binary scale, 2 symbols, 0 and 1; in the ternary scale, 3 symbols, 0, 1, and 2; the cipher being a symbol in every scale.

In the duodecimal scale, 12 symbols are required, and the numbers 10 and 11 are represented by the symbols *t* and *e*, respectively.

In the decimal scale, a digit in the second place represents tens; in the third place, squares of tens; in the fourth place, cubes of tens; and so on.

Thus, 3548 represents

$$3 \times 10^3 + 5 \times 10^2 + 4 \times 10 + 8.$$

In like manner, in any scale, a digit in the second place represents so many times the radix; in the third place, so many times the square of the radix; and so on.

Thus, in the nonary scale, 7524 represents

$$7 \times 9^3 + 5 \times 9^2 + 2 \times 9 + 4.$$

Example. Write in the senary scale the numbers from 1 to 19 inclusive in the common scale.

The symbols used in the senary scale are 0, 1, 2, 3, 4, 5.

The numbers from 1 to 5 inclusive are expressed in the same way in each scale.

The number 6, being 1 *six* and no *ones*, is expressed 10.

The number 7, being 1 *six* and 1 *one*, is expressed 11; etc.

Result: 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 30, 31.

To change from the Decimal to any other Scale.

Example. Change 77609 to the duodecimal scale.

$$\begin{array}{rl}
12)\underline{77609} & \\
12)\underline{6467} & 5 \\
12)\underline{538} & 11 \text{ or } e \\
12)\underline{44} & 10 \text{ or } t \\
3 & 8
\end{array}$$

$38te5$, *Ans.*

The radix of the duodecimal scale is 12.

Dividing 77609 by 12, the quotient is 6467, and the remainder 5.

That is, $77609 = 6467 \times 12 + 5.$

Dividing 6467 by 12, the quotient is 538, and the remainder 11.

That is, $77609 = (538 \times 12 + 11) \times 12 + 5$
$= 538 \times 12^2 + 11 \times 12 + 5.$

Dividing 538 by 12, the quotient is 44, and the remainder 10.

That is, $77609 = (44 \times 12 + 10) \times 12^2 + 11 \times 12 + 5$
$= 44 \times 12^3 + 10 \times 12^2 + 11 \times 12 + 5.$

Dividing 44 by 12, the quotient is 3, and the remainder 8.

That is, $77609 = (3 \times 12 + 8) \times 12^3 + 10 \times 12^2 + 11 \times 12 + 5$
$= 3 \times 12^4 + 8 \times 12^3 + 10 \times 12^2 + 11 \times 12 + 5.$

Thus, 77609 is expressed by $38te5$ in the duodecimal scale.

ANSWERS.

NOTE. In the following collection of answers, all those are omitted which, if given, would destroy the utility of the example.

Art. 26. Page 11.

1. 12.
2. 52.
3. 32.
4. 94.
5. 101.
6. 179.
7. 100.
8. 153.
9. 506.

Art. 39. Page 17.

1. 834588.
2. 10270000.
3. 3900383.
4. 4054799.
5. 2172500.
6. 4116328.
7. 228958488.
8. 148732062.
9. 433403622.
10. 37709424.
11. 15716910.
12. 602025849.
13. 229480867.
14. 248644665.
15. 512.
16. 598.
17. 1326.
18. 4440.
19. 3304.
20. 21424.
21. 291.
22. 12154.
23. 3907.
24. 255245.
25. 291146.

Art. 50. Page 23.

2. 537.
3. 10503.
4. 507.
5. 983.
6. 6039; 107, Rem.
8. 5062.
9. 42860.
10. 84; 347206, Rem.
11. 319; 72, Rem.
12. 800013.
13. 9284.
14. 86395.
15. 6.
16. 48.
17. 13.
18. 26.
19. 3.
20. 8.
21. 242.
22. 12.
23. 1855.
24. 54.

Art. 66.
Pages 34, 35.

2. 15.
3. 72.
4. 34.
5. 18.
6. 64.
7. 225.
8. 35.
9. 12.
10. 48.
11. 9.
12. 13.
13. 6.
14. 4.
15. 16.
16. 65.
17. 30.
18. 216.
19. 75.
20. 385.
21. 84.
22. 21 feet.
23. 96; in first, 5; in second, 7.

24. 28 inches.
25. 24.
26. 56 square rods.
27. 22 feet.

Art. 67. Page 37.

3. 43.
4. 23.
5. 31.
6. 19.
7. 41.
8. 43.
9. 118.
10. 161.
11. 37.
12. 295.
13. 73.
14. 83.
15. 79.
16. 89.

Art. 68. Page 37.

2. 13.
3. 17.
4. 19.
5. 23.

Art. 73. Page 40.

2. 180.
3. 252.
4. 3360.
5. 432.
6. 330.
7. 2310.
8. 1026.
9. 900.
10. 528.
11. 3872.
12. 5568.
13. 1296.
14. 588.
15. 1008.
16. 1155.
17. 2808.
18. 690.
19. 660.
20. 2592.
21. 8505.
22. 17640.
23. 42120.
24. 42840.
25. 91500.
26. 90.
27. 240 minutes.
28. $1080.
29. 252; A, 28 times; B, 21 times; C, 18 times.

Art. 74. Page 41.

2. 5491.
3. 11951.
4. 17081.
5. 20677.
6. 20071.
7. 6877.
8. 8303.
9. 11339.
10. 12493.
11. 15457.
12. 110837.
13. 485377.

Art. 75. Page 42.

2. 549072.
3. 3174045.

Art. 83. Page 45.

3. $3\frac{6}{13}$.
4. 9.
5. $14\frac{37}{40}$.
6. $47\frac{15}{22}$.
7. $35\frac{37}{38}$.
8. $28\frac{41}{78}$.
9. 46.
10. $98\frac{16}{29}$.
11. $69\frac{32}{45}$.
12. $65\frac{47}{92}$.
13. $91\frac{49}{74}$.
14. 85.
15. $145\frac{44}{85}$.
16. $82\frac{30}{151}$.
17. $70\frac{351}{362}$.
18. $53\frac{263}{380}$.

Art. 84. Page 45.

6. $\frac{396}{18}$.
7. $\frac{285}{13}$.
8. $\frac{744}{24}$.
9. $\frac{1776}{37}$.

Art. 85. Page 46.

6. $\frac{359}{48}$.
7. $\frac{702}{75}$.
8. $\frac{907}{15}$.
9. $\frac{1953}{50}$.
10. $\frac{1525}{28}$.
11. $\frac{1867}{64}$.
12. $\frac{2123}{36}$.
13. $\frac{2818}{5}$.
14. $\frac{4901}{68}$.
15. $\frac{5522}{57}$.
16. $\frac{10435}{82}$.
17. $\frac{14582}{91}$.

Art. 89. Page 47.

7. $\frac{1}{8}$.
8. $\frac{7}{4}$.
9. $\frac{21}{10}$.
10. $\frac{7}{12}$.
11. $\frac{25}{63}$.
12. $\frac{8}{11}$.
13. $\frac{64}{51}$.
14. $\frac{4}{11}$.
15. $\frac{12}{5}$.
16. $\frac{4}{9}$.
17. $\frac{5}{2}$.

Art. 90. Page 48.

2. $\frac{7}{10}$.
3. $\frac{3}{5}$.
4. $\frac{5}{42}$.
5. 10.
6. $\frac{21}{8}$.
7. $\frac{1}{3}$.
8. $\frac{4}{35}$.
9. $\frac{49}{3}$.
10. $\frac{1}{9}$.
11. $\frac{27}{35}$.

Art. 91. Page 49.

2. $\frac{7}{17}$.
3. $\frac{19}{13}$.
4. $\frac{17}{39}$.
5. $\frac{28}{13}$.
6. $\frac{29}{31}$.
7. $\frac{97}{113}$.
8. $\frac{31}{19}$.
9. $\frac{59}{79}$.
10. $\frac{61}{37}$.
11. $\frac{73}{127}$.

Art. 94. Pages 50, 51.

3. $\frac{70}{105}$, $\frac{63}{105}$, $\frac{90}{105}$.
4. $\frac{36}{45}$, $\frac{10}{45}$, $\frac{42}{45}$.
5. $\frac{105}{126}$, $\frac{108}{126}$, $\frac{112}{126}$.
6. $\frac{124}{96}$, $\frac{63}{96}$, $\frac{50}{96}$.
7. $\frac{21}{70}$, $\frac{65}{70}$, $\frac{44}{70}$.
8. $\frac{35}{60}$, $\frac{27}{60}$, $\frac{38}{60}$.
9. $\frac{77}{84}$, $\frac{45}{84}$, $\frac{60}{84}$.
10. $\frac{25}{120}$, $\frac{92}{120}$, $\frac{51}{120}$, $\frac{74}{120}$.
11. $\frac{81}{90}$, $\frac{132}{90}$, $\frac{95}{90}$, $\frac{47}{90}$.
12. $\frac{70}{150}$, $\frac{55}{150}$, $\frac{63}{150}$, $\frac{56}{150}$.
13. $\frac{378}{420}$, $\frac{175}{420}$, $\frac{450}{420}$, $\frac{320}{420}$, $\frac{276}{420}$.
14. $\frac{1750}{900}$, $\frac{756}{900}$, $\frac{380}{900}$, $\frac{185}{900}$, $\frac{96}{900}$.
16. $\frac{8}{17}$ is greater than $\frac{5}{11}$.
17. $\frac{3}{5}$, $\frac{2}{3}$, $\frac{7}{10}$.
18. $\frac{13}{21}$, $\frac{23}{33}$, $\frac{5}{7}$.
19. $\frac{3}{16}$, $\frac{7}{36}$, $\frac{2}{9}$.

Art. 95. Page 51.

2. $\frac{48}{75}$.
3. $\frac{35}{125}$.
4. $\frac{120}{224}$.
5. $\frac{255}{375}$.
6. $\frac{322}{504}$.
7. $\frac{312}{576}$.

Art. 96. Pages 52, 53.

3. $\frac{179}{104}$.
4. $\frac{13}{35}$.
5. $3\frac{3}{8}$.
6. $9\frac{19}{28}$.
7. $11\frac{119}{240}$.
8. $\frac{5}{4}$.
9. $\frac{23}{16}$.
10. $\frac{8}{15}$.
11. $\frac{862}{385}$.
12. $5\frac{7}{15}$.
13. $7\frac{5}{8}$.
14. $17\frac{3}{14}$.
15. $\frac{18}{7}$.
16. $\frac{28}{27}$.
17. $\frac{137}{70}$.
18. $\frac{32}{27}$.
19. $20\frac{1}{15}$.
20. $27\frac{1}{5}$.
21. $47\frac{1}{3}$.
22. $11\frac{1}{34}$.
23. $21\frac{25}{26}$.
24. $10\frac{7}{32}$.
25. $19\frac{17}{40}$.
26. $24\frac{55}{56}$.
27. $20\frac{59}{100}$.
28. $\frac{949}{360}$.
29. $\frac{205}{168}$.
30. $34\frac{127}{140}$.
31. $97\frac{3}{8}$.
32. $172\frac{7}{30}$.

Art. 97. Pages 54, 55.

5. $\frac{4}{15}$.
6. $\frac{11}{28}$.
7. $\frac{23}{90}$.
10. $\frac{10}{63}$.
13. $\frac{13}{36}$.
14. $\frac{11}{26}$.
15. $\frac{19}{60}$.
16. $\frac{1}{42}$.
17. $\frac{5}{28}$.
18. $6\frac{17}{35}$.
19. $\frac{19}{28}$.
20. $1\frac{1}{33}$.
21. $7\frac{13}{21}$.
22. $4\frac{19}{135}$.
23. $9\frac{31}{40}$.
24. $6\frac{71}{96}$.
25. $5\frac{19}{35}$.
26. $11\frac{23}{34}$.
27. $8\frac{13}{77}$.
28. $7\frac{22}{35}$.
29. $13\frac{2}{15}$.
30. $9\frac{8}{55}$.
31. $12\frac{38}{45}$.
33. $\frac{1}{8}$.
34. $4\frac{1}{4}$.
35. $2\frac{32}{45}$.
36. $3\frac{2}{5}$.
37. $9\frac{7}{32}$.
38. $6\frac{2}{21}$.
39. $17\frac{79}{252}$.
40. $18\frac{67}{150}$.
41. $26\frac{65}{72}$.
42. $32\frac{5}{42}$.

Art. 99. Page 56.

7. $\frac{51}{2}$.
8. $\frac{70}{3}$.

10. $\frac{44}{3}$.
11. $\frac{45}{2}$.
12. $\frac{57}{4}$.
13. $12\frac{4}{5}$.
14. $40\frac{5}{7}$.
15. $61\frac{1}{12}$.
16. 480.
17. $275\frac{2}{5}$.
18. $617\frac{1}{2}$.
19. $561\frac{2}{3}$.
20. 1080.
21. $1513\frac{1}{3}$.
22. $2593\frac{5}{7}$.

Art. 101. Page 58.

5. $\frac{9}{8}$.
6. $\frac{18}{5}$.
7. $\frac{9}{8}$.
8. $\frac{16}{49}$.
9. $\frac{56}{11}$.
10. $\frac{3}{8}$.
11. $\frac{11}{4}$.
12. 1.
13. $\frac{25}{4}$.
14. $\frac{16}{15}$.
15. $\frac{8}{9}$.
16. $\frac{10}{21}$.
17. $\frac{36}{5}$.
18. $\frac{1}{210}$.
19. $\frac{7}{66}$.
20. $\frac{121}{35}$.
21. $\frac{20}{3}$.
22. 18.
23. $\frac{45}{4}$.

Art. 103. Page 59.

9. $\frac{17}{25}$.
10. $\frac{9}{11}$.
11. $\frac{35}{72}$.
12. $2\frac{2}{5}$.
13. $3\frac{3}{8}$.
14. $3\frac{31}{44}$.
15. $2\frac{7}{17}$.
16. $3\frac{5}{7}$.

Art. 104. Page 61.

6. $\frac{13}{14}$.
8. $\frac{3}{88}$.
9. $\frac{33}{5}$.
10. $\frac{12}{157}$.
11. $\frac{10}{9}$.
12. $\frac{11}{84}$.
13. $\frac{3}{25}$.
14. $\frac{77}{153}$.
16. $\frac{22}{3}$.
17. $\frac{7}{180}$.
18. $\frac{13}{9}$.
19. $\frac{143}{72}$.
20. $\frac{5}{84}$.
21. $\frac{145}{3}$.
22. $\frac{91}{34}$.
23. $\frac{6}{65}$.
24. $\frac{75}{152}$.
25. $\frac{69}{28}$.
26. $\frac{15}{14}$.
27. $\frac{13}{60}$.
28. $\frac{28}{15}$.
29. $\frac{55}{56}$.
30. $\frac{24}{7}$.
31. $\frac{14}{15}$.
32. $\frac{7}{40}$.
33. $\frac{9}{16}$.
34. $\frac{85}{2}$.
35. $\frac{9}{14}$.
36. $\frac{21}{10}$.
37. $\frac{3}{10}$.
38. $\frac{13}{20}$.
39. $\frac{26}{45}$.

Art. 105. Page 63.

4. $\frac{2}{3}$.
5. $\frac{48}{85}$.
6. $\frac{3}{32}$.
7. $\frac{22}{5}$.
8. $\frac{10}{33}$.
9. $\frac{10}{21}$.
10. $\frac{33}{2}$.
11. $\frac{13}{7}$.
12. $\frac{44}{45}$.
13. $\frac{18}{25}$.
14. $\frac{14}{15}$.
15. $\frac{4}{25}$.
16. $\frac{10}{21}$.
17. $\frac{22}{5}$.
18. $\frac{3}{2}$.
19. $\frac{14}{75}$.
20. $\frac{33}{32}$.
21. $\frac{13}{34}$.
22. $\frac{3}{2}$.
23. $\frac{114}{49}$.
24. $\frac{5}{22}$.
25. $\frac{108}{35}$.

Art. 107. Page 64.

4. $\frac{16}{45}$.
5. $\frac{5}{6}$.
6. $\frac{20}{9}$.
7. $\frac{28}{15}$.
8. $\frac{28}{5}$.
9. $\frac{3}{22}$.
10. $\frac{9}{4}$.
11. $\frac{11}{7}$.
12. $\frac{27}{28}$.
13. $\frac{3}{65}$.
14. $\frac{45}{13}$.
15. $\frac{44}{9}$.
16. $\frac{25}{21}$.
17. $\frac{30}{11}$.
18. $\frac{9}{16}$.
19. $\frac{15}{7}$.
20. $\frac{2}{3}$.
21. $\frac{64}{27}$.
22. $\frac{18}{25}$.
23. $\frac{5}{17}$.
24. $\frac{34}{81}$.

25. $\frac{19}{7}$.
26. $\frac{1}{2}$.

Art. 108.
Pages 64, 65.

3. $\frac{14}{3}$.
4. $\frac{44}{5}$.
5. $\frac{57}{45}$.
6. $\frac{15}{4}$.
7. $\frac{5}{3}$.
8. $\frac{4}{15}$.
9. $\frac{9}{8}$.
10. $\frac{21}{20}$.

Art. 109. Page 65.

2. $\frac{4}{75}$.
3. $\frac{11}{45}$.
4. $\frac{19}{30}$.
5. $\frac{7}{52}$.
6. $\frac{27}{8}$.
7. $\frac{8}{315}$.
8. $\frac{81}{385}$.
9. $\frac{14}{2475}$.

Art. 110. Page 66.

2. $\frac{33}{4}$.
3. 48.
4. $\frac{18}{7}$.
5. $\frac{103}{15}$.
6. $\frac{225}{11}$.
7. $\frac{441}{8}$.
8. $\frac{315}{16}$.
9. $\frac{720}{49}$.

Art. 111.
Pages 66-68.

1. $160\frac{371}{578}$.
2. $\frac{8836}{137}$.
4. $104\frac{1}{5}$.
5. $\frac{30}{77}$.
6. $\frac{3}{41}$.
7. $\frac{2014}{75}$.
8. $\frac{43}{35}, \frac{37}{30}, \frac{53}{42}$.
9. $\frac{464}{785}$.
10. $2\frac{10}{13}$.
11. $\frac{5}{6}$.
12. $\frac{14}{25}$.
13. $\frac{6}{7}$.
14. 1144.
15. $\frac{13}{25}$.
16. $26\frac{71}{84}$.
17. $\frac{43}{28}$.
18. $\frac{214}{1575}$.
19. $\frac{69}{4}$.
20. $\frac{267}{140}$.
21. $\frac{10}{7}$.
22. $\frac{137}{32}$.
23. $\frac{81}{25}$.
24. $\frac{127}{181}$.
25. $9\frac{13}{15}$.
26. $\frac{21}{10}$.
27. $\frac{63}{10}$.
28. $30\frac{97}{840}$.
29. $\frac{61}{72}, \frac{85}{98}, \frac{73}{84}$.
30. $\frac{20}{81}$.
31. $\frac{22}{945}$.
32. $\frac{495}{14}$.
33. $\frac{63}{40}$.
34. $3\frac{83}{126}$.
35. $\frac{26}{33}$.
36. $\frac{21}{17}$.
37. $\frac{177}{40}$.
38. $\frac{41}{10}$.
39. $\frac{303}{50}$.
40. $\frac{6}{55}$.
41. $\frac{283}{240}$.
42. $\frac{66}{25}$.

Art. 112.
Pages 69-73.

4. $4\frac{16}{17}$.
5. $5\frac{1}{31}$.
6. $13\frac{1}{2}$.
7. $204.
8. $234.
9. $2\frac{2}{3}$.
10. $750.
11. Hind-wheel, 560; fore-wheel, 600.
12. $25000.
13. Carriage, $390; horse, $234.
14. $2\frac{7}{10}$ hours.
15. $3\frac{1}{2}$.
16. $5\frac{17}{33}$.
17. $450.
18. 36.
19. $2535.
20. 216.
21. $594.
22. $83\frac{7}{12}$.
23. $106\frac{1}{3}$ feet.
24. 32 cents.
25. $1200.
26. $87.
27. Second class, 42; third, 48; fourth, 63.
28. $\frac{25}{27}$ acre.
29. $5\frac{1}{4}$.
30. $37\frac{1}{2}$ miles.
31. $123.
32. $5400.
33. $9\frac{3}{8}$.
34. $2\frac{2}{3}$.
35. 10.
36. 26.
37. $\frac{21}{25}$.
38. $3\frac{3}{4}$.
39. $4\frac{4}{5}$ days.
40. 11.
41. $37\frac{5}{7}$.
42. $\frac{24}{55}$.
43. $3\frac{8}{9}$.
44. 32.
45. $402\frac{1}{12}$ feet.

46. $163\frac{1}{3}$ minutes; A will have gone around 20 times, B 21 times, and C 25 times.
47. $3\frac{5}{24}$ miles.
48. $3\frac{7}{11}$ days.
49. $\frac{13}{18}$ rod.
50. $\frac{33}{14}$.
51. $3\frac{3}{11}$.
52. $4\frac{4}{5}$.
53. $44\frac{1}{3}$ feet.
54. The sum divided was $252; A received $105, B $$33\frac{3}{5}$, and C $48.
55. $10\frac{2}{3}$.
56. $3\frac{57}{65}$ days; A performs $\frac{21}{65}$, B $\frac{18}{65}$, C $\frac{14}{65}$, and D $\frac{12}{65}$.

Art. 118. Page 76.

1. $\frac{14}{5}$.
2. $\frac{1}{200}$.
3. $\frac{1886}{25}$.
4. $\frac{171}{250}$.
5. $\frac{37}{20}$.
6. $\frac{1064}{125}$.
7. $\frac{123}{4}$.
8. $\frac{79}{400}$.
10. $\frac{1}{40}$.
11. $\frac{47}{1250}$.
12. $\frac{6}{625}$.
13. $\frac{247}{80}$.
15. $\frac{4}{3125}$.
16. $\frac{11}{500}$.
17. $\frac{71}{10}$.
18. $\frac{5}{32}$.
19. $\frac{127}{15625}$.
20. $\frac{135}{64}$.

Art. 119. Pages 76, 77.

2. 27.37096.
3. 38.65775.
4. 225.17578.
5. 11.20697.
6. 5.742231.
7. 82.451502.
8. 901.180486.
9. 5341.02012.

Art. 120. Page 77.

3. 4.8536.
4. .25673.
6. .01054.
7. 9.26771.
8. .0037881.
10. 2.44671188.
11. 781.7513.
12. .0765716.

Art. 121. Page 79.

3. 242.311.
4. .0541926.
5. .00025842.
6. 6.38357.
7. .00463136.
8. 62.8245.
9. .0945162.
10. .20727042.
11. .01088352.
12. 2.615338.
13. 13.539.
14. 104.9688.
15. 4.9298795.
16. .00076075064.

Art. 125. Page 80.

6. 1219.86.
8. 6530.97.
12. 4320.06.
15. 1431894.5.
16. 143602200.

Art. 127. Page 83.

7. .102.
8. 7480.
9. .936.
10. 1.63.
11. .897.
12. .00359.
13. 62510.
14. .3731.
15. .00587.
16. 2.35.
17. 45.6.
18. 50.83.
19. 73.32.
20. .9225.
21. 490300.
22. 6812.5.
23. .003961.

Art. 131. Page 84.

5. .325.
7. .02531.
11. .000132.

Art. 132. Pages 85, 86.

4. .08.
5. .6875.
8. .01176.
9. .1424.
10. .078125.
11. .00672.
12. .96875.
13. .33828125.
14. .004672.
17. .38462.
18. .85185.
19. .03123.
20. .07209.
21. 1.58416.
22. .55008.

Art. 136. Page 87.

4. $.\dot{5}1\dot{3}$.
5. $.12\dot{4}$.
6. $.6\dot{8}\dot{4}$.
7. $10.\dot{7}22\dot{7}$.
8. $.32\dot{0}7\dot{4}$.
9. $.\dot{4}612\dot{5}$.
10. $.8\dot{4}65\dot{3}$.
11. $.00\dot{6}9230\dot{7}$.
12. $.6421\dot{2}9\dot{6}$.
13. $7.1\dot{9}8412\dot{6}$.

Art. 137. Page 88.

3. $\frac{13}{33}$.
4. $\frac{43}{90}$.
5. $\frac{9}{11}$.
6. $\frac{11}{27}$.
7. $2\frac{25}{37}$.
8. $\frac{18}{55}$.
9. $\frac{105}{1111}$.
10. $48\frac{43}{75}$.
11. $\frac{45}{74}$.
12. $\frac{559}{1500}$.
13. $\frac{97}{185}$.
14. $\frac{562}{909}$.
15. $\frac{137}{660}$.
16. $7\frac{2083}{2475}$.
17. $\frac{347}{740}$.
18. $\frac{137}{505}$.
19. $\frac{3381}{6600}$.
20. $\frac{131}{540}$.
21. $6\frac{583}{12375}$.
22. $\frac{73}{202}$.

Art. 139.
Pages 88, 89.

4. 21.
5. .262.
6. .175.
7. 410.
8. .05875.
9. 6.1.
10. 14600.
11. 14.
12. .202.
13. .00228.
14. 15.42.
15. 1.652.
16. .0784.
17. 782.88.
18. 12.8464.

Art. 140.
Pages 89, 90.

1. 18.305665.
2. .004486873.
3. $\frac{113}{3125}$.
5. .615625.
6. .0136.
7. $.28\dot{6}2\dot{1}$.
8. $\frac{41}{100}$.
9. .061464585.
10. .03378.
11. 3.3.
12. .0390625.
13. 6245.8.
14. 28500.
15. $\frac{98}{101}$.
17. .14087.
18. .000868.
19. .025587.
20. $.\dot{1}2345679\dot{0}$.
21. .09671875.
22. 90.38.
23. $\frac{277}{510}$.
24. $\frac{21}{500}$.
25. .0459.
26. $\frac{5}{9}$.
27. $\frac{86}{25}$.
28. $\frac{29}{75}$.
29. .006.
30. $\frac{11}{112}$.

Art. 146.
Pages 92, 93.

7. $226.463.
8. $13.7582.
9. $5.785.
10. 275.42 c.
11. $141.687.
12. .0397377 e.
17. $27850.2.
18. $3809.532.
19. $89.537022.
20. 2928.076 d.
21. $3.58.
22. $.039.
23. 30.8.
24. .732.
25. 4.37.
26. .0688.

Art. 147.
Pages 94–97.

3. $21.60.
4. $24.50.
5. $11.76.
6. 291.
7. $1.68.
8. $105.
9. $728.18.
10. $12.07½.
11. $4149.03.
12. $9.21.
13. $60.48.
14. 36, and 21 gallons remaining.
15. 75.
16. $37.50.
17. $21\frac{2}{3}$.
18. $75.55.
19. $120.75.
20. $6.66⅔.
21. .0083.
22. $5.25.

23. A, $7.25;
B, $4.64;
C, $5.51.
24. $1.44.
25. 4.6903575 miles.
26. $88.75.
27. $\frac{5}{6}$ of $17.67 is the greater by $.62.
28. $17.46.
29. $35.18.
30. $23\frac{1}{3}$.
31. $974.05.
32. $14\frac{7}{8}$.
33. $1\frac{13}{35}$ days.
34. $37493.40.
35. $171.60.
36. $47.25.
37. Wife, $874.80;
son, $546.75;
daughter, $328.05.
38. $180.81.
39. 8 hours; A, $5.00;
B, $3.75;
C, $2.50.
40. $52.07.
41. $.97.
42. Elder, $12.60;
younger, $9.45.
43. 16.
44. $1113.21.
45. $25\frac{1}{2}$.
46. Real estate, $562.50;
railway shares, $281.25;
city bonds, $656.25.
47. Wife, $2465;
eld. son, $1848.75;
younger son, $1479;
dau., $1232.50.
48. A, $12.75;
B, $24.75;
C, $16.50.
49. $1\frac{5}{76}$.

Art. 161.
Pages 104, 105.

5. 6720 pt.
6. 6480 pwt.
7. 6930 in.
8. 1105920 cu. in.
9. 4088 gi.
10. 7476 far.
11. 916960 dr.
12. 453912 gr.
13. 67073 gr.
14. $146832\frac{4}{7}$ sq. ft.
15. 31556929.7 sec.
16. 176957 in.
17. 438 gi.
18. 81920 dr.
19. $1518\frac{3}{4}''$.
20. 88704 in.
21. $274909\frac{1}{11}$ sec.
22. 627264 sq. in.
23. $787\frac{1}{2}$ far.
24. 4672 ♏.
25. $594055\frac{5}{7}$ gr.
26. 609.92 pt.
27. 338688 sec.
28. 127.7952 dr.
29. 7.09038 in.
30. 2342.439 sq. ft.

Art. 162. Page 106.

3. 15 bu. 0 pk. 7 qt. 1 pt.
4. 1 cwt. 47 lb. 15 oz.
5. £6 16*s*. 10*d*. 3 far.
6. 1 lb 4 ℥ 5 ʒ 2 ℈ 7 gr.
7. 4 O. 10 f ℥ 4 f ʒ 38 ♏.
8. 92 rd. 1 yd. 2 ft.
9. 81 gal. 2 qt. 0 pt. 1 gi.
10. £31.
11. 3 d. 17 h. 51 min. 55 sec.
12. 34° 19′ 9″.
13. 8 lb. 6 oz. 15 pwt. 6 gr.
14. 228 rd. 5 yd. 1 ft. 6 in.
15. 2 A. 96 sq. rd. 4 sq. yd.
16. 1 wk. 2 d. 6 h. 13 min. 20 sec.
17. 2 cd. 47 cu. ft. 1502 cu. in.
18. 1 T. 17 cwt. 14 lb. 13 oz. 8 dr.
19. 5 mi. 89 rd. 1 yd. 1 ft. 6 in.
20. 1 sq. rd. 1 sq. yd. 5 sq. ft. 33 sq. in.
21. 1 mi. 33 rd. 2 yd. 2 ft. 10 in.
22. 2 A. 9 sq. rd. 1 sq. yd. $2\frac{3}{4}$ sq. ft.

Art. 163. Page 108.

3. 44 gal. 3 qt. 1 pt. 1 gi.
4. 19 cwt. 1 lb. 5 oz. 4 dr.
5. 164 bu. 3 pk. 7 qt. 1 pt.
6. 275 d. 6 h. 0 min. 50 sec.
7. 313° 27′ 20″.
8. 80 lb 0 ℥ 0 ʒ 2 ℈ 10 gr.

9. 26 mi. 119 rd. 0 yd. 1 ft. 6 in.
10. £451 0*s*. 5*d*. 1 far.
11. 69 T. 13 cwt. 67 lb. 10 oz. 5 dr.
12. 146 cd. 30 cu. ft. 1142 cu. in.
13. 68 lb. 9 oz. 0 pwt. 10 gr.
14. 14 sq. mi. 66 A. 5 sq. rd. 12 sq. yd. 0 sq. ft. 108 sq. in.
15. 129 mi. 223 rd. 3 yd. 0 ft. 2 in.

Art. 164. Pages 109, 110.

3. 10 bu. 2 pk. 6 qt. 1 pt.
4. £17 19*s*. 11*d*. 2 far.
5. 37° 46′ 41″.
6. 9 gal. 3 qt. 1 pt. 3 gi.
7. 9 lb. 11 oz. 1 pwt. 20 gr.
8. 7 cu. yd. 9 cu. ft. 970 cu. in.
9. 9 O. 7 f ℥ 4 f ʒ 35 ♏.
10. 7 mi. 203 rd. 4 yd. 0 ft. 6 in.
11. 14 cwt. 18 lb. 2 oz. 1 dr.
12. 5 A. 12 sq. rd. 10 sq. yd. 6 sq. ft. 36 sq. in.
13. 4 d. 19 h. 16 min. 4 sec.
14. 10 mi. 114 rd. 5 yd. 1 ft.
15. 19 A. 0 sq. rd. 7 sq. yd. 3 sq. ft. 9 sq. in.

Art. 165. Page 111.

4. 258.
5. 295.
6. 265.
7. 695.
8. 627.
9. 3513.
10. 7 y. 10 mo. 16 d.
11. 5 y. 3 mo. 20 d.
12. 1 y. 10 mo. 22 d.
13. 5 y. 8 mo. 5 d.
14. 8 y. 11 mo. 15 d.
15. 1 y. 4 mo. 19 d.

Art. 166. Page 112.

2. 41 bu. 3 pk. 0 qt. 1 pt.
3. 213° 10′ 50.56″.
4. 154 gal. 1 qt. 1 pt. 2 gi.
5. 61 O. 15 f ℥ 6 f ʒ 37 ♏.
6. 75 mi. 144 rd. 0 yd. 2 ft.
7. 87 d. 6 h. 25 min. 30 sec.
8. £1503 2*s*. 0*d*. 3 far.
9. 82 T. 9 cwt. 13 lb. 6 oz. 8.5 dr.
10. 40 cd. 64 cu. ft. 987 cu. in.
11. 253 lb. 2 oz. 10 pwt. 20 gr.
12. 6 mi.
13. 56 A. 17 sq. rd. 13 sq. yd. 2 sq. ft. 66 sq. in.

Art. 167. Page 113.

2. 11 bu. 3 pk. 7 qt. 1 pt.
3. 5 gal. 1 qt. 1 pt. 3 gi.
4. 2° 48′ 45″.
5. 23 T. 9 cwt. 78 lb. 15 oz.
6. 25 cu. yd. 16 cu. ft. 843 cu. in.
7. 5 lb. 9 oz. 15 pwt. $12\frac{1}{2}$ gr.
8. 136 rd. 4 yd. 2 ft. 8 in.
9. £9 10*s*. 7*d*. $3\frac{1}{3}$ far.
10. 34 gal. 2 qt. 1 pt. 3 gi.
11. 6 d. 13 h. 41 min. 33.2 sec.
12. 4 lb 9 ℥ 0 ʒ 2 ℈ 12 gr.
13. 2 mi. 45 rd. 3 yd. 2 ft. 10 in.
14. 3 A. 132 sq. rd. 10 sq. yd. 8 sq. ft. 31 sq. in.

Art. 168. Page 114.

2. £18 2*s*. 9*d*. 1 far.
3. 46 lb. 3 oz. 17 pwt. 22 gr.
4. 5 T. 1 cwt. 42 lb. 9 oz. 14 dr.
5. 5 bu. 2 pk. 6 qt. 1 pt.
6. 1 d. 23 h. 37 min. 15 sec.
7. 3 mi. 20 rd. 1 yd. 2 ft. 10.8 in.
8. 7 gal. 2 qt. 1 pt. 3 gi.

9. 6 cd. 124 cu. ft. 750 cu. in.
10. 22° 55′ 27$\frac{1}{11}$″.
11. 16 ℔ 1 ℥ 6 ʒ 1 ℈ 5 gr.

Art. 169. Page 115.

2. 25.
3. 31.
4. 8.
5. 12.
6. 13.

Art. 170. Page 115.

3. 6 d. 3 h.
4. 3 oz. 6 pwt. 16 gr.
5. 2 qt. 1 pt. 3$\frac{7}{15}$ gi.
6. 1 pk. 1 qt. 1.2 pt.
7. 52′ 15.6″.
8. 12 cwt. 10 lb.
9. 6 f℥ 5 fʒ 20 ♏.
10. 4 yd. 2 ft. 8 in.
11. 21 sq. yd. 1 sq. ft. 82.8 sq. in.
12. 3 ℥ 4 ʒ 0 ℈ 7.68 gr.
13. 9*s*. 2*d*. 3$\frac{1}{15}$ far.
14. 58 cu. ft. 314$\frac{2}{11}$ cu. in.
15. 163 rd. 1 yd. 0 ft. 3.6 in.
16. 1*s*. 10*d*. 3.68 far.
17. 91 sq. rd. 12 sq. yd. 8 sq. ft. 97$\frac{5}{7}$ sq. in.

Art. 171. Pages 116, 117.

3. $\frac{5}{9}$ lb.
4. £$\frac{5}{32}$.
5. $\frac{38}{75}$°.
6. $\frac{37}{64}$ bu.
7. $\frac{17}{25}$ d.
8. $\frac{2}{3}$ gal.
9. $\frac{13}{24}$ cd.
10. $\frac{4}{5}$ rd.
11. $\frac{5}{7}$ cwt.
12. $\frac{8}{25}$ O.
13. $\frac{3}{11}$ A.
14. .8 bu.
15. .4275°.
16. .96875 gal.
17. .69 wk.
18. £.148.
19. .625 cu. yd.
20. .855 lb.
21. .7532 T.
22. .3 sq. rd.
23. .81 mi.

Art. 172. Pages 117, 118.

3. $\frac{1}{2}$.
4. $\frac{5}{16}$.
5. $\frac{7}{15}$.
6. $\frac{37}{41}$.
7. $\frac{2}{3}$.
8. $\frac{4}{5}$.
9. $\frac{5}{9}$.
10. $\frac{5}{11}$.
11. $\frac{4}{7}$.
12. .52.
13. .7.
14. .9616.
15. .75.
16. .875.
17. .625.
18. .27.
19. .4.

Art. 173. Pages 119, 120.

3. 3 h. 13 min. 48 sec.
4. 6 h. 48 min. 4 sec.
5. 11 h. 54 min. 24$\frac{1}{3}$ sec.
6. 9 h. 15 min. 40$\frac{3}{5}$ sec.
7. 5 h. 33 min. 28 sec. A.M.
8. 9 h. 5 min. 29 sec. A.M.
9. 9 h. 59 min. 3$\frac{1}{3}$ sec. P.M.
10. 10 h. 53 min. 57$\frac{4}{5}$ sec. P.M.
11. 89° 15′.
12. 38° 40′ 30″.
13. 168° 16′ 15″.
14. 131° 24′ 30″.
15. 74° 1′ W.
16. 88° 12′ W.
17. 18° 30′ 6″ E.
18. 73° 25′ 57″ W.
19. 72° 53′ 10″ E.

Art. 174. Pages 120-124.

1. 17.
2. $97.44.
3. A train that runs a mile in 85 sec.
4. 1 h. 30 min. 50 sec.
5. 1 y. 8 mo. 18 d.
6. 5 A. 133 sq. rd. 10 sq. yd. 0 sq. ft. 108 sq. in.
7. 40 rd. 1 yd. 0 ft. 9 in.
8. 5.
9. $3470.625.
10. 15.
11. $160.
12. 16$\frac{1}{15}$.
13. 7.
14. 18$\frac{11}{48}$.
15. £4 9*s*. 10$\frac{13}{60}$*d*.
16. 2023.

17. 14 rd. 3 yd. 1 ft.
18. \$13599.72.
19. 2 ℔ 11 ℥ 7 ʒ 0 ℈ 2 gr.
20. $17\frac{87}{175}$.
21. \$315.15$\frac{6}{33}$.
22. \$31.25.
23. 2.6.
24. $80\frac{3}{8}$ cents.
25. $52\frac{4}{5}$ ft. per sec.
26. 4 ℔ 8 ℥ 0 ʒ 1 ℈ $6\frac{1}{4}$ gr.
27. $1\frac{11}{45}$.
28. $95\frac{7}{40}$ cents.
29. \$39.69.
30. 155.
31. 8 oz. $15\frac{18}{35}$ dr.
32. $\frac{2}{3}$.
33. $8\frac{27}{64}$.
34. $8663\frac{1}{8}$ miles.
35. 87° 48′ 45″ W.
36. \$209.15.
37. $\frac{5}{8}$ of 3 gal. 2 qt. is greater by 1 gi.
38. .8228$\frac{4}{7}$ lb. av.
39. 150.
40. $12\frac{3}{5}$.
41. 148 lb. 13.4 oz.
42. $\frac{11}{31}$.
43. 3984 mi.
44. \$8471.77$\frac{47}{64}$.
45. 8 oz. 13 pwt. $4\frac{1}{4}$ gr.
46. $86\frac{2}{5}$.
47. 11*s.* 1*d.* $1\frac{1}{3}$ far.
48. $15\frac{6}{11}$.
49. .859$\frac{39441}{107521}$.
50. $111\frac{21}{64}$ lb.
51. 799.92 ft.
52. A, 2 A. 90 sq. rd.; B, 6 A. 65 sq. rd.
53. $21\frac{57}{83}$.
54. $110\frac{33}{43}$.

Art. 185. Page 133.

3. 361.2400$^{\text{l}}$.
4. 63.43126$^{\text{g}}$.
5. 989.293$^{\text{m}}$.
6. 314.41948$^{\text{sq m}}$.
7. 191.94$^{\text{Dg}}$.
8. .499198$^{\text{m}}$.
9. 934.37$^{\text{cu cm}}$.
10. 4246.567$^{\text{dl}}$.
11. 1.3581$^{\text{Hm}}$.
12. .2543772$^{\text{sq m}}$.
13. 2.26893$^{\text{cg}}$.
14. .2086118$^{\text{dl}}$.
15. .937.
16. .1821$^{\text{Dg}}$.
17. .568.
18. 93.6$^{\text{cu mm}}$.

Art. 186. Pages 134, 135.

3. 0.144$^{\text{dm}}$.
4. .8898$^{\text{Dl}}$.
5. .22046 cwt.
6. .1308 cu. yd.
7. 9.29$^{\text{sq dm}}$.
8. .035316 cu. ft.
9. .981536 gi.
10. 19.764 sq. ft.
11. .1772$^{\text{Dg}}$.
12. 32.808 ft.
13. 4.7315$^{\text{dl}}$.
14. 3.73242$^{\text{Hg}}$.
15. 1.308 cu. yd.
16. 616.4288 rd.
17. 739.93225$^{\text{cg}}$.
18. 45.827522 cu. in.
19. 166.278$^{\text{Ha}}$.
20. 8.213$^{\text{Hl}}$.
21. .1829818 cwt.
22. 5.248176 dr.
23. 141.58368$^{\text{cu dm}}$.
24. 81.54$^{\text{dst}}$.

25. .842265$^{\text{cl}}$.
26. 2.309136 sq. rd.
27. 37.5441$^{\text{Dl}}$.
28. .51562$^{\text{Dm}}$.
29. 7.22624$^{\text{sq m}}$.
30. .589932$^{\text{st}}$.
31. 128283.75$^{\text{cg}}$.
32. 34.6815$^{\text{cu dm}}$.
33. 163.23675$^{\text{dl}}$.
34. .946206$^{\text{cu Dm}}$.
35. 3.99470825$^{\text{Kg}}$.
36. 2.1964347$^{\text{Ha}}$.
37. 2523.1344$^{\text{m}}$.

Art. 187. Pages 135-137.

1. 2.9116.
2. 305.4.
3. .164$^{\text{mm}}$.
4. \$1992.
5. .18162.
6. .1296.
7. 5.286.
8. \$39.37.
9. \$243.8387.
10. 2.679.
11. 4784$^{\text{Dg}}$.
12. 38.7.
13. .03696.
14. 4.047.
15. .4971.
16. \$3.77+.
17. 4 min. 39 sec.
18. 5.39712.
19. 11.82875.
20. \$2.57 nearly.
21. 134.1$^{\text{Hm}}$.
22. 15.55175.
23. 5.6432.
24. 28.316736$^{\text{Kg}}$.
25. 9 h. 28 min.
26. 47.315$^{\text{Dg}}$.

27. 169.0208 rods a minute.
28. 162.30912.
29. 2.09 + cents per mile.
30. 10.35+.
31. 6.103.
32. 264.17.
33. 32.362512.
34. $35.07.
35. 2.076+ Kg.
36. .0839+.
37. 13.63+ Kg.
38. 1.27+.
39. 34.22+.
40. 13490.7619 Km.

Art. 203. Page 143.

1. 78.
2. .97.
3. 21.4.
4. 523.
5. .286.
6. 80.9.
7. .497.
8. .0722.
9. 5.70.
10. .1082.
11. 21.12.
12. .8253.
13. 900.8.
14. 5783.
15. 7.041.
16. .04738.
17. 850.35.
18. .98657.

Art. 204. Page 144.

2. 2.6457+.
3. 5.5677+.
4. 4.1593+.
5. .2828+.
6. .3162+.
7. 1.4443+.
8. 19.3864+.
9. 20.1053+.
11. 1.6583+.
12. .4330+.
13. 1.2018+.
14. .4472+.
15. .6454+.
16. .9045+.
17. .6230+.
18. .4240+.
19. 1.1319+.
20. .8552+.

Art. 212. Page 149.

1. 31.
2. 4.6.
3. .88.
4. 123.
5. 1.14.
6. .098.
7. 2.02.
8. 372.
9. 21.6.
10. .803.
11. 4.89.
12. .317.
13. .898.
14. 101.3.
15. .0534.
16. 73.4.
17. 5.815.
18. .6523.

Art. 213. Pages 149, 150.

2. 1.259+.
3. 1.817+.
4. 1.930+.
5. 3.448+.
6. 5.528+.
8. .854+.
9. 1.077+.
10. .637+.
11. .873+.
12. .912+.
13. 1.1304+.
14. .7802+.

Art. 214. Page 150.

1. 33.
2. 76.
3. 88.
4. 514.
5. 49.
6. 65.

Art. 226. Pages 154-156.

4. 588 sq. in.
5. 14 sq. ft. 100 sq. in.
6. $4\frac{4}{9}$ sq. yd.
7. 4 ft.
8. 12 yd.
9. 75 ft.
10. 3780 sq. in.
11. $\frac{8}{9}$ yd.
12. 75 in.
13. $3\frac{3}{5}$ A.
14. 196 sq. ft. 112 sq. in.
15. $31\frac{17}{25}$ rd.
16. 32 rd.
17. $320.
18. 9 ft.
19. 6600.
20. 630.
21. 1200.
22. 280 yd.
23. 24 rd.
24. $93.65.
25. 3421440.
26. $562.50.

27. 20 rd. 3 yd.
28. 22 rd. 4 yd. 1 ft.
29. $1878.80.

Art. 228.
Pages 157, 158.

4. 25 in.
5. 7 rd. 2$\frac{1}{2}$ yd.
6. 32 in.
7. 1 yd. 2 ft.
8. 10.6066+ in.
9. 12.2065+ in.
10. 22 ft. 1 in.
11. 12 ft.
12. 60 ft. 9 in.
13. 105.4 mi.
14. 40.5 ft.
15. 8 ft. 2 in.

Art. 231.
Pages 159, 160.

3. 43.9824 in.; 153.9384 sq. in.
4. 52.36 yd.
5. 5.25+ rd.
6. 13.54+ in.
7. 24856.3392 mi.
8. 2.67+.
9. 4.507+.
10. 346.3614.
11. 61.11+ in.
12. 110 sq. ft. 104.5776 sq. in.
13. 1306.9056 sq. ft.
14. 9075+.
15. 8.862+ in.
16. 28.337232 ft.
17. 120.96 ft.

Art. 242.
Pages 164–166.

4. 330 sq. in.; 330 cu. in.
5. 34$\frac{21}{25}$ cu. in.; 63$\frac{3}{5}$ sq. in.
6. 144 sq. ft.
7. 7 ft.; 364 sq. ft.
8. 420 cu. in.
9. 392 sq. ft.
10. 11 in.
11. 650 cu. in.
12. 32 in.
13. 1092 cu. in.; 662 sq. in.
14. 3 ft.; 122 sq. ft.
15. $8.47.
16. 19$\frac{1}{2}$ in.
17. 2400 sq. in.
18. $55.38.
19. 5951$\frac{1}{4}$ lb.
20. 1944.
21. 32 ft.
22. 53$\frac{1}{3}$ cu. ft.
23. 15 in.
24. 3$\frac{191}{432}$.
25. 700 sq. in.; 1568 cu. in.
26. 2090880.

Art. 248.
Pages 169, 170.

5. 131.9472 sq. in.; 197.9208 cu. in.
6. 201.0624 sq. in.; 268.0832 cu. in.
7. 483.8064 sq. ft.
8. 204.204 sq. in.; 314.16 cu. in.
9. 1847.2608 cu. ft.
10. 10 in.; 8 in.
11. 10 in.; 523.6 cu. in.
12. 8 in.
13. 12 in.
14. 196067256 sq. mi.; 258155220400 cu. mi.
15. 67.0208.
16. 6 ft.
17. $184.07+.
18. 5$\frac{22}{25}$ in.
19. 14.32+.
20. 134.0416.
21. 2.3562.
22. 4 ft.
23. 12 in.
24. 6 in.
25. 67.3698+ lb.
26. 8.7593+.
27. 6.109 ft.
28. 8.169+ in.

Art. 249.
Pages 171, 172.

2. 67$\frac{1}{3}$.
3. 7.
4. 65$\frac{3}{8}$.
5. 552.5.
6. 4$\frac{3}{8}$ ft.
7. 5 ft.
8. 83$\frac{1}{3}$ in.
9. 65.35+ in.
10. 48.23+.
11. $117.
12. 58.5+ in.
13. 4 ft. 7.98+ in.
14. $111.13+.
15. 61.91+ in.
16. 53.856+.
17. 41.0502+.
18. 4.12+ ft.
19. 53.71+.

Art. 250. Page 173.

2. 34.
3. $7.50.
4. 48$\frac{22}{33}$; 49.
5. $10.45.
6. 21.14\frac{7}{12}$.

7. $28\frac{13}{24}$.
8. Across the room.
9. \$51.64.
10. With oil-cloth.

Art. 251.
Pages 174, 175.

3. \$35.88.
4. \$48.48.
5. $8\frac{7}{10}$.
6. \$5.40.
7. \$27.53$\frac{2}{3}$.
8. \$6.
9. \$314.16.
10. $6\frac{2}{3}$.
11. \$37.71$\frac{1}{3}$.
12. \$10.97$\frac{2}{3}$.

Art. 252.
Pages 176, 177.

2. $19\frac{1}{4}$.
3. $9\frac{1}{6}$.
4. $154\frac{1}{2}$.
5. $92\frac{5}{9}$.
6. $107\frac{13}{16}$.
7. \$17.64.
8. \$13.34$\frac{3}{5}$.
9. \$66.85$\frac{1}{4}$.
16. \$5.90$\frac{5}{8}$.
11. \$33.68$\frac{3}{4}$.

Art. 253. Page 177.

2. 8.484 in.; 108.
3. 11.312 in.; 224.
4. 10.605 in.; $162\frac{1}{2}$.
5. 9.898 in.; $128\frac{1}{3}$.
6. 13.433 in.; $358\frac{71}{14}$.

Art. 254.
Pages 178, 179.

3. 550.625 lb.
4. 3037.5 lb.
5. $148\frac{13}{14}$ lb.
6. $7\frac{125}{144}$ oz.
7. 25.
8. 40.
9. 432.
10. 54.
11. 2.72.
12 .88.
13. 10.5.
14. 8.3469312.

Art. 256.
Pages 181–188.

MENSURATION
OF PLANE FIGURES.

1. $6541.5^{\text{sq dm}}$.
2. 35^{m}.
3. $.4^{\text{Hm}}$.
4. $.0162^{\text{Dm}}$.
5. 144513.6^{m}.
6. $.094776^{\text{sq Hm}}$.
7. 1.83^{Dm}.
8. $13273.26^{\text{sq cm}}$.
9. 3.79^{dm}.
16. $1233.395^{\text{sq dm}}$.
11. 2.5^{Dm}.
12. $.3819+^{\text{cm}}$.
13. 276^{cm}.
14. 3.52782^{s}.
15. 25.67566^{cs}.
16. 166.311744^{cs}.
17. 65.9^{m}.
18. 87.9^{Dm}.
19. 221^{m}.
20. 5500.
21. \$6653.36.
22. 65.1^{Km}.
23. 16.23256536.
24. 1.6193925.
25. 11.5^{m}.
26. $5.939+^{\text{Hm}}$.
27. 7.6^{Dm}.
28. 235361.94.
29. 37^{m}.
30. $11.2+^{\text{Dm}}$.
31. $53.74+^{\text{m}}$.

MENSURATION OF
SOLIDS.

1. $84^{\text{sq dm}}$; $42^{\text{cu dm}}$.
2. $113.0976^{\text{sq cm}}$; $113.0976^{\text{cu cm}}$.
3. $549.78^{\text{sq m}}$; $1231.5672^{\text{cu m}}$.
4. 16^{cm}; $976^{\text{sq cm}}$.
5. 33^{dm}.
6. $301.5936^{\text{sq m}}$.
7. 4^{cm}; $268.0832^{\text{cu cm}}$.
8. $9.4956^{\text{sq m}}$.
9. $425.6^{\text{cu Dm}}$.
16. 9^{m}.
11. $45804.528^{\text{sq mm}}$; $2661.26376^{\text{cu cm}}$.
12. 15^{m}; 9^{m}; $1357.1712^{\text{cu m}}$.
13. $407.52^{\text{sq m}}$.
14. 268.149326; 268149.326^{dg}.
15. \$13.65.
16. \$199.0989.
17. 7824.
18. \$113.40.
19. 1.38^{m}.
20. 1.5372.
21. 48.55344^{Kg}.
22. 5309.304.
23. 638.4.
24. 37000.
25. 19373.2.
26. 3650.5392^{Dg}.
27. 13.8984384^{Kg}.
28. 579.63588.
29. $.33^{\text{m}}$.
30. 3.8050012.

CAPACITY OF BINS, TANKS, AND CISTERNS, CARPETING, PLASTERING, AND PAPERING.

1. 5801.6.
2. 49.68.
3. 46.13.
4. $27.63.
5. 1.92^{m}.
6. 26.9^{dm}.
7. 134.3034; 134303.4^{Hg}.
8. 6.5.
9. 1.7^{m}.
10. $471.24.
11. $60.42.
12. 11 h. 30 min.
13. 10.31.
14. 15.927912.
15. 3.5^{m}.
16. $40.026.
17. 13.5.
18. Across the room.
19. $102.971; 3107.852^{Kg}.
20. 1.8^{m}.
21. $34.104; $34.104.
22. 75^{cm}.
23. .997+^{m}.
24. $7.98.
25. 1.5^{m}.

Art. 257.
Pages 189, 190.

4. 1200^{g}.
5. 1317^{Hg}.
6. 18.7.
7. 12.9.
8. .92.
9. 13.596.
10. 2.30679.
11. 11568.72^{dg}.
12. 65^{cm}.
13. 7.1.
14. 2.08.
15. 9^{dm}.
16. 20.5633428^{Kg}.

Art. 263.
Pages 191, 192.

1. 2 : 5.
2. 8 : 15.
3. 23 : 37.
4. 9 : 10.
5. 9 : 13.
6. 22 : 15.
7. 11 : 14.
8. 5 : 3.
9. 7 : 9.
10. 25 : 27.
11. 8 : 5.
12. 2 : 3.
13. 16 : 25.
14. 64 : 45.

Art. 270. Page 194.

3. 112.
4. 36.
5. 24.
6. $\frac{25}{27}$.
7. $\frac{13}{25}$.
8. $\frac{21}{2}$.
9. $1\frac{2}{3}$.
10. $\frac{16}{9}$.

Art. 271.
Pages 196, 197.

3. $4.51.
4. $15.96.
5. $51\frac{1}{4}$.
6. $55\frac{1}{2}$.
7. $648\frac{3}{5}$.
8. $78\frac{4}{7}$.
9. $4.60.
10. $49\frac{7}{8}$.
11. $331.25.
12. $1125.
13. 280.
14. $20.40.
15. 159 ft. $4\frac{1}{2}$ in.
16. $8\frac{8}{9}$.
17. 42 mi. an hour.
18. $29.80.
19. $8\frac{4}{15}$.
20. 10 min. 30 sec.
21. $2\frac{1}{6}$ h.
22. 51.
23. 276 lb. $\frac{2}{3}$ oz.
24. 105 d.
25. $\$161\frac{1}{3}$.

Art. 273.
Pages 198-201.

2. 135.
3. $3\frac{1}{3}$.
4. 8.
5. 20.
6. $40\frac{2}{3}$.
7. 5.
8. 12.
9. $2\frac{2}{9}$.
10. 9.
11. 216 mi.
12. $42\frac{2}{3}$.
13. $3\frac{3}{16}$ oz.
14. $8\frac{3}{4}$.
15. 72.
16. 660 lb.
17. 10.
18. $6\frac{4}{13}$ ft.
19. $7\frac{1}{2}$.
20. $78\frac{3}{4}$ lb.
21. $13\frac{1}{3}$.
22. 9 ft.
23. 4.
24. $4\frac{1}{2}$ ft.

25. 26$\frac{1}{4}$.
26. 1150 lb.
27. 28.
28. 81.
29. 8.
30. 5$\frac{1}{4}$.

Art. 275.
Pages 202, 203.

3. 14, 35, 56.
4. 140, 184.
5. 39, 65, 91, 117, 143.
6. 672, 630, 588, 576.
7. 50$\frac{3}{4}$, 355$\frac{1}{4}$.
8. 510 lb. copper,
   306 lb. zinc,
   119 lb. tin.
9. A, \$375 ; B, \$480 ;
   C, \$612.
10. 182$\frac{2}{3}$ parts saltpetre,
    34$\frac{1}{4}$ parts charcoal,
    57$\frac{1}{12}$ parts sulphur.
11. 42, 84, 252. 1008.
12. \$23.36, \$35.04,
    \$43.80, \$61.32.
13. 20$\frac{23}{30}$ cu. ft. oxygen, 99$\frac{7}{30}$ cu. ft. nitrogen.
14. 30, 20, 16.
15. \$115.
16. 15, 30, 55, 90.

Art. 276.
Pages 204, 205.

3. 540 sq. in.
4. 8 in.
5. 703$\frac{1}{8}$ cu. in.
6. 6 ft.
7. 1 ft. 4$\frac{1}{2}$ in.
8. 7 in.
9. 4 ft. 6 in.
10. 9 in.
11. \$37.80.
12. 218 lb. 12 oz.
13. 7.93+ in.
14. 2 h. 6 min.
15. 1 ft. 4 in.

Art. 280.
Pages 206, 207.

2. A, \$600; B, \$800.
3. Allen, \$1250 ;
   Brown, \$1500 ;
   Cole, \$800.
4. A, \$192.75 ;
   B, \$64.25.
5. A, \$49.24 ;
   B, \$61.55 ;
   C, \$73.86.
6. A, \$17120 ;
   B, \$8560 ;
   C, \$1712.
7. Hale, \$1725 ;
   Hunt, \$2070.
8. A, \$1677.36 ;
   B, \$1258.02 ;
   C, \$3145.05.
9. A, \$5.25 ;
   B, \$3.75 ;
   C, \$6.00 ;
   D, \$4.50.
10. A, \$867.84 ;
    B, \$578.56 ;
    C, \$1084.80.

Art. 282.
Pages 208, 209.

2. A, \$162 ;
   B, \$194.40.
3. A, \$82.50 ;
   B, \$247.50 ;
   C, \$137.50.
4. A, \$26.40 ;
   B, \$13.20 ;
   C, \$33 ; D, \$44.
5. Adams, \$1200 ;
   Burke, \$1120.
6. A, \$1275 ;
   B, \$1020;
   C, \$1360.
7. Rand, \$704 ;
   Sears, \$720 ;
   Thomas, \$768.
8. A, \$532 ;
   B, \$148.
9. A, \$48.75 ;
   B, \$39; C, \$22.75.
10. Fuller, \$200 ;
    Gray, \$250.
11. A, \$145 ;
    B, \$185 ;
    C, \$200.
12. A, \$1500 ;
    B, \$1800 ;
    C, \$3375.
13. Lowe, \$2035 ;
    Martin, \$2255 ;
    Neal, \$770.

Art. 287. Page 210.

8. .056.
9. .008.
10. .001875.
11. .0044.

Art. 288. Page 211.

1. $\frac{12}{25}$.
2. $\frac{7}{4}$.
3. $\frac{1}{15}$.
4. $\frac{7}{16}$.
5. $\frac{17}{24}$.
6. $\frac{23}{48}$.
7. $\frac{5}{9}$.
8. $\frac{1}{150}$.

9. $\frac{1}{120}$.
10. $\frac{4}{125}$.
11. $\frac{14}{475}$.
12. $\frac{1}{32}$.
13. $\frac{3}{40}$.
14. $\frac{8}{625}$.
15. $\frac{1}{160}$.

Art. 289. Page 211.

2. $57\frac{1}{7}$%.
3. $11\frac{1}{9}$%.
4. $112\frac{1}{2}$%.
5. $58\frac{1}{3}$%.
6. 85%.
7. 16%.
8. $7\frac{1}{2}$%.
9. $131\frac{1}{4}$%.
10. $73\frac{1}{3}$%.
11. $171\frac{7}{8}$%.
12. $54\frac{1}{6}$%.
13. $\frac{25}{32}$%.
14. $\frac{4}{5}$%.
15. $\frac{16}{21}$%.
16. $\frac{5}{9}$%.
17. $\frac{6}{7}$%.
18. $123\frac{5}{13}$%.
19. $1\frac{3}{32}$%.
20. $21\frac{7}{48}$%.
21. $49\frac{33}{35}$%.

Art. 290.
Pages 212-214.

3. 4.291.
4. $7.77.
5. 62 bu.
6. $\frac{23}{8}$.
7. $\frac{4}{9}$.
8. 1.
9. $2.27.
10. $\frac{5}{102}$.
11. £100 2s.
12. $\frac{21}{5}$.
13. $\frac{13}{160}$.
14. 3 ft. $6\frac{1}{2}$ in.
15. $413.
16. $415.
17. $953.04.
18. $498.27.
19. $1132.25.
20. $378.42.
21. $2257.57.
22. 14093.
23. $891.
24. $9.45.
25. $.03.
26. 1040 lb. lead; 16 lb. silver.
27. $1815.
28. $344.75.
29. 703.
30. 1 in.
31. $.46$\frac{10}{57}$.
32. $1573.

Art. 291.
Pages 215–218.

3. 32.4.
4. $18.50.
5. $\frac{28}{9}$.
6. 17° 45′.
7. $2\frac{6}{7}$.
8. 47 ft. 8 in.
9. $\frac{175}{13}$.
10. 3.
11. $\frac{16}{9}$.
12. 187 lb. 4 oz.
13. $2\frac{6}{7}$.
14. $193.75.
18. $83.50.
19. $1328.
20. 384.
21. 656.25.
22. $425.
23. $62.50.
24. 1038.54\frac{6}{11}$.
25. $\frac{3}{5}$.
26. $45.75.
27. 146 ft. 8 in.
28. 7608.
29. $1286.40.
30. $107.80.
31. 2928.
32. 135 girls, 162 boys.
33. $17500.
34. 1241, 816.
35. $2220.35.
36. $5688.
37. $1000.50.
38. $1015.30.
39. 57615.
40. $1250.

Art. 292.
Pages 219-221.

4. $6\frac{2}{3}$%.
5. 40%.
6. $54\frac{1}{6}$%.
7. 74%.
8. $91\frac{2}{3}$%.
9. 75%.
10. 108%
11. $63\frac{5}{9}$%.
12. $31\frac{1}{4}$%.
13. $\frac{5}{8}$%.
14. $42\frac{6}{7}$%.
15. $\frac{1}{8}$%.
16. $41\frac{2}{3}$%.
17. $12\frac{1}{2}$%.
18. 76% silver; 24% copper; $31\frac{11}{15}$%.
19. $19\frac{4}{9}$%.
20. $26\frac{46}{79}$%.
21. $22\frac{2}{9}$%.
22. $18\frac{3}{4}$%.
23. $87\frac{1}{2}$%.
24. 20%.

25. $\frac{3}{4}$%.
26. $2\frac{1}{2}$%.
27. $83\frac{1}{3}$%.
28. 28%.
29. 17%.
30. $16\frac{2}{3}$%.
31. 19%.
32. 8.9%.
33. $6\frac{2}{3}$%; $0\frac{1}{4}$%.
34. $15\frac{5}{8}$%.
35. $2\frac{2}{25}$%.
36. $19\frac{1}{6}$%.
37. $37\frac{1}{2}$%.

Art. 294.
Pages 222-224.

3. $45.75.
4. $203.28.
5. $14.70.
6. $4.00.
7. $8.00.
8. $4.35.
9. $14.28.
10. Loses $3\frac{3}{4}$%.
11. $1.26.
12. $2653.56.
13. $2.10.
14. $9\frac{3}{8}$%.
15. $24.75.
16. $13.32.
18. $33\frac{1}{3}$%.
19. 50%.
20. $27\frac{7}{9}$%.
21. $36\frac{1}{4}$%.
22. $37\frac{1}{2}$%.

Art. 296.
Pages 225-227.

3. $18.72.
4. $4964.82.
5. 5%.
6. $21.25.
7. $520; $31.20.
8. $990.099+; $9.901.
9. $3\frac{3}{4}$%.
10. 400.
11. $2496.
12. $7840.
13. $2364.18.
14. $438 18.
15. $2444.98+; $55.02.
16. $8400.
17. $2157.40.
18. 336.
19. $632.
20. $0390.
21. 350.
22. 13; $8.84.
23. 96.

Art. 298.
Pages 228, 229.

3. $47.97.
4. $415.25.
5. $1860.
6. $\frac{1}{3}$%.
7. $1720.
8. $3475.50.
9. $2009.60.
10. $\frac{2}{5}$%.
11. $5760.
12. $9600.
13. $2625.
14. $2653.75.
15. $108.50.
16. $103.50.
17. $7980.
18. $3\frac{3}{4}$%.

Art. 301.
Pages 230, 231.

2. $13.50 on $1000.
3. $17.50 on $1000.
4. $54.28.
5. $1200.
6. $7000.
7. $215.79.
8. $12.75 on $1000; $107.10.
9. $35700.
10. $332.
11. $11.30 on $1000.
12. $16 on $1000.
13. $22401.

Art. 303.
Pages 232, 233.

3. $7.98.
4. $100.30.
5. $157.0746.
6. $151.20.
7. $82.40.
8. $61.11.
9. $1614.60.
10. $690.
11. 55%.
12. $77.50494.
13. $293.443375.
14. $904.8553125.

Art. 307.
Pages 235, 236.

4. $217.50.
5. $33.78.
6. $10.90.
7. $4.74.
8. $108.35.
9. $2.67.
10. $21.28.
11. $14.11.
12. $50.36.
13. $18.08.
14. $2.61.
15. $14.44.

16. $197.95.
17. $76.39.
18. $276.37.
19. $2214.36.
20. $467.99.
21. $3794.82.
22. $1102.58.
23. $636.46.

Art. 308.
Pages 238, 239.

4. $161.35.
5. $10.33.
6. $6.42.
7. $1.05.
8. $8.12.
9. $7.08.
10. $66.68.
11. $20.17.
12. $36.80.
13. $333.60.
14. $5183.46.
15. $1047.72.
17. $162.17.
18. $13.65.
19. $1.24.
20. $24.25.
21. $38.23.
22. $1.73.
23. $142.06.
24. $157.11.
25. $9179.74.
26. $328.47.

Art. 309. Page 240.

2. $921.18.
3. $1.26.
4. $19.38.
5. $62.59.
6. $10.53.
7. $1.92.
8. $6689.71.
9. $250.61.
10. $197.45.
11. $845.54.

Art. 310.
Pages 240, 241.

2. 2½ %.
3. 3⅓ %.
4. 6 %.
5. 2 %.
6. 3¼ %.
7. 2⅔ %.
8. 7 %.
9. 1⅔ %.
10. 2¼ %.
11. 4 %.
12. 3 %.
13. 4¼ %.
14. 3½ %.
15. 8 %.
16. 7½ %.
17. 3 %.

Art. 311.
Pages 242, 243.

3. 4 y. 3 mo.
4. 2 y. 10 mo.
5. 6 mo.
6. 3 y. 1 mo.
7. 5 mo. 24 d.
8. 11 mo. 12 d.
9. 1 y. 8 mo. 12 d.
10. 6 mo. 3 d.
11. 9 mo. 6 d.
12. 4 mo. 7 d.
13. 2 mo. 11 d.
14. 8 mo.
15. 21 d.
16. 9 mo. 29 d.
17. 5 y. 5 mo. 0 d.
18. 7 mo. 12 d.
19. 16 y. 8 mo.
20. 22 y. 2 mo. 20 d.

Art. 312. Page 244.

3. $310.
4. $197.75.
5. $196.
6. $297.20.
7. $178.80.
8. $657.16.
9. $86.34.
10. $947.93.
11. $74.16.
12. $305.48.
13. $325.85.
14. $250.91.
15. $342.86.
16. $10.74.
17. $573.25.
18. $4191.04.
19. $589.66.
20. $136.08.

Art. 314. Page 246.

2. $188.24.
3. $27.21.
4. $31.06.
5. $103.02.
6. $52.88.
7. $24.38.
8. $37.05.
9. $38.43.

Art. 323.
Pages 249, 250.

2. $318.35.
3. $244.24.
4. $606.91.
5. $90.66.
6. $371.65.
7. $183.05.
8. $521.25.

Art. 324.
Pages 251, 252.

2. $309.85.

3. $251.39.
4. $754.70.
5. $766.71.
6. $244.06.
7. $436.79.
8. $327.70.
9. $345.68.

Art. 326.
Pages 253, 254.

2. $156.63.
3. $258.81.
4. $59.85.
5. $167.24.
6. $464.16.
7. $180.89.
8. $801.87.
9. $624.48.
10. $143.42.
11. $1790.82.

Art. 327. Page 255.

2. $566.34.
3. $1696.18.
4. $429.25.
5. $639.00.
6. $788.13.
7. $696.71.
8. $771.80.

Art. 328. Page 257.

2. $133.49.
3. $1562.79.
4. $422.37.
5. $272.07.
6. $265.16.
7. $298.30.

Art. 329. Page 258.

2. $1059.62.
3. $469.21.
4. $96 86.
5. $1016.86.

6. $1196.76.

Art. 331. Page 260.

2. $340.43 ; $59.57.
3. $868.29 ; $21.71.
4. $595.47 ; $129.53.
5. $1549.25 ; $180.75.
6. $662.46 ; $19.54.
7. $267.46 ; $1.74.
8. $2454.19 ; $45.81.
9. $907.50 ; $42.50.
10. $127.07 ; $8.68.
11. $342.21 ; $5.47.

Art. 334.
Pages 261-263.

3. $492.25.
4. $941.69.
5. $36.17.
6. $5.58.
7. $996.33.
8. $5.35.
9. $6009.95.
10. $420.05.
11. $802.98.
12. $3.33.
13. $6.43.
14. $2980.75.
15. $191.44.
16. $1.13.
17. $1128.73.
18. $199.88.
19. $4.67.
20. $279.76.
21. $396.59.
22. $3.71.

Art. 335. Page 264.

2. $609.45.
3. $341.57.
4. $227.39.
5. $8245.30.

6. $554.81.
7. $1535.12.
8. $430.27.
9. $921.80.
10. $377.48.

Art. 343.
Pages 267, 268.

3. $506.25.
4. $278.60.
5. $7874.
6. $474.56.
7. $1928.16.
8. $693.61.
9. $344.97.
10. $606.80.
11. $1333.59.

Art. 344.
Pages 269, 270.

3. $440.
4. $649.60.
5. $918.46.
6. $241.81.
7. $752.10.
8. $192.
9. $584.65.
10. $2985.07.
11. $2346.71.

Art. 348.
Pages 272, 273.

4. $1693.89.
5. $201.93.
6. $581.40.
7. £39 4*s*.
8. 29942.55 fcs.
9. $78.89375.
10. $515.20.
11. $139.83.
12. 29368 mks.
13. $2557.697625.
14. £176.

15. 4914.70125 fcs.
16. \$466.4109375.
17. £132 8*s.* 9*d.*
18. 1501.03 mks.
19. \$5248.19.

Art. 350.
Pages 275, 276.

4. 7 mo. 10 d.
5. 8 mo. 27 d.
6. 73 d.
7. April 23.
8. Oct. 10.
9. Nov. 18.
10. Jan. 8, 1890.
11. June 5, 1891.
12. Dec. 24.
13. July 29, 1892.
14. After 9 mo. 15 d.
15. Oct. 2.
16. 5 mo. 27 d. after it becomes due.
17. Oct. 14, 1892.

Art. 352.
Pages 279-281.

2. Nov. 12.
3. Jan. 11, 1891.
4. Feb. 13.
5. April 26.
6. Nov. 5, 1890.
7. Dec. 2.
8. Dec. 27, 1890.

Art. 363.
Pages 286-291.

5. \$17741.25.
6. \$8011.50.
7. \$5466.75.
8. \$2588.25.
9. \$1354.50.
10. \$4977.
11. 46.
12. \$7500.
13. 43.
14. 626.
15. \$15000.
16. 75.
17. 153.
18. $160\frac{3}{8}$.
19. $8\frac{1}{2}$%.
20. $80\frac{7}{8}$.
21. $12\frac{3}{8}$%.
22. $36\frac{3}{4}$%.
23. 186.
24. \$395.50.
25. \$75000.
26. \$3982.75.
27. $1\frac{7}{8}$%.
28. \$7900.
29. 38.
30. 153.
31. 375.
36. \$160.
37. \$150.
38. \$271.25.
39. \$484.50.
40. \$4431.
41. \$5797.50.
42. \$12060.
43. \$7449.75.
44. 5%.
45. $4\frac{1}{2}$%.
46. 6.4%.
47. $3\frac{1}{3}$%.
48. 224.
49. 74.
50. $146\frac{7}{8}$.
51. $67\frac{3}{4}$.
52. $150\frac{1}{2}$.
53. A $4\frac{1}{2}$% stock at 90.
54. Increased \$11.
55. $61\frac{1}{4}$.
56. The investments are equally good.
57. Diminished \$23.50.
58. \$6757.50 invested in 6% bonds at $112\frac{3}{8}$.
59. $4\frac{1}{4}$%.
60. A $3\frac{3}{5}$% stock at a disc't of $28\frac{1}{2}$%.
61. 129.
62. Increased \$36.75.

Art. 367.
Pages 293-295.

2. 72.
3. $32\frac{1}{2}$.
4. 85.
5. 51.6.
6. $\frac{17}{3}$.
7. 70, 444.
8. 289, 3129.
9. 69, 5249.
10. 31, 9306.
11. $81\frac{2}{3}$, 2040.
12. 9.9, 748.3.
13. $\frac{11}{2}$, $\frac{2183}{12}$.
14. $\frac{1}{6}$, $\frac{391}{36}$.
15. 4950.
16. 2550.
17. The last term is $\frac{14}{9}$.
18. 1197.
19. 44550.
20. $498\frac{7}{12}$ ft., $4117\frac{1}{3}$ ft.
21. $15\frac{5}{8}$ mi., $325\frac{5}{12}$ mi.
22. \$3850.

Art. 372.
Pages 297, 298.

3. 2500.

4. $\frac{125}{81}$.
5. $\frac{64}{215}$.
6. $\frac{75}{512}$.
7. 1024, 2047.
8. 13122, 19680.
9. $\frac{6}{125}$, $\frac{5116}{125}$.
10. $\frac{729}{4096}$, $\frac{10101}{4096}$.
11. $\frac{213}{64}$, $\frac{2039}{192}$.
12. $\frac{8}{25}$, $\frac{1031}{50}$.
13. $\frac{2101}{20}$.
14. $\frac{31525}{729}$.
15. $103.83.
16. $\frac{3}{4}$ mi., 767$\frac{1}{4}$ mi.
17. The last term is $\frac{9}{8}$.
18. 17576.
19. $121.550625.

Art. 373. Page 299.

3. $7203.26.
4. $9733.22.
5. $78.74.
6. $132.42.
7. $24000.
8. $4800.
9. $992.

Art. 375. Page 301.

4. $1620; $1350.
5. $783.75; $690.53.
6. $7520; $5371.43.
7. $5607; $4441.19.
8. $408.
9. $620.
10. $272.
11. $642.

Art. 376. Page 302.

4. $630.50; $544.65.
5. $1312.38; $1039.53.
6. $2160.53; $1780.73.
7. $280.90.
8. $351.52.
9. $169.21.

Art. 377. Pages 303–317.

1. 4930095.
2. $9\frac{181}{253}$.
3. $\frac{166803}{917767}$.
4. $39.20.
5. $602.98.
6. $570.42.
7. 19 rd. 5 yd. $\frac{1}{2}$ ft.
8. $\frac{1}{8}$.
9. Hind-wheel, 5580; fore-wheel, 6600.
10. .853976.
11. $149.60.
12. 198 mi. 195 rd. 2 yd. 0 ft. 9 in.
13. 122° 27′ 17″ W.
14. $\frac{107}{103}$.
15. $3\frac{15}{17}$.
16. $\frac{11}{56}$.
17. 6876.
18. $1500.
19. 6 y. 3 mo. 12 d.
20. $.7\dot{3}142857\dot{7}$.
21. 11.312 in.; $208\frac{5}{9}$.
22. $4934\frac{12}{13}$.
23. 18 in.
24. $\frac{533}{61000}$.
25. $273.75.
26. $32\frac{1}{7}$%.
27. 18744264.
28. 1 sq. mi. 468 A. 96 sq. rd. 7 sq. yd. 8 sq. ft. 29 sq. in.
29. $253.50.
30. 6 h. 37 min. $30\frac{5}{8}$ sec. P.M.
31. $12\frac{37}{38}$.
32. $1466.9424.
33. $3151.82.
34. $1\frac{3}{8}$%.
35. 504245 in.
36. 5 h. $40\frac{36}{37}$ min.
37. $447.15; $31.80.
38. 2088.77 lks.
39. $1\frac{72}{247}$.
40. 4420 sq. in.; 18928 cu. in.
41. $21356.25.
42. $738.92.
43. 1008.
44. $\frac{13}{18}$.
45. 4032.
46. $11059.58.
48. .0875.
49. $54.81; $54.23.
50. $3\frac{21}{22}$.
51. $987.05; $12.35.
52. 523.
53. $5\frac{1}{2}$%.
54. $\frac{231}{325}$.
55. $34.79.
56. 2.375.
57. $2801.25.
58. $1\frac{369}{378}$.
59. 7.6985.
60. A, $248.82; B, $317.46; C, $197.34; D, $351.78.
61. 23.
62. 336 in.
63. 10 ft. 1 in.
64. 2 mi. 159 rd. 3 yd. $1\frac{1}{8}$ ft.
65. 304.
66. $4\frac{389}{399}$; A, $25.20; B, $21; C, $18; D, $15.75.

67. $1472.06.
68. 2 d. 8 h. 18 min. 45 sec.
69. $10.60.
70. 9 oz. 11 pwt. 14.7264 gr.
71. $85\frac{1}{2}$.
72. $9\frac{1}{15}$; $200\frac{1}{4}$.
73. $\frac{27}{139}$.
74. $68\frac{1}{32}$; $201\frac{137}{238}$.
75. 74 sq. mi. 380 A. 102 sq. rd. 14 sq. yd. 3 sq. ft. 61 sq. in.
76. $9135.75.
77. £4 9*s.* 7*d.* $\frac{4}{5}$ far.
78. A, $30.50; B, $38.71; C, $52.14.
79. 4 ft. $10\frac{1}{35}$ in.
80. .06444625.
81. $874.80.
82. $2906.14.
83. $10\frac{11}{12}$.
84. $3\frac{3}{4}$.
85. $949.62.
86. $1744.688015.
87. $\frac{12}{17}$ mi.
88. Wife, $3150; son, $2940; daught'r, $2866.50.
89. 826.
90. 70 d.
91. $\frac{53}{78}$, $\frac{187}{273}$, $\frac{21}{42}$.
92. 11 y. 9 mo. 21 d.
93. 12 d. 14 h. 0 min. 25 sec.
94. $20\frac{65}{105}$.
95. $17\frac{43}{72}$.
96. $53.90.
97. $9986\frac{77}{96}$ mi.
98. 2800733.
99. 19.584.
100. $118070.75.
101. $81\frac{50}{633}$.
102. 8010, 12460, 17355, 22428, 27590.
103. $5038.20; $3650.87.
104. .860855.
105. $6.44.
106. $1\frac{339}{1071}$.
107. 21 ft. 9.629 + in.
108. $470332\frac{1}{2}$ gr.
109. $1040.72.
110. $40\frac{3}{13}$.
111. £88 16*s.* 7*d.* 3 far.
112. May 20.
113. $547.18.
114. $27\frac{7}{9}$%.
115. $861.69.
116. 3 lb. 2 oz. 5.376 dr.
117. $\frac{2432}{861}$.
118. $67.31.
119. $863.95.
120. 2 ft. 8 in.
121. $\frac{7}{30}$.
122. $.03$\frac{1}{3}$.
123. $\frac{168}{12163}$.
124. $13.75 on $1000; $122.75.
125. 4.753 + in.
126. $2376.36.
127. $37\frac{1}{33}$.
128. 393 A. 135 sq. rd. 11 sq. yd. 5 sq. ft. $102\frac{6}{13}$ sq. in.
129. $389.91.
130. $209\frac{5}{11}$.
131. 16794.50112 cu. in.
132. .8796 +.
133. 605 ft.
134. .897216796875 lb.
135. 7 lb. 1 oz. 3 pwt. $6\frac{29}{32}$ gr.
136. $179.96.
137. $\frac{99}{200}$.
138. .0738 cd.
139. 26880.
140. $\frac{67184}{105}$.
141. $3\frac{7}{31}$%; $3\frac{1}{8}$%.
142. $25709.25.
143. $1\frac{113}{125}$ mi.; $170\frac{782}{1125}$ mi.
144. 175.76.
145. 7 mo. 14 d. after it becomes due.
146. $29\frac{1}{3}$%.
147. 19 T. 12 cwt. 91 lb. 5 oz. $12\frac{12}{23}$ dr.
148. 4 ft. 6.2 + in.
149. $1093.00.
150. $4\frac{27}{128}$.
151. 23.7.
152. 190.5904.
153. $19\frac{5}{8}$.
154. $21068.
155. A, $449.75; B, $428.75; C, $458.50.
156. $582.
157. $568.91.
158. $2\frac{1}{18}$ rd.
159. 8028979200 sq. in.
160. $\frac{59}{432}$.
161. $\frac{116}{77}$.
162. $8\frac{1}{2}$%.
163. $2155.06; $1772.98.
164. $2\frac{8}{39}$.
165. Gained $15.78.
166. .71875.
167. $1\frac{5}{8}$.
168. 287.35 + lb.

169. $335.05.
170. Diminished $179.25.
171. 1054.326528 lb.
172. $1\frac{1}{2}$.
173. 3 h. $19\frac{178}{379}$ min.
174. $13120\frac{2}{3}$.
175. $546.09.
176. 3128.4.
177. $14\frac{2}{3}$.
178. $1\frac{7}{125}$.
179. $\$41.94\frac{3}{4}$; $\$11.67\frac{1}{5}$.
180. $218.35.
181. March 7, 1890.

Art. 378.
Pages 317-320.

1. $.527^{\text{cu cm}}$; $.0527^{\text{Dg}}$.
2. $965.7^{\text{cu mm}}$.
3. $.3425^{\text{Dm}}$.
4. $49.83.
5. 9.6558^{Hm} a min.
6. 141.696^{Dg}.
7. 4.3443906^{Km}.
8. $156.77+.
9. 112.373125^{dl}.
10. 4.97096.
11. 2.01168.
12. 2.674224^{a}.
13. 230 gal. 3 qt. 1 pt. .30656 gi.; 24 bu. 3 pk. 1 qt. .90432 pt.
14. 1.491+c. per kilometer.
15. 226.0737; 2260.737.
16. 562500.
17. $95.496.
18. 4158.
19. $1542.03.
20. 2.6753^{T}.
21. $40.08; $39.42.
22. $.5809^{\text{Hm}}$.
23. $53.0079^{\text{cu dm}}$.
24. 96.19830528.
25. $35.01485526.
26. $.178^{\text{Dm}}$.
27. 964.5.
28. 18^{dm}.
29. 23 A. 28 sq. rd. 14 sq. yd. 3 sq. ft. 116.4672 sq. in.
30. 2.2763736.
31. 109.1674584^{Hg}.
32. 590.64552^{Dg}.
33. 5822.4075.
34. $9.079224^{\text{sq dm}}$.
35. $.1432+^{\text{m}}$.
36. $.175^{\text{Dm}}$.
37. $.21^{\text{m}}$; $.00494802^{\text{sq Dm}}$.
38. 1306^{mm}.
39. 201.0624.
40. 2.57.
41. 8 lb. 5 oz. 5.347328 dr.; 10 lb. 1 oz. 10 pwt. 12.96 gr.
42. 655.02+ lb.
43. 62566400.
44. 72.572+.
45. 3 h. 19 min.
46. $343308.42.
47. 156.0872544.

Appendix.
Page 324.

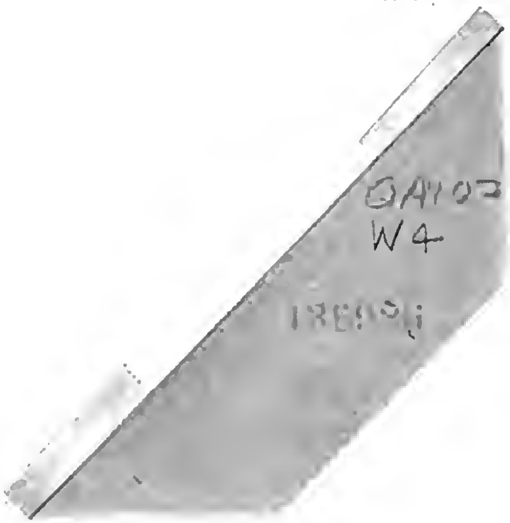
4. 25° C.; 20° R.
5. 60° C.; 48° R.
6. $-13\frac{1}{3}$° C.; $-10\frac{2}{3}$° R.
7. $-36\frac{2}{3}$° C.; $-29\frac{1}{3}$° R.
8. 131° F.; 44° R.
9. 158° F.; 56° R.
10. $10\frac{2}{5}$° F.; $-9\frac{3}{5}$° R.
11. −13° F.; −20° R.
12. 149° F.; 65° C.
13. $88\frac{1}{4}$° F.; $31\frac{1}{4}$° C.
14. $9\frac{1}{2}$° F.; $-12\frac{1}{2}$° C.
15. $-17\frac{1}{2}$° F.; $-27\frac{1}{2}$° C.

Page 332.

2. £7 14*s.* 2*d.* 2.5 far.; £63 19*s.* 2*d.* 2.5 far.
3. 18*s.* 2*d.* 1.072 far.; £32 12*s.* 8*d.* 1.072 far.
4. 15*s.* 11*d.* 3.55 far.; £28 4*s.* 2*d.* 3.55 far.
5. £2 0*s.* 11*d.* 2.05 far.; £43 0*s.* 1*d.* 3.05 far.
6. $6\frac{1}{4}$%.
7. 2 y. 1 mo. 15 d.
8. £14 13*s.* 9*d.*

Page 339.

7. 10010111.
8. 114144.
9. 1576*t*2.
10. 100120100112; 557663; 90*ete*.
11. 3230.
12. 1826.
13. 16046.
14. 51692.
15. 100000000000.
16. 547771.
17. 1433423.
18. 1032*e*.
19. 13122*t*5.
20. 8536.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